Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/26-1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [385]. This is test number [26].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (385)	0.00 (0)
Mathematica	99.48 (383)	0.52 (2)
Maple	66.23 (255)	33.77 (130)
Fricas	56.62 (218)	43.38 (167)
Mupad	44.16 (170)	55.84 (215)
Maxima	43.12 (166)	56.88 (219)
Sympy	37.40 (144)	62.60 (241)
Giac	33.77 (130)	66.23 (255)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

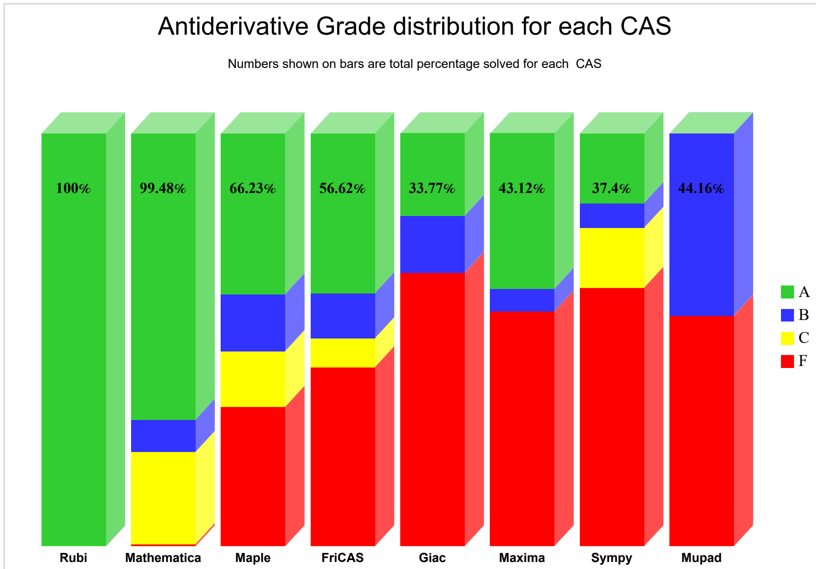
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

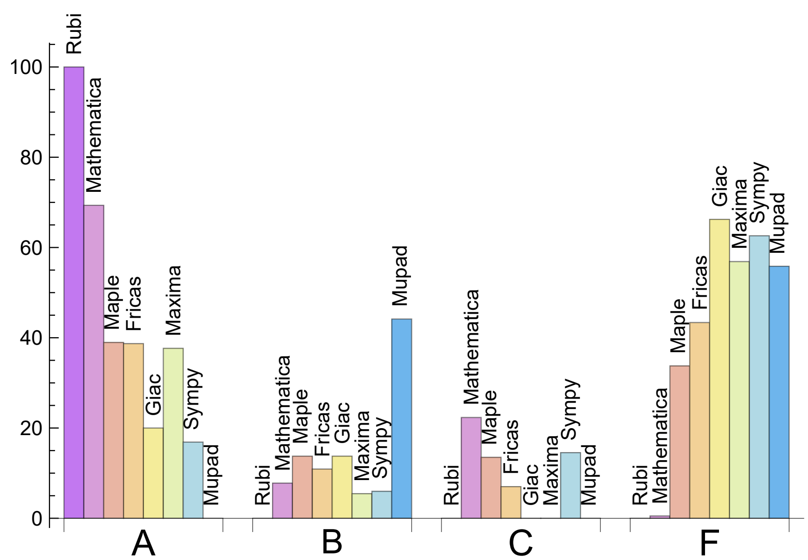
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	69.351	7.792	22.338	0.519
Maple	38.961	13.766	13.506	33.766
Fricas	38.701	10.909	7.013	43.377
Maxima	37.662	5.455	0.000	56.883
Giac	20.000	13.766	0.000	66.234
Sympy	16.883	5.974	14.545	62.597
Mupad	0.000	44.156	0.000	55.844

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	130	100.00	0.00	0.00
Fricas	167	56.29	41.92	1.80
Mupad	215	0.00	100.00	0.00
Maxima	219	99.54	0.46	0.00
Sympy	241	55.60	36.93	7.47
Giac	255	88.63	0.00	11.37

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.11
Maxima	0.28
Giac	0.31
Fricas	2.21
Mathematica	3.22
Maple	3.74
Mupad	8.12
Sympy	11.60

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	170.35	1.00	122.00	1.00
Maxima	185.84	1.28	143.00	1.14
Mathematica	204.42	1.44	140.00	0.97
Maple	217.27	1.30	155.00	1.08
Giac	294.09	1.84	211.00	1.40
Sympy	381.10	3.33	149.50	1.19
Fricas	557.84	2.80	239.50	2.11
Mupad	1166.19	4.53	148.50	1.22

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

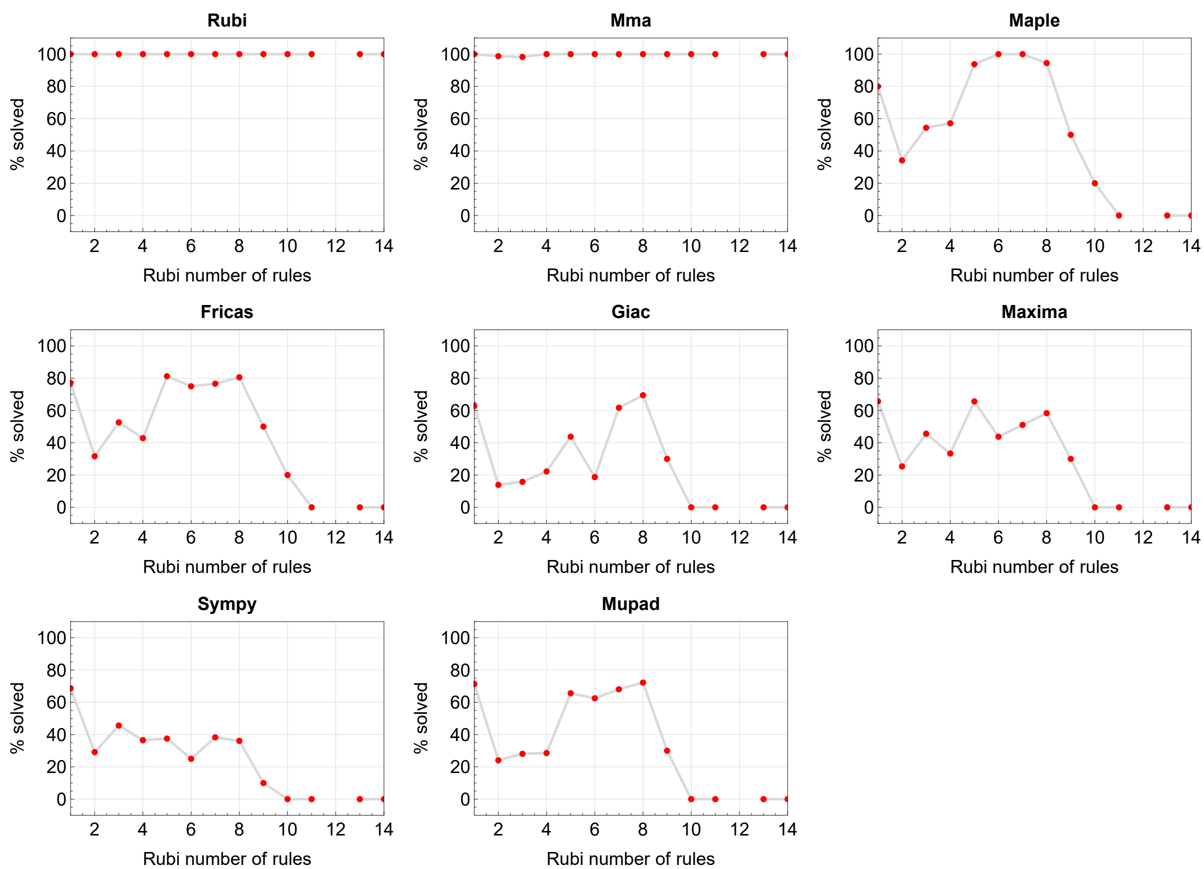


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

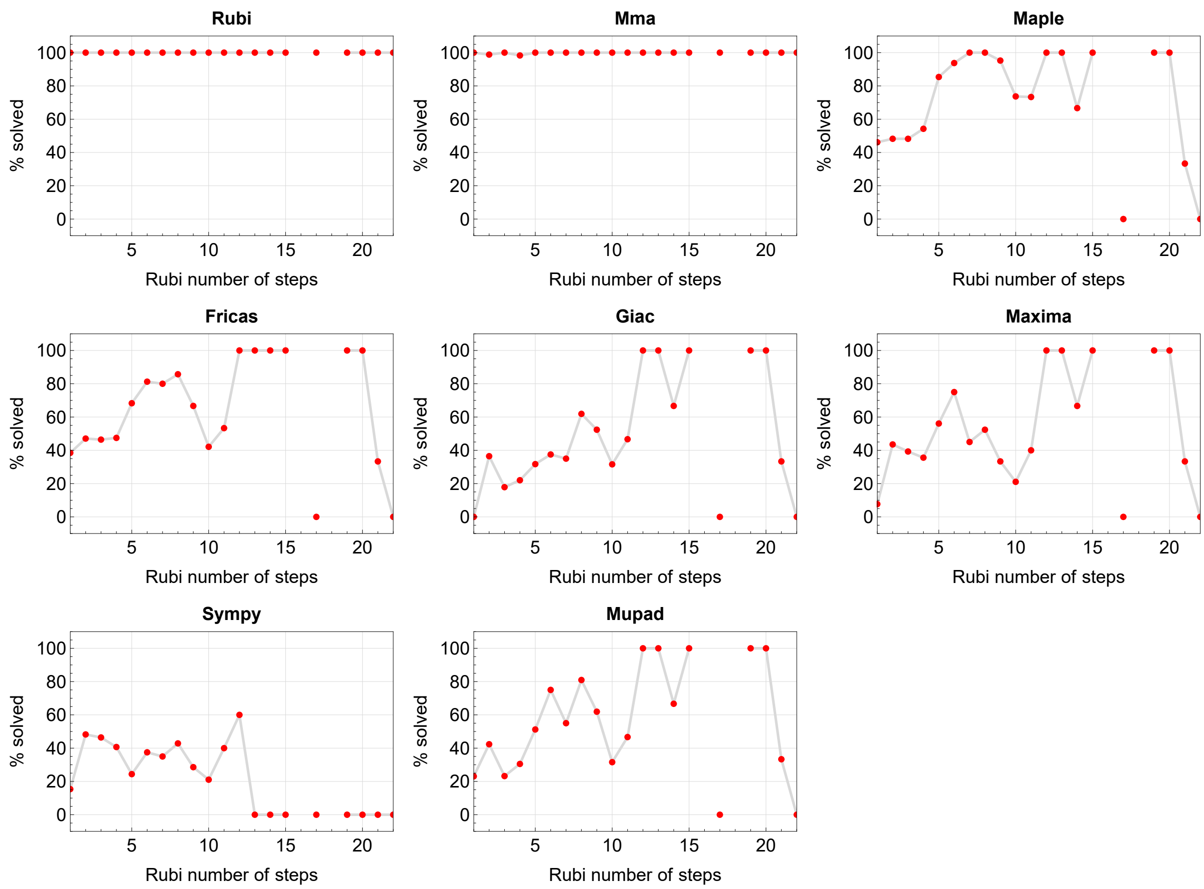


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

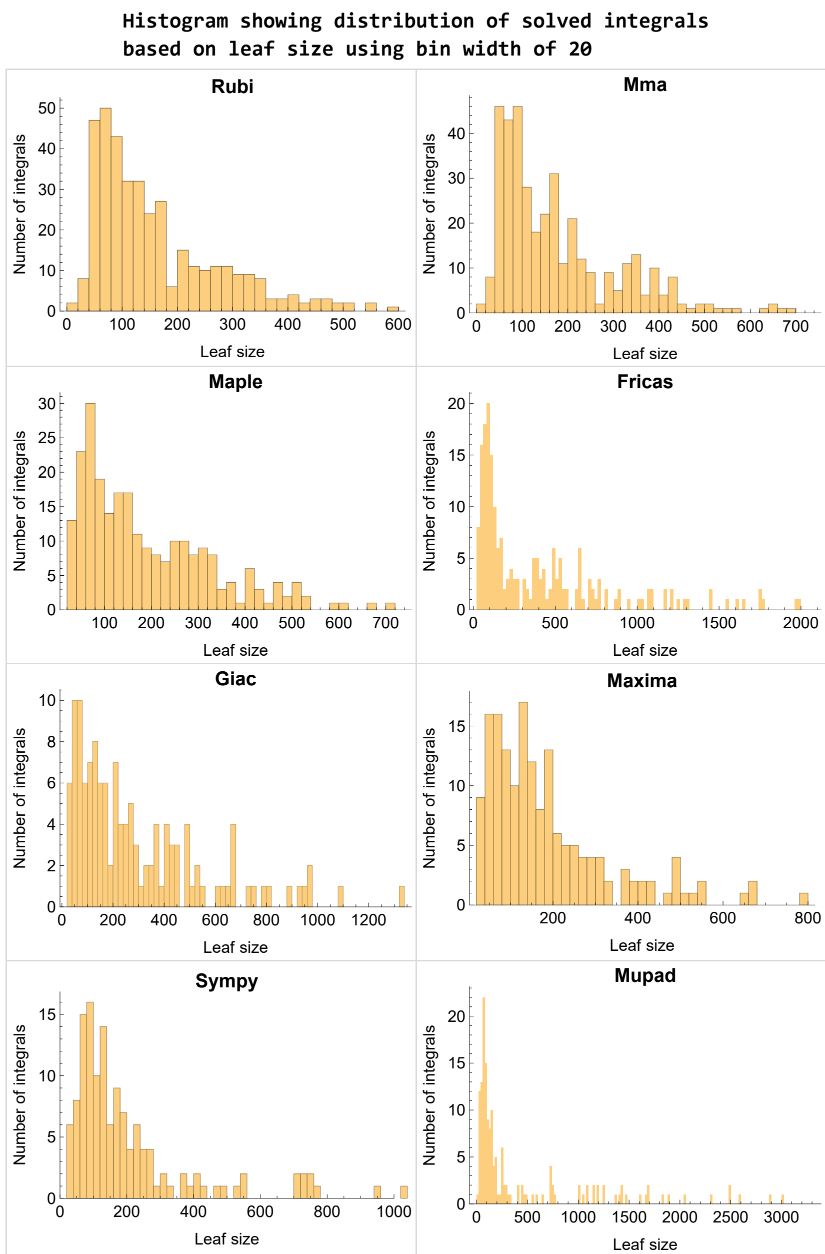


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

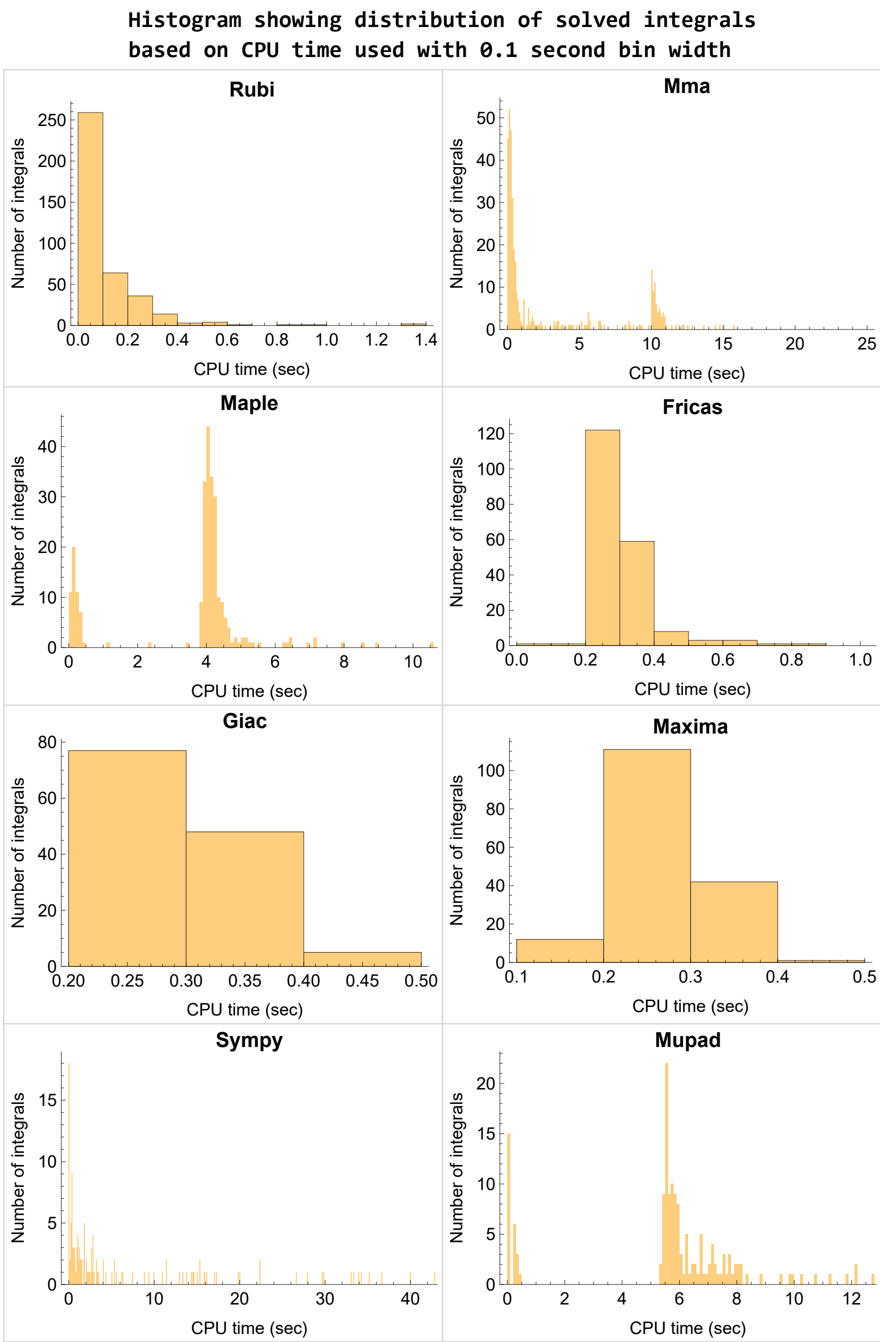


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

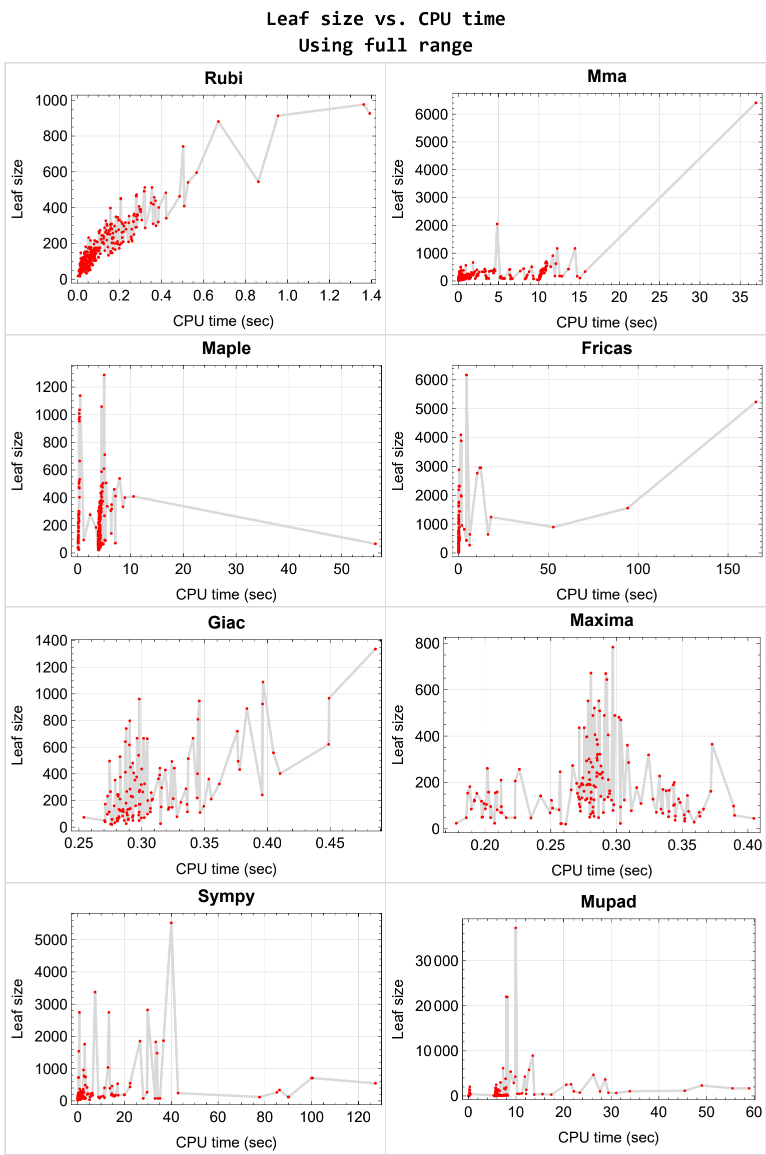


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {34, 35, 37, 38, 39, 79, 80, 83, 86, 87, 93, 94, 95, 96, 97, 98, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 199, 200, 201, 202, 203, 204, 211, 212, 213, 214, 215, 218, 221, 222, 269, 306, 307, 312, 317, 318, 319, 320, 336, 348, 350, 352, 355, 357, 384}

Maple {173, 174, 175, 176, 179, 180, 181, 184, 185, 186, 187}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

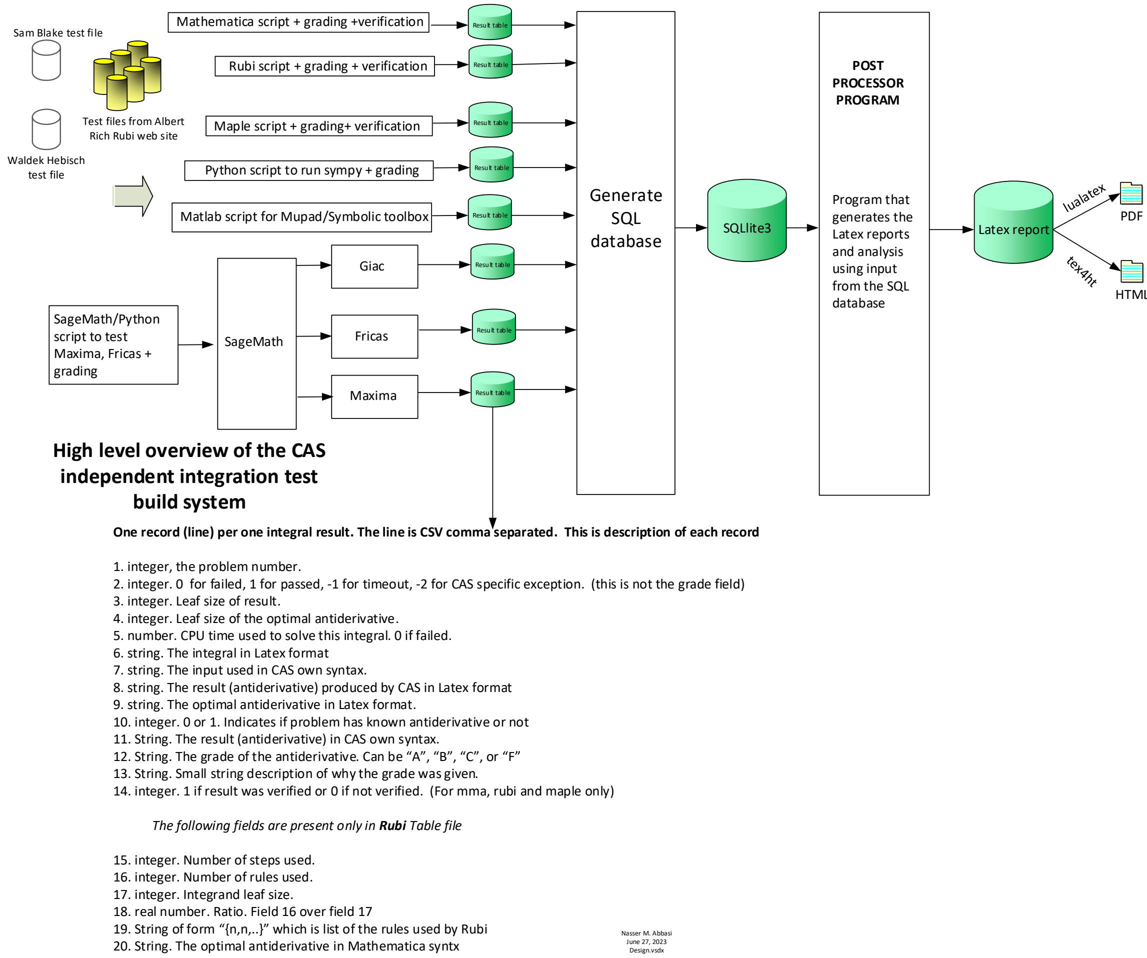
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 383, 385 }

B grade { 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142, 143, 144, 218, 221, 222, 269, 312, 317, 318, 319, 384 }

C grade { 34, 35, 37, 38, 39, 86, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 271, 273, 298, 299, 300, 306, 307, 323, 330, 331, 332, 333, 334, 335, 336, 379, 380 }

F normal fail { 328, 382 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 18, 19, 25, 26, 27, 28, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 56, 57, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 116, 145, 146, 147, 148, 149, 153, 154, 155, 156, 164, 165, 171, 172, 191, 192, 198, 210, 216, 217, 223, 224, 225, 226, 231, 232, 233, 234, 238, 239, 240, 241, 245, 246, 247, 248, 252, 270, 271, 272, 273, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 370, 371, 374, 376, 377, 378, 379 }

B grade { 29, 58, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 227, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 321, 322, 323, 324, 329, 330, 331, 340, 352, 363, 373, 375 }

C grade { 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 275, 354, 364, 372 }

F normal fail { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 199, 200, 201, 202, 203, 204, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 272, 273, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade { 7, 12, 13, 20, 21, 22, 23, 26, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 109, 110, 111, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade { 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192, 193, 194, 205, 206, 216, 380 }

F normal fail { 35, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 218, 219, 220, 221, 222, 269, 271, 274, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

F(-1) timedout fail { 34, 37, 38, 39, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217 }

F(-2) exception fail { 223, 281, 282 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 44, 45, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 85, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 241, 245, 248, 252, 253, 255, 259, 260, 261, 262, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379 }

B grade { 26, 27, 28, 32, 33, 40, 41, 42, 43, 46, 47, 56, 57, 58, 70, 71, 72, 73, 246, 247, 254 }

C grade { }

F normal fail { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { 240 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 168, 169, 170, 171, 227, 248, 270, 275, 276, 277, 278, 279, 280, 340, 341, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 370, 378 }

B grade { 160, 161, 166, 167, 172, 229, 230, 236, 237, 243, 244, 247, 250, 251, 253, 254, 255, 258, 259, 260, 261, 262, 264, 265, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 342, 343, 344, 345, 347, 355, 356, 357, 365, 366, 367, 369, 371, 372, 373, 374, 375, 376, 377 }

C grade { }

F normal fail { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 332, 333, 334, 335, 336, 379, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { }

F(-2) exception fail { 224, 225, 226, 228, 231, 232, 233, 234, 235, 238, 239, 240, 241, 242, 245, 246, 249, 252, 256, 257, 263, 313, 314, 315, 321, 322, 323, 330, 331 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 316, 324, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 374, 376, 378, 379 }

C grade { }

F normal fail { }

F(-1) timedout fail { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 368, 370, 372, 373, 375, 377, 380, 381, 382, 383, 384, 385 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280 }

B grade { 30, 31, 60, 61, 234, 254, 261, 262, 270, 284, 285, 286, 287, 292, 293, 294, 321, 322, 323, 324, 330, 331, 332 }

C grade { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 134, 135, 139, 140, 141, 219, 220, 288, 289, 295, 298, 299, 300, 301, 308, 313, 314, 315, 316, 349, 351, 353, 354, 356, 359, 361, 363, 364, 366, 378, 379 }

F normal fail { 34, 35, 36, 37, 38, 39, 42, 43, 44, 51, 52, 73, 74, 75, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 113, 118, 125, 126, 127, 128, 129, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 223, 228, 229, 230, 235, 236, 242, 243, 249, 250, 252, 253, 256, 257, 259, 260, 263, 264, 266, 267, 268, 269, 271, 272, 273, 281, 282, 283, 296, 302, 305, 306, 307, 311, 319, 329, 337, 338, 339, 340, 341, 343, 344, 345, 346, 380, 381 }

F(-1) timedout fail { 18, 19, 25, 26, 32, 33, 45, 46, 47, 53, 54, 55, 62, 63, 76, 77, 78, 85, 98, 109, 110, 111, 112, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 130, 131, 132, 133, 136, 137, 138, 142, 143, 144, 145, 164, 165, 166, 167, 171, 172, 184, 189, 205, 218, 221, 222, 237, 244, 251, 258, 265, 274, 290, 291, 297, 342, 348, 350, 352, 355, 357, 358, 360, 362, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 385 }

F(-2) exception fail { 303, 304, 309, 310, 312, 317, 318, 320, 325, 326, 327, 328, 333, 334, 335, 336, 347, 384 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	94	96	96	104	97	87
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.03	0.93
time (sec)	N/A	0.048	0.024	5.121	0.212	0.271	0.025	0.273	0.055

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	80	74	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.14	1.06	0.94
time (sec)	N/A	0.031	0.019	3.997	0.213	0.278	0.022	0.254	0.035

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.020	0.011	3.866	0.223	0.275	0.021	0.270	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.010	0.008	0.231	0.178	0.292	0.018	0.288	0.037

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	128	42	128	369	71	133	123
N.S.	1	1.00	0.89	0.29	0.89	2.56	0.49	0.92	0.85
time (sec)	N/A	0.073	0.082	4.047	0.281	0.292	0.209	0.321	0.267

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	65	158	537	97	160	143
N.S.	1	1.00	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.061	0.092	3.851	0.281	0.329	0.309	0.278	5.529

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	175	84	192	743	133	180	173
N.S.	1	1.00	0.89	0.43	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.078	0.131	3.854	0.279	0.321	0.398	0.287	5.537

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.052	0.022	3.915	0.192	0.285	0.030	0.282	5.334

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	90	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.10	1.11	0.91
time (sec)	N/A	0.034	0.014	3.916	0.209	0.282	0.025	0.289	0.045

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.020	0.009	3.872	0.186	0.276	0.020	0.281	0.048

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	189	505	156	211	152
N.S.	1	1.00	0.97	0.45	1.09	2.92	0.90	1.22	0.88
time (sec)	N/A	0.088	0.096	3.899	0.277	0.299	0.342	0.283	5.547

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	210	99	226	771	189	233	191
N.S.	1	1.00	1.03	0.49	1.11	3.80	0.93	1.15	0.94
time (sec)	N/A	0.163	0.212	3.911	0.285	0.336	0.653	0.273	5.593

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	217	131	267	1067	233	264	249
N.S.	1	1.00	0.84	0.51	1.03	4.14	0.90	1.02	0.97
time (sec)	N/A	0.168	0.272	3.890	0.282	0.323	1.018	0.301	5.591

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	253	201	364	873	371	391	250
N.S.	1	1.00	1.00	0.80	1.44	3.46	1.47	1.55	0.99
time (sec)	N/A	0.154	0.133	4.149	0.286	0.322	0.870	0.314	5.596

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	203	131	273	700	257	296	192
N.S.	1	1.00	0.98	0.63	1.31	3.37	1.24	1.42	0.92
time (sec)	N/A	0.121	0.096	3.923	0.267	0.316	0.495	0.316	5.592

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	190	507	156	211	152
N.S.	1	1.00	0.97	0.45	1.10	2.93	0.90	1.22	0.88
time (sec)	N/A	0.083	0.110	3.906	0.273	0.316	0.335	0.309	0.245

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	42	128	390	71	133	123
N.S.	1	1.00	0.89	0.29	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.064	0.065	4.166	0.272	0.319	0.211	0.285	5.533

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	293	254	0	278	1364
N.S.	1	1.00	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.129	0.121	4.026	0.277	0.351	0.000	0.298	12.176

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	336	246	489	432	0	443	2589
N.S.	1	1.00	0.97	0.71	1.41	1.25	0.00	1.28	7.48
time (sec)	N/A	0.191	0.217	4.037	0.299	4.481	0.000	0.314	21.570

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	313	304	509	1619	546	529	416
N.S.	1	1.00	0.98	0.95	1.59	5.06	1.71	1.65	1.30
time (sec)	N/A	0.209	0.259	3.925	0.287	0.318	127.217	0.283	0.443

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	260	218	397	1316	405	412	302
N.S.	1	1.00	0.97	0.82	1.49	4.93	1.52	1.54	1.13
time (sec)	N/A	0.184	0.225	3.930	0.277	0.312	11.467	0.287	5.744

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	227	153	306	1027	291	319	240
N.S.	1	1.00	0.97	0.65	1.31	4.39	1.24	1.36	1.03
time (sec)	N/A	0.171	0.164	3.899	0.285	0.291	1.307	0.300	0.308

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	205	101	220	768	189	227	191
N.S.	1	1.00	1.01	0.50	1.08	3.78	0.93	1.12	0.94
time (sec)	N/A	0.169	0.216	3.907	0.274	0.303	0.601	0.293	5.664

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	65	158	537	97	160	143
N.S.	1	1.00	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.062	0.098	3.878	0.281	0.296	0.316	0.295	5.536

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	337	247	489	440	0	443	2492
N.S.	1	1.00	0.97	0.71	1.41	1.27	0.00	1.28	7.20
time (sec)	N/A	0.190	0.233	4.025	0.291	4.460	0.000	0.326	20.561

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	381	285	784	897	0	664	3637
N.S.	1	1.00	0.91	0.68	1.87	2.14	0.00	1.58	8.68
time (sec)	N/A	0.361	0.632	4.068	0.297	52.873	0.000	0.304	28.736

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	156	144	322	399	80	0	0
N.S.	1	1.00	1.39	1.29	2.88	3.56	0.71	0.00	0.00
time (sec)	N/A	0.023	0.518	4.067	0.286	0.350	1.874	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	140	118	244	363	76	0	0
N.S.	1	1.00	1.54	1.30	2.68	3.99	0.84	0.00	0.00
time (sec)	N/A	0.014	0.443	4.179	0.279	0.300	1.207	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	142	142	130	372	70	0	0
N.S.	1	1.00	1.67	1.67	1.53	4.38	0.82	0.00	0.00
time (sec)	N/A	0.010	0.433	3.931	0.278	0.280	2.808	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	50	44	190	0	27
N.S.	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57
time (sec)	N/A	0.007	0.306	4.057	0.198	0.287	19.872	0.000	5.492

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	37	85	69	709	0	44
N.S.	1	1.00	0.73	0.67	1.55	1.25	12.89	0.00	0.80
time (sec)	N/A	0.010	0.379	4.077	0.190	0.274	100.311	0.000	5.453

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	51	48	119	91	0	0	58
N.S.	1	1.00	0.69	0.65	1.61	1.23	0.00	0.00	0.78
time (sec)	N/A	0.013	0.515	3.935	0.197	0.289	0.000	0.000	5.494

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	62	59	153	113	0	0	73
N.S.	1	1.00	0.67	0.63	1.65	1.22	0.00	0.00	0.78
time (sec)	N/A	0.018	0.723	3.909	0.194	0.298	0.000	0.000	5.502

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	483	483	232	0	0	0	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	10.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	464	464	217	0	0	0	0	0	0
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	10.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	398	398	428	0	0	644	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.156	3.288	0.000	0.000	16.545	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	452	452	153	0	0	0	0	0	0
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	473	473	213	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	10.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	492	492	240	0	0	0	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	10.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	167	164	552	421	126	0	0
N.S.	1	1.00	1.20	1.18	3.97	3.03	0.91	0.00	0.00
time (sec)	N/A	0.037	0.763	4.129	0.287	0.443	5.341	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	165	144	436	399	121	0	0
N.S.	1	1.00	1.38	1.20	3.63	3.32	1.01	0.00	0.00
time (sec)	N/A	0.027	0.618	4.124	0.272	0.358	2.823	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	154	156	296	412	0	0	0
N.S.	1	1.00	1.36	1.38	2.62	3.65	0.00	0.00	0.00
time (sec)	N/A	0.027	0.678	4.200	0.285	0.356	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	137	128	180	521	0	0	0
N.S.	1	1.00	1.25	1.16	1.64	4.74	0.00	0.00	0.00
time (sec)	N/A	0.027	0.549	4.377	0.284	0.348	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	40	37	105	67	0	0	44
N.S.	1	1.00	0.53	0.49	1.38	0.88	0.00	0.00	0.58
time (sec)	N/A	0.015	0.456	4.126	0.201	0.443	0.000	0.000	5.484

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	51	48	155	91	0	0	56
N.S.	1	1.00	0.49	0.46	1.48	0.87	0.00	0.00	0.53
time (sec)	N/A	0.023	0.658	4.062	0.208	0.353	0.000	0.000	5.509

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	59	206	113	0	0	71
N.S.	1	1.00	0.63	0.60	2.10	1.15	0.00	0.00	0.72
time (sec)	N/A	0.023	0.883	3.993	0.223	0.344	0.000	0.000	5.518

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	70	257	135	0	0	86
N.S.	1	1.00	0.62	0.60	2.20	1.15	0.00	0.00	0.74
time (sec)	N/A	0.029	1.345	4.002	0.226	0.364	0.000	0.000	5.500

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	97	0	0	0	168	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.79	0.00	0.00
time (sec)	N/A	0.024	8.400	0.000	0.000	0.000	2.098	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	126	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	1.34	0.00	0.00
time (sec)	N/A	0.023	6.710	0.000	0.000	0.000	1.456	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	121	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.29	0.00	0.00
time (sec)	N/A	0.024	10.047	0.000	0.000	0.000	1.565	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	10.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	95	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	10.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	106	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	10.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	208	180	406	482	170	0	0
N.S.	1	1.00	1.20	1.03	2.33	2.77	0.98	0.00	0.00
time (sec)	N/A	0.052	1.003	4.123	0.284	0.306	6.364	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	180	151	322	424	82	0	0
N.S.	1	1.00	1.28	1.07	2.28	3.01	0.58	0.00	0.00
time (sec)	N/A	0.032	0.828	4.081	0.287	0.314	1.825	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	163	225	244	362	78	0	0
N.S.	1	1.00	1.47	2.03	2.20	3.26	0.70	0.00	0.00
time (sec)	N/A	0.022	0.622	4.048	0.289	0.286	1.218	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	150	161	134	488	71	0	0
N.S.	1	1.00	1.52	1.63	1.35	4.93	0.72	0.00	0.00
time (sec)	N/A	0.016	0.529	3.985	0.274	0.309	2.835	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	51	54	190	0	33
N.S.	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70
time (sec)	N/A	0.007	0.374	3.904	0.198	0.267	20.029	0.000	5.459

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	52	86	87	709	0	87
N.S.	1	1.00	0.66	0.57	0.95	0.96	7.79	0.00	0.96
time (sec)	N/A	0.019	0.525	3.931	0.200	0.323	100.012	0.000	5.495

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	80	71	120	121	0	0	105
N.S.	1	1.00	0.66	0.59	0.99	1.00	0.00	0.00	0.87
time (sec)	N/A	0.025	0.755	3.930	0.192	0.301	0.000	0.000	5.514

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	100	90	154	155	0	0	132
N.S.	1	1.00	0.66	0.60	1.02	1.03	0.00	0.00	0.87
time (sec)	N/A	0.032	1.126	4.091	0.187	0.364	0.000	0.000	5.545

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	0.00
time (sec)	N/A	0.017	9.342	0.000	0.000	0.000	2.666	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	0.00
time (sec)	N/A	0.016	8.283	0.000	0.000	0.000	1.816	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.016	6.488	0.000	0.000	0.000	1.040	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.016	10.044	0.000	0.000	0.000	0.916	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.84	0.00	0.00
time (sec)	N/A	0.018	10.042	0.000	0.000	0.000	3.419	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	78	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.83	0.00	0.00
time (sec)	N/A	0.019	10.047	0.000	0.000	0.000	35.161	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	293	244	672	717	270	0	0
N.S.	1	1.00	1.12	0.93	2.56	2.74	1.03	0.00	0.00
time (sec)	N/A	0.132	1.615	4.262	0.281	0.300	29.662	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	256	203	552	634	131	0	0
N.S.	1	1.00	1.17	0.93	2.52	2.89	0.60	0.00	0.00
time (sec)	N/A	0.139	1.125	4.219	0.278	0.328	5.304	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	223	166	436	554	126	0	0
N.S.	1	1.00	1.27	0.95	2.49	3.17	0.72	0.00	0.00
time (sec)	N/A	0.083	0.994	4.194	0.275	0.329	2.814	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	199	170	301	652	0	0	0
N.S.	1	1.00	1.25	1.07	1.89	4.10	0.00	0.00	0.00
time (sec)	N/A	0.073	1.191	4.276	0.279	0.325	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	186	176	190	719	0	0	0
N.S.	1	1.00	1.22	1.16	1.25	4.73	0.00	0.00	0.00
time (sec)	N/A	0.047	1.101	4.129	0.274	0.401	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	71	109	103	0	0	148
N.S.	1	1.00	0.94	0.91	1.40	1.32	0.00	0.00	1.90
time (sec)	N/A	0.015	0.853	4.082	0.199	0.296	0.000	0.000	5.577

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	106	96	159	152	0	0	176
N.S.	1	1.00	0.61	0.55	0.91	0.87	0.00	0.00	1.01
time (sec)	N/A	0.052	1.140	4.204	0.209	0.363	0.000	0.000	5.575

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	138	126	210	200	0	0	217
N.S.	1	1.00	0.65	0.60	1.00	0.95	0.00	0.00	1.03
time (sec)	N/A	0.090	1.561	4.064	0.212	0.307	0.000	0.000	5.580

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	169	156	261	246	0	0	257
N.S.	1	1.00	0.67	0.62	1.03	0.97	0.00	0.00	1.02
time (sec)	N/A	0.149	2.260	4.213	0.202	0.342	0.000	0.000	5.595

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	177	0	0	0	418	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	3.10	0.00	0.00
time (sec)	N/A	0.048	12.595	0.000	0.000	0.000	4.007	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	176	0	0	0	270	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	2.03	0.00	0.00
time (sec)	N/A	0.060	11.999	0.000	0.000	0.000	2.604	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	179	0	0	0	131	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.046	9.279	0.000	0.000	0.000	1.465	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	104	0	0	0	126	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.058	15.098	0.000	0.000	0.000	1.518	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	12.873	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	171	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	14.757	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	120	110	182	166	0	0	271
N.S.	1	1.00	1.10	1.01	1.67	1.52	0.00	0.00	2.49
time (sec)	N/A	0.031	1.403	4.119	0.189	0.292	0.000	0.000	5.859

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	655	461	0	643	0	0	0
N.S.	1	1.00	1.98	1.39	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.306	10.833	6.922	0.000	6.475	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	443	402	0	535	0	0	0
N.S.	1	1.00	1.62	1.47	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.152	10.506	4.573	0.000	0.765	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	423	339	0	469	0	0	0
N.S.	1	1.00	1.82	1.45	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.052	4.463	4.159	0.000	0.303	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	255	168	0	0	0	0	0
N.S.	1	1.00	1.72	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	1.909	4.216	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	328	243	0	0	0	0	0
N.S.	1	1.00	1.83	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	3.232	4.091	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	364	269	0	0	0	0	0
N.S.	1	1.00	1.61	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	4.085	5.065	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	420	321	0	0	0	0	0
N.S.	1	1.00	1.50	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	6.425	4.423	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	10.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	0
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	10.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	0
N.S.	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	10.776	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	698	496	0	819	0	0	0
N.S.	1	1.00	1.99	1.41	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	0.279	10.938	4.674	0.000	3.351	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	510	435	0	631	0	0	0
N.S.	1	1.00	1.69	1.45	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.141	9.120	4.355	0.000	0.516	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	319	229	0	0	0	0	0
N.S.	1	1.00	1.75	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	2.605	4.162	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	336	261	0	0	0	0	0
N.S.	1	1.00	1.55	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	2.987	4.193	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	370	303	0	0	0	0	0
N.S.	1	1.00	1.42	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	4.585	4.461	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	443	371	0	0	0	0	0
N.S.	1	1.00	1.37	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	8.108	4.551	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	0
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	10.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	0
N.S.	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	0
N.S.	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	10.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	0
N.S.	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	10.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	0
N.S.	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.976	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	541	541	1171	711	0	1555	0	0	0
N.S.	1	1.00	2.16	1.31	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.526	12.279	5.124	0.000	94.183	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	908	609	0	1246	0	0	0
N.S.	1	1.00	1.98	1.33	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.365	11.742	4.972	0.000	18.185	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	651	505	0	954	0	0	0
N.S.	1	1.00	1.66	1.29	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.296	11.002	4.803	0.000	1.954	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	344	259	0	0	0	0	0
N.S.	1	1.00	1.59	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	3.845	4.321	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	366	297	0	0	0	0	0
N.S.	1	1.00	1.37	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	4.298	4.320	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	407	307	0	0	0	0	0
N.S.	1	1.00	1.33	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	6.353	4.374	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	428	426	0	0	0	0	0
N.S.	1	1.00	1.14	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	13.679	4.462	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	337	475	0	0	0	0	0
N.S.	1	1.00	0.73	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	15.745	4.687	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	285	0	0	0	0	0	0
N.S.	1	1.00	4.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	10.467	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	407	0	0	0	0	0	0
N.S.	1	1.00	6.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.624	0.000	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	418	0	0	0	0	0	0
N.S.	1	1.00	7.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.679	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	0
N.S.	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.918	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	0
N.S.	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	11.481	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	5.738	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	5.460	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	5.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	3.783	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	3.504	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	5.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	3.489	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	5.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	5.681	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	5.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	5.720	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	121	0	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.72	0.00	0.00
time (sec)	N/A	0.090	0.210	0.000	0.000	0.000	90.092	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	75	0	0
N.S.	1	1.01	1.07	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.027	0.143	0.000	0.000	0.000	34.200	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.442	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	8.446	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	121	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.69	0.00	0.00
time (sec)	N/A	0.101	5.186	0.000	0.000	0.000	89.965	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.035	0.140	0.000	0.000	0.000	33.081	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	203	0	0	0	34	0	41
N.S.	1	1.00	4.61	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.009	0.179	0.000	0.000	0.000	5.070	0.000	5.426

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.014	0.562	7.138	0.000	0.303	0.000	0.000	5.925

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.063	0.024	3.955	0.204	0.266	0.033	0.305	5.409

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	76	74	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.09	1.06	0.94
time (sec)	N/A	0.040	0.017	3.905	0.212	0.246	0.025	0.293	5.394

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.023	0.013	4.159	0.204	0.257	0.022	0.293	0.050

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.011	0.008	0.178	0.207	0.282	0.024	0.315	0.038

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	196	42	212	560	87	245	720
N.S.	1	1.00	0.88	0.19	0.95	2.51	0.39	1.10	3.23
time (sec)	N/A	0.112	0.133	4.324	0.294	0.283	0.284	0.281	0.235

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	212	65	236	648	112	266	740
N.S.	1	1.00	0.87	0.27	0.96	2.64	0.46	1.09	3.02
time (sec)	N/A	0.119	0.168	4.087	0.288	0.280	0.399	0.275	5.688

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	243	84	271	739	151	286	762
N.S.	1	1.00	0.89	0.31	0.99	2.71	0.55	1.05	2.79
time (sec)	N/A	0.137	0.208	3.934	0.288	0.301	0.522	0.291	5.934

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	160	158	158	185	173	146
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.20	1.12	0.95
time (sec)	N/A	0.083	0.034	3.949	0.203	0.275	0.031	0.271	5.647

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	0.95
time (sec)	N/A	0.053	0.024	3.876	0.276	0.260	0.036	0.289	5.590

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.035	0.019	4.004	0.256	0.350	0.028	0.298	0.046

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.029	0.010	3.949	0.216	0.270	0.026	0.286	0.044

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	78	286	1092	187	353	1081
N.S.	1	1.00	0.91	0.31	1.13	4.32	0.74	1.40	4.27
time (sec)	N/A	0.142	0.106	3.986	0.309	0.307	0.584	0.295	5.840

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	298	101	319	1210	219	376	1254
N.S.	1	1.00	1.02	0.35	1.10	4.16	0.75	1.29	4.31
time (sec)	N/A	0.281	0.175	3.965	0.324	0.283	2.016	0.296	5.950

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	319	131	361	1294	264	407	1401
N.S.	1	1.00	0.91	0.38	1.03	3.71	0.76	1.17	4.01
time (sec)	N/A	0.183	0.212	4.014	0.308	0.306	85.141	0.294	6.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	322	201	489	2190	435	617	1822
N.S.	1	1.00	0.97	0.61	1.47	6.60	1.31	1.86	5.49
time (sec)	N/A	0.199	0.210	4.205	0.282	0.295	22.351	0.290	5.745

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	271	131	385	1642	303	481	1433
N.S.	1	1.00	0.94	0.45	1.34	5.70	1.05	1.67	4.98
time (sec)	N/A	0.159	0.163	3.980	0.285	0.299	1.296	0.293	0.236

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	78	287	1093	187	353	1081
N.S.	1	1.00	0.91	0.31	1.13	4.32	0.74	1.40	4.27
time (sec)	N/A	0.148	0.103	4.109	0.281	0.287	0.530	0.279	5.706

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	196	42	212	560	87	245	720
N.S.	1	1.00	0.88	0.19	0.95	2.51	0.39	1.10	3.23
time (sec)	N/A	0.112	0.115	3.941	0.280	0.290	0.309	0.281	0.223

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	365	1171	0	437	6153
N.S.	1	1.00	0.76	0.50	0.81	2.61	0.00	0.97	13.70
time (sec)	N/A	0.206	0.126	4.086	0.373	0.392	0.000	0.300	7.296

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	498	262	481	2955	0	667	21975
N.S.	1	1.00	0.97	0.51	0.94	5.76	0.00	1.30	42.84
time (sec)	N/A	0.354	0.307	4.097	0.302	12.016	0.000	0.296	8.177

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	391	304	644	2884	0	798	2490
N.S.	1	1.00	0.96	0.75	1.58	7.09	0.00	1.96	6.12
time (sec)	N/A	0.293	0.366	4.025	0.293	0.401	0.000	0.290	5.856

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	341	219	521	2315	0	642	2043
N.S.	1	1.00	0.96	0.61	1.46	6.48	0.00	1.80	5.72
time (sec)	N/A	0.267	0.306	3.987	0.283	0.331	0.000	0.287	0.318

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	301	151	405	1741	337	496	1616
N.S.	1	1.00	0.95	0.48	1.28	5.49	1.06	1.56	5.10
time (sec)	N/A	0.228	0.243	4.011	0.294	0.323	86.251	0.274	5.719

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	297	101	319	1210	219	376	1254
N.S.	1	1.00	1.02	0.35	1.10	4.16	0.75	1.29	4.31
time (sec)	N/A	0.277	0.178	4.264	0.285	0.319	1.147	0.283	0.304

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	212	65	236	648	112	266	740
N.S.	1	1.00	0.87	0.27	0.96	2.64	0.46	1.09	3.02
time (sec)	N/A	0.115	0.167	4.001	0.287	0.293	0.457	0.298	5.701

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	499	263	470	2955	0	667	21975
N.S.	1	1.00	0.97	0.51	0.92	5.76	0.00	1.30	42.84
time (sec)	N/A	0.320	0.309	4.121	0.303	12.457	0.000	0.302	7.956

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	561	298	670	5234	0	967	37266
N.S.	1	1.00	0.94	0.50	1.12	8.78	0.00	1.62	62.53
time (sec)	N/A	0.567	0.945	4.195	0.292	165.512	0.000	0.449	9.949

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	321	321	290	350	0	0	0	0	0
N.S.	1	1.00	0.90	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	10.698	6.409	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	277	277	341	306	0	0	0	0	0
N.S.	1	1.00	1.23	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	10.352	6.264	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0	0
N.S.	1	1.00	0.65	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	10.183	4.139	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	0
N.S.	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	10.224	4.233	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	381	301	0	0	0	0	0
N.S.	1	1.00	1.36	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	10.263	4.152	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	422	361	0	0	0	0	0
N.S.	1	1.00	1.26	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	10.716	4.184	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	926	926	346	319	0	0	0	0	0
N.S.	1	1.00	0.37	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.392	10.473	6.444	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	881	881	161	273	0	0	0	0	0
N.S.	1	1.00	0.18	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	10.186	4.142	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	742	742	161	191	0	0	0	0	0
N.S.	1	1.00	0.22	0.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	10.057	4.165	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	913	913	331	313	0	0	0	0	0
N.S.	1	1.00	0.36	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.955	10.278	4.184	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	976	976	429	371	0	0	0	0	0
N.S.	1	1.00	0.44	0.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.363	10.818	4.164	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	426	426	477	539	0	0	0	0	0
N.S.	1	1.00	1.12	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	10.818	7.936	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	365	365	396	411	0	0	0	0	0
N.S.	1	1.00	1.08	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	10.584	7.161	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	309	309	342	328	0	0	0	0	0
N.S.	1	1.00	1.11	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.332	4.033	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	276	276	233	293	0	0	0	0	0
N.S.	1	1.00	0.84	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	10.166	4.351	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	386	321	0	0	0	0	0
N.S.	1	1.00	1.25	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	10.281	4.477	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	374	374	0	0	0	0	0
N.S.	1	1.00	1.03	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	10.520	4.751	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	0
N.S.	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	10.879	4.427	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	81	89	0	437	0	0	0
N.S.	1	1.00	0.79	0.86	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.037	0.553	5.254	0.000	0.578	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	88	141	0	482	0	0	0
N.S.	1	1.00	0.76	1.22	0.00	4.16	0.00	0.00	0.00
time (sec)	N/A	0.015	0.519	6.369	0.000	0.576	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	288	401	0	1962	0	0	0
N.S.	1	1.00	1.36	1.90	0.00	9.30	0.00	0.00	0.00
time (sec)	N/A	0.190	1.796	8.934	0.000	1.706	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	255	338	0	733	0	0	0
N.S.	1	1.00	1.47	1.95	0.00	4.24	0.00	0.00	0.00
time (sec)	N/A	0.080	1.199	5.585	0.000	0.405	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	178	244	0	0	0	0	0
N.S.	1	1.00	1.70	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.879	4.252	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	229	325	0	0	0	0	0
N.S.	1	1.00	1.71	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	1.788	4.146	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	621	329	0	0	0	0	0
N.S.	1	1.00	3.45	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	12.117	4.390	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	1172	381	0	0	0	0	0
N.S.	1	1.00	5.03	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	14.498	4.440	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	294	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	10.647	0.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	346	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	10.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	10.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	10.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	332	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	10.273	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	357	357	430	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	10.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	349	507	0	2764	0	0	0
N.S.	1	1.00	1.25	1.81	0.00	9.87	0.00	0.00	0.00
time (sec)	N/A	0.258	3.364	5.364	0.000	10.518	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	341	408	0	1440	0	0	0
N.S.	1	1.00	1.48	1.77	0.00	6.26	0.00	0.00	0.00
time (sec)	N/A	0.124	2.076	4.509	0.000	0.859	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	238	293	0	0	0	0	0
N.S.	1	1.00	1.76	2.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	1.464	4.429	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	251	343	0	0	0	0	0
N.S.	1	1.00	1.55	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	2.396	4.401	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	285	355	0	0	0	0	0
N.S.	1	1.00	1.39	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	4.372	4.476	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	357	445	0	0	0	0	0
N.S.	1	1.00	1.34	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	7.694	4.661	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	353	353	392	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	10.481	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	341	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	10.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	233	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	10.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	337	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	10.370	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	390	390	387	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	10.539	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	61	0	270	0	0	0
N.S.	1	1.00	0.83	1.15	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.012	0.302	4.669	0.000	6.297	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	65	0	0	0	0	0
N.S.	1	1.00	0.84	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.565	4.867	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	119	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.68	0.00	0.00
time (sec)	N/A	0.106	0.282	0.000	0.000	0.000	77.687	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.031	0.172	0.000	0.000	0.000	27.993	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.408	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.510	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	48	409	0	0	0	0	0
N.S.	1	1.00	0.09	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.861	10.031	10.584	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	118	160	164	306	461	0	173
N.S.	1	1.00	0.83	1.12	1.15	2.14	3.22	0.00	1.21
time (sec)	N/A	0.083	0.329	0.101	0.295	0.322	14.651	0.000	6.787

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	84	115	126	208	129	0	99
N.S.	1	1.00	0.85	1.16	1.27	2.10	1.30	0.00	1.00
time (sec)	N/A	0.047	0.277	0.085	0.291	0.260	9.907	0.000	6.066

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	84	106	128	95	0	92
N.S.	1	1.00	0.70	1.14	1.43	1.73	1.28	0.00	1.24
time (sec)	N/A	0.031	0.168	0.081	0.272	0.249	11.491	0.000	6.128

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	72	50	99	42	62	58
N.S.	1	1.00	1.00	1.85	1.28	2.54	1.08	1.59	1.49
time (sec)	N/A	0.015	0.019	0.052	0.280	0.263	0.979	0.289	5.469

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	100	232	0	482	0	0	149
N.S.	1	1.00	0.96	2.23	0.00	4.63	0.00	0.00	1.43
time (sec)	N/A	0.076	0.322	0.222	0.000	0.280	0.000	0.000	5.736

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	122	477	0	801	0	401	1195
N.S.	1	1.00	0.83	3.24	0.00	5.45	0.00	2.73	8.13
time (sec)	N/A	0.164	0.533	0.235	0.000	0.305	0.000	0.344	6.434

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	189	969	0	1749	0	809	1895
N.S.	1	1.00	0.89	4.55	0.00	8.21	0.00	3.80	8.90
time (sec)	N/A	0.260	1.428	0.261	0.000	0.408	0.000	0.345	7.936

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	159	214	190	380	1828	0	327
N.S.	1	1.00	0.97	1.30	1.16	2.32	11.15	0.00	1.99
time (sec)	N/A	0.099	0.399	0.104	0.280	0.280	33.365	0.000	8.027

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	115	153	152	268	546	0	197
N.S.	1	1.00	0.91	1.21	1.21	2.13	4.33	0.00	1.56
time (sec)	N/A	0.060	0.367	0.091	0.295	0.259	22.390	0.000	6.706

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	105	132	164	177	0	81
N.S.	1	1.00	0.73	1.05	1.32	1.64	1.77	0.00	0.81
time (sec)	N/A	0.046	0.258	0.086	0.281	0.275	14.378	0.000	6.705

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	78	63	100	92	0	34
N.S.	1	1.00	0.85	1.44	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.021	0.014	0.060	0.280	0.340	1.393	0.000	5.602

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	103	247	0	519	0	0	556
N.S.	1	1.00	0.97	2.33	0.00	4.90	0.00	0.00	5.25
time (sec)	N/A	0.087	0.395	0.208	0.000	0.309	0.000	0.000	5.817

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	144	502	0	769	0	514	448
N.S.	1	1.00	0.92	3.22	0.00	4.93	0.00	3.29	2.87
time (sec)	N/A	0.158	0.513	0.234	0.000	0.287	0.000	0.337	6.332

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	169	1008	0	1765	0	720	1664
N.S.	1	1.00	0.81	4.82	0.00	8.44	0.00	3.44	7.96
time (sec)	N/A	0.243	0.658	0.273	0.000	0.341	0.000	0.376	7.515

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	201	273	219	494	5523	0	487
N.S.	1	1.00	1.02	1.38	1.11	2.49	27.89	0.00	2.46
time (sec)	N/A	0.124	0.502	0.154	0.289	0.275	39.992	0.000	10.276

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	145	194	181	350	1853	0	271
N.S.	1	1.00	0.95	1.28	1.19	2.30	12.19	0.00	1.78
time (sec)	N/A	0.080	0.405	0.135	0.272	0.277	26.636	0.000	8.108

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	94	129	0	222	534	0	99
N.S.	1	1.00	0.75	1.03	0.00	1.78	4.27	0.00	0.79
time (sec)	N/A	0.052	0.305	0.106	0.000	0.283	17.071	0.000	7.754

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	94	78	139	99	0	34
N.S.	1	1.00	0.90	1.32	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.025	0.060	0.069	0.311	0.278	2.148	0.000	5.878

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	117	279	0	659	0	0	1427
N.S.	1	1.00	0.87	2.08	0.00	4.92	0.00	0.00	10.65
time (sec)	N/A	0.147	0.376	0.249	0.000	0.376	0.000	0.000	6.260

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	146	517	0	1001	0	667	1153
N.S.	1	1.00	0.88	3.11	0.00	6.03	0.00	4.02	6.95
time (sec)	N/A	0.179	0.482	0.284	0.000	0.334	0.000	0.341	6.586

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	205	1035	0	1445	0	947	1476
N.S.	1	1.00	0.86	4.37	0.00	6.10	0.00	4.00	6.23
time (sec)	N/A	0.255	0.653	0.339	0.000	0.488	0.000	0.346	7.775

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	130	166	233	391	0	107
N.S.	1	1.00	0.75	1.03	1.32	1.85	3.10	0.00	0.85
time (sec)	N/A	0.067	0.324	0.146	0.340	0.265	13.729	0.000	5.926

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	105	129	158	119	0	63
N.S.	1	1.00	0.90	1.44	1.77	2.16	1.63	0.00	0.86
time (sec)	N/A	0.035	0.250	0.148	0.347	0.249	8.879	0.000	5.801

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	84	109	115	85	87	88
N.S.	1	1.00	1.00	1.65	2.14	2.25	1.67	1.71	1.73
time (sec)	N/A	0.022	0.150	0.134	0.319	0.268	9.371	0.293	6.242

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	98	44	69	66
N.S.	1	1.00	1.00	1.65	1.56	2.28	1.02	1.60	1.53
time (sec)	N/A	0.015	0.021	0.110	0.335	0.265	1.163	0.300	5.742

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	104	228	0	542	0	0	1183
N.S.	1	1.00	0.96	2.11	0.00	5.02	0.00	0.00	10.95
time (sec)	N/A	0.064	0.365	0.261	0.000	0.298	0.000	0.000	6.234

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	149	469	0	1163	0	493	3813
N.S.	1	1.00	0.87	2.73	0.00	6.76	0.00	2.87	22.17
time (sec)	N/A	0.168	0.776	0.309	0.000	0.356	0.000	0.324	7.824

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	215	953	0	2307	0	890	2890
N.S.	1	1.00	0.86	3.81	0.00	9.23	0.00	3.56	11.56
time (sec)	N/A	0.269	1.790	0.328	0.000	0.600	0.000	0.384	9.552

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	113	226	200	336	0	0	172
N.S.	1	1.00	0.86	1.71	1.52	2.55	0.00	0.00	1.30
time (sec)	N/A	0.077	0.319	0.185	0.344	0.294	0.000	0.000	6.033

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	89	182	164	272	0	187	120
N.S.	1	0.99	0.95	1.94	1.74	2.89	0.00	1.99	1.28
time (sec)	N/A	0.052	0.300	0.171	0.338	0.345	0.000	0.331	5.896

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	159	144	210	224	149	71
N.S.	1	1.00	0.92	2.09	1.89	2.76	2.95	1.96	0.93
time (sec)	N/A	0.034	0.218	0.166	0.354	0.258	14.550	0.324	6.286

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	116	85	156	71	114	34
N.S.	1	1.00	0.95	1.93	1.42	2.60	1.18	1.90	0.57
time (sec)	N/A	0.019	0.063	0.133	0.334	0.267	1.725	0.307	5.632

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	297	0	1075	0	0	3000
N.S.	1	1.00	0.98	2.02	0.00	7.31	0.00	0.00	20.41
time (sec)	N/A	0.158	0.684	0.291	0.000	0.355	0.000	0.000	6.745

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	215	533	0	2321	0	0	4274
N.S.	1	1.00	0.96	2.38	0.00	10.36	0.00	0.00	19.08
time (sec)	N/A	0.256	1.157	0.379	0.000	0.618	0.000	0.000	9.869

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	298	983	0	4093	0	1089	8936
N.S.	1	1.00	0.93	3.07	0.00	12.79	0.00	3.40	27.92
time (sec)	N/A	0.384	1.879	0.354	0.000	1.453	0.000	0.397	13.527

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	134	302	228	483	0	429	194
N.S.	1	1.00	0.94	2.11	1.59	3.38	0.00	3.00	1.36
time (sec)	N/A	0.109	0.350	0.199	0.333	0.271	0.000	0.319	6.097

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	121	112	287	190	407	0	363	144
N.S.	1	0.99	0.92	2.35	1.56	3.34	0.00	2.98	1.18
time (sec)	N/A	0.082	0.281	0.193	0.343	0.272	0.000	0.313	6.291

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	91	254	170	331	1479	259	87
N.S.	1	1.00	0.88	2.47	1.65	3.21	14.36	2.51	0.84
time (sec)	N/A	0.043	0.243	0.182	0.335	0.267	33.930	0.308	7.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	157	101	225	774	171	34
N.S.	1	1.00	0.91	1.99	1.28	2.85	9.80	2.16	0.43
time (sec)	N/A	0.029	0.043	0.155	0.343	0.276	2.715	0.298	5.872

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	200	403	0	1990	0	0	5387
N.S.	1	1.00	1.00	2.00	0.00	9.90	0.00	0.00	26.80
time (sec)	N/A	0.209	0.773	0.334	0.000	1.524	0.000	0.000	8.883

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	305	665	0	3887	0	924	5789
N.S.	1	1.00	1.06	2.32	0.00	13.54	0.00	3.22	20.17
time (sec)	N/A	0.322	1.602	0.367	0.000	1.659	0.000	0.396	12.726

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	404	1138	0	6171	0	1336	4284
N.S.	1	1.00	0.99	2.78	0.00	15.09	0.00	3.27	10.47
time (sec)	N/A	0.508	2.323	0.464	0.000	4.570	0.000	0.486	11.877

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	176	218	0	890	0	0	4674
N.S.	1	1.00	1.43	1.77	0.00	7.24	0.00	0.00	38.00
time (sec)	N/A	0.080	0.516	0.097	0.000	0.601	0.000	0.000	26.273

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	113	155	0	247	0	0	478
N.S.	1	1.00	1.40	1.91	0.00	3.05	0.00	0.00	5.90
time (sec)	N/A	0.036	0.363	0.126	0.000	0.414	0.000	0.000	10.767

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	126	280	0	319	0	0	0
N.S.	1	1.00	1.03	2.30	0.00	2.61	0.00	0.00	0.00
time (sec)	N/A	0.063	0.473	0.156	0.000	0.387	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.021	0.030	0.070	0.352	0.308	0.145	0.279	0.069

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0	0
N.S.	1	1.00	0.88	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	2.471	2.353	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	116	0	0	0
N.S.	1	1.00	0.37	0.41	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.165	2.116	1.137	0.000	0.084	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	191	185	0	164	0	0	0
N.S.	1	1.00	0.73	0.71	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.211	3.409	3.468	0.000	0.114	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	42	128	390	71	133	123
N.S.	1	1.00	0.89	0.29	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.088	0.085	0.090	0.328	0.262	0.227	0.282	0.273

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	48	47	48	82	49	49
N.S.	1	1.00	1.04	0.98	0.96	0.98	1.67	1.00	1.00
time (sec)	N/A	0.035	0.050	3.916	0.235	0.248	0.177	0.298	0.076

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	20
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.011	0.022	3.921	0.261	0.350	0.070	0.276	0.033

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	23	23	27	24	13
N.S.	1	1.00	1.00	1.41	1.35	1.35	1.59	1.41	0.76
time (sec)	N/A	0.011	0.024	3.966	0.258	0.242	0.081	0.276	0.051

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	66	71	71	102	71	90
N.S.	1	1.00	1.00	0.63	0.68	0.68	0.98	0.68	0.87
time (sec)	N/A	0.062	0.192	56.369	0.330	0.256	1.875	0.302	5.582

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	22	22	26	23	22
N.S.	1	1.00	0.93	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.013	0.006	3.932	0.258	0.303	0.066	0.275	0.039

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	128	186	527	2744	740	131
N.S.	1	1.00	0.83	0.97	1.41	3.99	20.79	5.61	0.99
time (sec)	N/A	0.082	0.682	3.993	0.283	0.286	0.733	0.288	5.691

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	90	96	140	319	1540	450	99
N.S.	1	1.00	0.91	0.97	1.41	3.22	15.56	4.55	1.00
time (sec)	N/A	0.052	0.226	4.093	0.288	0.253	0.490	0.291	5.651

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.032	0.151	3.931	0.282	0.253	0.359	0.281	5.610

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	39	48	69	236	83	38
N.S.	1	1.00	0.92	0.98	1.20	1.72	5.90	2.08	0.95
time (sec)	N/A	0.016	0.115	0.051	0.286	0.250	0.257	0.285	5.580

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	110	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	2.56	0.00	0.00
time (sec)	N/A	0.015	0.135	0.000	0.000	0.000	1.046	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	741	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	10.15	0.00	0.00
time (sec)	N/A	0.021	0.083	0.000	0.000	0.000	3.385	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	149	154	242	667	3376	962	157
N.S.	1	1.00	0.94	0.97	1.53	4.22	21.37	6.09	0.99
time (sec)	N/A	0.098	0.773	4.243	0.285	0.262	7.409	0.298	5.914

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	105	109	168	370	1760	539	108
N.S.	1	1.00	0.94	0.97	1.50	3.30	15.71	4.81	0.96
time (sec)	N/A	0.057	0.763	4.000	0.266	0.251	2.956	0.298	5.792

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.033	0.157	4.096	0.303	0.262	0.355	0.292	5.694

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	235	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.80	0.00	0.00
time (sec)	N/A	0.068	0.216	0.000	0.000	0.000	1.762	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	133	0	0	0	488	0	0
N.S.	1	1.00	0.43	0.00	0.00	0.00	1.57	0.00	0.00
time (sec)	N/A	0.360	3.528	0.000	0.000	0.000	3.214	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	360	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	2.08	0.00	0.00
time (sec)	N/A	0.198	1.488	0.000	0.000	0.000	2.385	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	235	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	2.80	0.00	0.00
time (sec)	N/A	0.068	0.486	0.000	0.000	0.000	1.810	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	110	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.62	0.00	0.00
time (sec)	N/A	0.012	0.128	0.000	0.000	0.000	1.047	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	6.394	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	2050	0	0	0	0	0	0
N.S.	1	1.00	10.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	4.826	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	666	0	0	0	0	0	0
N.S.	1	1.00	5.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	1.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	741	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	10.29	0.00	0.00
time (sec)	N/A	0.025	0.088	0.000	0.000	0.000	3.216	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.286	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.447	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	243	0	0
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.386	5.309	0.000	0.000	0.000	42.878	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	175	0	0
N.S.	1	0.98	0.69	0.00	0.00	0.00	0.87	0.00	0.00
time (sec)	N/A	0.192	5.198	0.000	0.000	0.000	17.299	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	107	0	0
N.S.	1	0.91	0.96	0.00	0.00	0.00	1.09	0.00	0.00
time (sec)	N/A	0.034	0.081	0.000	0.000	0.000	2.688	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	46	0	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	1.02
time (sec)	N/A	0.009	0.004	0.000	0.000	0.000	0.944	0.000	6.462

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.467	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	218	1288	0	478	2822	0	0
N.S.	1	1.00	1.22	7.24	0.00	2.69	15.85	0.00	0.00
time (sec)	N/A	0.065	0.212	5.015	0.000	0.274	29.868	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	588	0	231	1035	0	0
N.S.	1	1.00	0.97	5.07	0.00	1.99	8.92	0.00	0.00
time (sec)	N/A	0.023	0.140	4.506	0.000	0.248	12.920	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	82	199	0	85	311	0	0
N.S.	1	1.00	1.41	3.43	0.00	1.47	5.36	0.00	0.00
time (sec)	N/A	0.013	0.167	4.212	0.000	0.252	2.214	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	47	0	31	42	0	75
N.S.	1	1.00	1.00	2.61	0.00	1.72	2.33	0.00	4.17
time (sec)	N/A	0.002	0.055	4.077	0.000	0.248	0.678	0.000	5.883

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	193	179	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	334	0	108	0	0	0
N.S.	1	1.00	0.96	5.86	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.018	0.571	8.574	0.000	0.264	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	136	1059	0	400	2746	0	0
N.S.	1	1.00	0.42	3.24	0.00	1.22	8.40	0.00	0.00
time (sec)	N/A	0.135	0.442	4.525	0.000	0.267	13.315	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	94	434	0	173	959	0	0
N.S.	1	1.00	0.74	3.42	0.00	1.36	7.55	0.00	0.00
time (sec)	N/A	0.049	0.207	4.398	0.000	0.264	2.479	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	250	0	64
N.S.	1	1.00	1.10	0.00	0.00	1.36	5.00	0.00	1.28
time (sec)	N/A	0.011	0.050	0.000	0.000	0.254	0.768	0.000	5.934

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	6.530	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	212	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	8.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	333	0	0	0	0	0	0
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	8.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	6405	0	0	0	0	0	0
N.S.	1	1.00	48.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	36.956	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	164	97	92	178	114	0	621	152
N.S.	1	1.08	0.64	0.61	1.17	0.75	0.00	4.09	1.00
time (sec)	N/A	0.086	0.300	4.043	0.315	0.301	0.000	0.449	5.921

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	118	75	68	124	90	0	495	118
N.S.	1	1.08	0.69	0.62	1.14	0.83	0.00	4.54	1.08
time (sec)	N/A	0.063	0.215	4.186	0.306	0.267	0.000	0.377	5.791

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	49	44	70	66	0	361	83
N.S.	1	1.07	0.73	0.66	1.04	0.99	0.00	5.39	1.24
time (sec)	N/A	0.035	0.163	4.146	0.290	0.355	0.000	0.353	5.754

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	174	52	80	0	78	248
N.S.	1	1.00	0.94	2.18	0.65	1.00	0.00	0.98	3.10
time (sec)	N/A	0.058	0.223	4.021	0.340	0.260	0.000	0.328	7.769

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	114	74	138	98	85	0	157	584
N.S.	1	1.19	0.77	1.44	1.02	0.89	0.00	1.64	6.08
time (sec)	N/A	0.067	0.287	4.064	0.389	0.258	0.000	0.349	11.295

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	164	99	150	162	100	0	324	1004
N.S.	1	1.36	0.82	1.24	1.34	0.83	0.00	2.68	8.30
time (sec)	N/A	0.066	0.281	4.201	0.372	0.270	0.000	0.362	22.149

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	142	184	246	138	0	558	2314
N.S.	1	1.00	0.68	0.88	1.18	0.66	0.00	2.68	11.12
time (sec)	N/A	0.094	0.456	4.186	0.257	0.292	0.000	0.405	49.014

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	116	159	192	112	0	432	1681
N.S.	1	1.00	0.73	1.00	1.21	0.70	0.00	2.72	10.57
time (sec)	N/A	0.085	0.363	4.196	0.278	0.268	0.000	0.378	55.455

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	92	135	137	88	0	288	734
N.S.	1	1.00	0.81	1.18	1.20	0.77	0.00	2.53	6.44
time (sec)	N/A	0.039	0.263	4.202	0.294	0.254	0.000	0.335	23.405

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	74	117	105	83	0	110	243
N.S.	1	1.00	0.71	1.12	1.01	0.80	0.00	1.06	2.34
time (sec)	N/A	0.056	0.296	4.199	0.344	0.259	0.000	0.346	7.592

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	124	75	100	0	171	236
N.S.	1	1.00	0.96	1.48	0.89	1.19	0.00	2.04	2.81
time (sec)	N/A	0.066	0.269	4.204	0.340	0.273	0.000	0.336	7.534

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	96	129	153	96	0	172	1154
N.S.	1	1.00	0.77	1.03	1.22	0.77	0.00	1.38	9.23
time (sec)	N/A	0.055	0.314	4.223	0.270	0.292	0.000	0.299	45.444

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	61	57	95	55	216	108	108
N.S.	1	1.00	0.59	0.55	0.92	0.53	2.10	1.05	1.05
time (sec)	N/A	0.049	0.151	4.317	0.272	0.261	5.567	0.288	6.643

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	111	113	77	0	121	720
N.S.	1	1.00	0.89	1.28	1.30	0.89	0.00	1.39	8.28
time (sec)	N/A	0.069	0.253	4.283	0.280	0.259	0.000	0.284	29.408

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	38	54	37	202	59	66
N.S.	1	1.00	0.66	0.58	0.83	0.57	3.11	0.91	1.02
time (sec)	N/A	0.031	0.119	4.070	0.283	0.249	4.077	0.276	6.574

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	58	91	74	55	0	69	293
N.S.	1	1.00	1.23	1.94	1.57	1.17	0.00	1.47	6.23
time (sec)	N/A	0.015	0.166	4.029	0.282	0.304	0.000	0.282	17.424

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	62	29	48	162	45	77
N.S.	1	1.00	0.98	1.35	0.63	1.04	3.52	0.98	1.67
time (sec)	N/A	0.049	0.124	4.012	0.359	0.261	15.947	0.280	7.661

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	48	77	44	56	148	58	61
N.S.	1	1.00	1.45	2.33	1.33	1.70	4.48	1.76	1.85
time (sec)	N/A	0.043	0.137	4.030	0.352	0.272	15.319	0.276	6.848

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	71	45	57	0	114	297
N.S.	1	1.00	0.92	1.18	0.75	0.95	0.00	1.90	4.95
time (sec)	N/A	0.041	0.157	4.192	0.404	0.248	0.000	0.274	13.843

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	42	37	54	52	146	116	53
N.S.	1	1.00	0.68	0.60	0.87	0.84	2.35	1.87	0.85
time (sec)	N/A	0.038	0.141	4.018	0.363	0.251	10.918	0.285	6.725

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	78	94	85	78	0	268	650
N.S.	1	1.00	0.79	0.95	0.86	0.79	0.00	2.71	6.57
time (sec)	N/A	0.049	0.290	4.050	0.366	0.269	0.000	0.288	31.048

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	119	160	196	115	0	203	1682
N.S.	1	1.00	0.73	0.98	1.20	0.70	0.00	1.24	10.26
time (sec)	N/A	0.079	0.377	4.076	0.274	0.271	0.000	0.308	58.965

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	74	68	124	66	240	124	130
N.S.	1	1.00	0.63	0.58	1.05	0.56	2.03	1.05	1.10
time (sec)	N/A	0.064	0.191	4.191	0.296	0.267	6.166	0.283	7.120

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	92	136	142	90	0	140	1048
N.S.	1	1.00	0.78	1.15	1.20	0.76	0.00	1.19	8.88
time (sec)	N/A	0.075	0.286	4.210	0.242	0.273	0.000	0.291	33.909

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	43	69	42	223	65	76
N.S.	1	1.00	0.67	0.60	0.96	0.58	3.10	0.90	1.06
time (sec)	N/A	0.035	0.142	4.194	0.249	0.263	4.396	0.281	7.142

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	111	89	63	0	79	417
N.S.	1	1.00	1.00	1.63	1.31	0.93	0.00	1.16	6.13
time (sec)	N/A	0.022	0.178	4.167	0.251	0.251	0.000	0.282	15.566

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	108	37	61	178	55	108
N.S.	1	1.00	0.96	1.93	0.66	1.09	3.18	0.98	1.93
time (sec)	N/A	0.046	0.147	4.272	0.345	0.261	16.137	0.289	8.310

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	97	55	68	165	66	77
N.S.	1	1.00	1.00	1.70	0.96	1.19	2.89	1.16	1.35
time (sec)	N/A	0.057	0.166	4.217	0.345	0.265	15.379	0.280	7.168

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	123	60	73	0	141	457
N.S.	1	1.00	0.92	1.62	0.79	0.96	0.00	1.86	6.01
time (sec)	N/A	0.057	0.192	4.226	0.352	0.267	0.000	0.307	12.131

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	49	75	67	170	137	79
N.S.	1	1.00	0.72	0.65	1.00	0.89	2.27	1.83	1.05
time (sec)	N/A	0.046	0.168	4.205	0.355	0.253	15.094	0.290	7.002

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	101	151	114	100	0	325	1005
N.S.	1	1.00	0.82	1.23	0.93	0.81	0.00	2.64	8.17
time (sec)	N/A	0.065	0.287	4.068	0.348	0.257	0.000	0.302	27.648

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	137	275	196	190	0	214	0
N.S.	1	1.00	0.85	1.71	1.22	1.18	0.00	1.33	0.00
time (sec)	N/A	0.102	0.389	4.240	0.270	0.258	0.000	0.325	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	72	68	123	80	0	200	90
N.S.	1	1.00	0.63	0.59	1.07	0.70	0.00	1.74	0.78
time (sec)	N/A	0.079	0.200	4.074	0.251	0.260	0.000	0.305	7.252

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	108	251	138	159	0	147	0
N.S.	1	1.00	0.71	1.65	0.91	1.05	0.00	0.97	0.00
time (sec)	N/A	0.076	0.288	4.256	0.271	0.266	0.000	0.322	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	43	69	56	0	152	67
N.S.	1	1.00	0.59	0.57	0.91	0.74	0.00	2.00	0.88
time (sec)	N/A	0.047	0.166	4.245	0.282	0.257	0.000	0.315	6.930

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	71	166	76	129	0	113	0
N.S.	1	1.00	1.13	2.63	1.21	2.05	0.00	1.79	0.00
time (sec)	N/A	0.024	0.286	4.067	0.273	0.258	0.000	0.298	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	70	194	58	101	0	115	0
N.S.	1	1.00	1.08	2.98	0.89	1.55	0.00	1.77	0.00
time (sec)	N/A	0.055	0.238	4.206	0.390	0.247	0.000	0.336	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	48	71	103	0	219	73
N.S.	1	1.00	0.76	0.72	1.06	1.54	0.00	3.27	1.09
time (sec)	N/A	0.050	0.175	4.244	0.363	0.263	0.000	0.344	7.109

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	93	239	104	138	0	211	0
N.S.	1	1.00	0.79	2.04	0.89	1.18	0.00	1.80	0.00
time (sec)	N/A	0.081	0.329	4.253	0.298	0.258	0.000	0.355	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	77	73	125	132	0	242	104
N.S.	1	1.00	0.65	0.61	1.05	1.11	0.00	2.03	0.87
time (sec)	N/A	0.069	0.222	4.596	0.283	0.265	0.000	0.396	7.366

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	122	267	162	165	0	402	0
N.S.	1	1.00	0.73	1.61	0.98	0.99	0.00	2.42	0.00
time (sec)	N/A	0.084	0.443	4.263	0.275	0.367	0.000	0.410	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	53	23	39	148	40	72
N.S.	1	1.00	1.00	1.32	0.58	0.98	3.70	1.00	1.80
time (sec)	N/A	0.036	0.108	4.025	0.303	0.241	15.828	0.271	8.068

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	244	66	79	65	1867	0	96
N.S.	1	1.00	4.60	1.25	1.49	1.23	35.23	0.00	1.81
time (sec)	N/A	0.060	10.294	4.058	0.297	0.278	36.677	0.000	7.451

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	44	0	0	69	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	0.013	9.814	0.000	0.000	0.247	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	50	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.043	10.038	0.000	0.000	0.262	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	0
N.S.	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.478	0.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	104	103	0	0	180	0	0	0
N.S.	1	1.08	1.07	0.00	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.086	1.417	0.000	0.000	0.299	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [34] had the largest ratio of [.6363999999999999966]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	7	7	1.00	17	0.412
6	A	7	7	1.00	17	0.412
7	A	8	8	1.00	17	0.471
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	8	7	1.00	19	0.368
12	A	9	8	1.00	19	0.421
13	A	8	8	1.00	19	0.421
14	A	8	7	1.00	19	0.368
15	A	8	7	1.00	19	0.368
16	A	8	7	1.00	19	0.368
17	A	7	7	1.00	17	0.412
18	A	13	7	1.00	19	0.368
19	A	14	8	1.00	19	0.421
20	A	9	8	1.00	19	0.421
21	A	9	8	1.00	19	0.421
22	A	9	8	1.00	19	0.421
23	A	9	8	1.00	19	0.421
24	A	7	7	1.00	17	0.412

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	14	8	1.00	19	0.421
26	A	15	9	1.00	19	0.474
27	A	3	3	1.00	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100
30	A	2	2	1.00	20	0.100
31	A	3	3	1.00	20	0.150
32	A	4	3	1.00	20	0.150
33	A	5	3	1.00	20	0.150
34	A	22	14	1.00	22	0.636
35	A	21	13	1.00	22	0.591
36	A	14	8	1.00	22	0.364
37	A	17	11	1.00	22	0.500
38	A	21	13	1.00	22	0.591
39	A	22	14	1.00	22	0.636
40	A	4	4	1.00	22	0.182
41	A	3	3	1.00	22	0.136
42	A	3	3	1.00	22	0.136
43	A	3	3	1.00	22	0.136
44	A	3	2	1.00	22	0.091
45	A	4	3	1.00	22	0.136
46	A	5	4	1.00	22	0.182
47	A	6	4	1.00	22	0.182
48	A	4	4	1.00	22	0.182
49	A	4	4	1.00	22	0.182
50	A	4	4	1.00	22	0.182
51	A	4	4	1.00	22	0.182
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	22	0.182
54	A	4	4	1.00	22	0.182
55	A	4	4	1.00	22	0.182
56	A	4	3	1.00	19	0.158
57	A	3	3	1.00	19	0.158
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	2	2	1.00	19	0.105
61	A	3	3	1.00	19	0.158
62	A	4	3	1.00	19	0.158
63	A	5	3	1.00	19	0.158
64	A	3	3	1.00	19	0.158
65	A	3	3	1.00	19	0.158
66	A	3	3	1.00	19	0.158
67	A	3	3	1.00	19	0.158
68	A	3	3	1.00	19	0.158
69	A	3	3	1.00	19	0.158
70	A	5	4	1.00	21	0.190
71	A	4	4	1.00	21	0.190
72	A	3	3	1.00	21	0.143
73	A	3	3	1.00	21	0.143
74	A	3	3	1.00	21	0.143
75	A	3	2	1.00	21	0.095
76	A	4	3	1.00	21	0.143
77	A	5	4	1.00	21	0.190
78	A	6	4	1.00	21	0.190
79	A	4	4	1.00	21	0.190
80	A	4	4	1.00	21	0.190
81	A	4	4	1.00	21	0.190
82	A	4	4	1.00	21	0.190
83	A	4	4	1.00	21	0.190
84	A	4	4	1.00	21	0.190
85	A	4	2	1.00	21	0.095
86	A	5	5	1.00	21	0.238
87	A	4	4	1.00	21	0.190
88	A	3	3	1.00	21	0.143
89	A	1	1	1.00	21	0.048
90	A	2	2	1.00	21	0.095
91	A	4	4	1.00	21	0.190
92	A	5	4	1.00	21	0.190
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	A	5	5	1.00	21	0.238
99	A	4	4	1.00	21	0.190
100	A	2	2	1.00	21	0.095
101	A	2	2	1.00	21	0.095
102	A	4	4	1.00	21	0.190
103	A	5	4	1.00	21	0.190
104	A	2	2	1.00	21	0.095
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	21	0.095
109	A	7	6	1.00	21	0.286
110	A	6	6	1.00	21	0.286
111	A	5	5	1.00	21	0.238
112	A	3	2	1.00	21	0.095
113	A	3	3	1.00	21	0.143
114	A	4	4	1.00	21	0.190
115	A	5	4	1.00	21	0.190
116	A	6	4	1.00	21	0.190
117	A	2	2	1.00	21	0.095
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	21	0.095
120	A	2	2	1.00	21	0.095
121	A	2	2	1.00	21	0.095
122	A	3	2	1.00	23	0.087
123	A	3	2	1.00	23	0.087
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	2	2	1.00	23	0.087

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	23	0.087
131	A	3	2	1.00	23	0.087
132	A	3	2	1.00	23	0.087
133	A	3	2	1.00	19	0.105
134	A	4	4	1.05	19	0.210
135	A	3	3	1.01	17	0.176
136	A	2	2	1.00	19	0.105
137	A	2	2	1.00	19	0.105
138	A	5	5	1.00	19	0.263
139	A	4	4	1.00	19	0.210
140	A	3	3	0.91	17	0.176
141	A	2	2	1.00	9	0.222
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	2	2	1.00	19	0.105
145	A	1	1	1.00	50	0.020
146	A	2	1	1.00	17	0.059
147	A	2	1	1.00	17	0.059
148	A	2	1	1.00	17	0.059
149	A	2	1	1.00	15	0.067
150	A	10	7	1.00	17	0.412
151	A	10	7	1.00	17	0.412
152	A	11	8	1.00	17	0.471
153	A	2	1	1.00	19	0.053
154	A	2	1	1.00	19	0.053
155	A	2	1	1.00	19	0.053
156	A	2	1	1.00	17	0.059
157	A	11	7	1.00	19	0.368
158	A	12	8	1.00	19	0.421
159	A	11	8	1.00	19	0.421
160	A	11	7	1.00	19	0.368
161	A	11	7	1.00	19	0.368
162	A	11	7	1.00	19	0.368
163	A	10	7	1.00	17	0.412
164	A	19	7	1.00	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	20	8	1.00	19	0.421
166	A	12	8	1.00	19	0.421
167	A	12	8	1.00	19	0.421
168	A	12	8	1.00	19	0.421
169	A	12	8	1.00	19	0.421
170	A	10	7	1.00	17	0.412
171	A	20	8	1.00	19	0.421
172	A	21	9	1.00	19	0.474
173	A	10	8	1.00	23	0.348
174	A	9	7	1.00	23	0.304
175	A	8	6	1.00	23	0.261
176	A	5	3	1.00	23	0.130
177	A	9	7	1.00	23	0.304
178	A	10	8	1.00	23	0.348
179	A	10	6	1.00	21	0.286
180	A	9	5	1.00	21	0.238
181	A	7	4	1.00	21	0.190
182	A	10	6	1.00	21	0.286
183	A	11	7	1.00	21	0.333
184	A	11	8	1.00	23	0.348
185	A	10	8	1.00	23	0.348
186	A	9	7	1.00	23	0.304
187	A	9	7	1.00	23	0.304
188	A	9	7	1.00	23	0.304
189	A	10	8	1.00	23	0.348
190	A	11	8	1.00	23	0.348
191	A	4	4	1.00	25	0.160
192	A	1	1	1.00	25	0.040
193	A	10	9	1.00	21	0.429
194	A	9	8	1.00	21	0.381
195	A	4	4	1.00	21	0.190
196	A	5	5	1.00	21	0.238
197	A	7	7	1.00	21	0.333
198	A	8	7	1.00	21	0.333
199	A	11	10	1.00	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	10	9	1.00	21	0.429
201	A	4	3	1.00	21	0.143
202	A	9	8	1.00	21	0.381
203	A	10	9	1.00	21	0.429
204	A	11	10	1.00	21	0.476
205	A	11	10	1.00	21	0.476
206	A	10	9	1.00	21	0.429
207	A	5	5	1.00	21	0.238
208	A	5	5	1.00	21	0.238
209	A	7	7	1.00	21	0.333
210	A	8	7	1.00	21	0.333
211	A	11	10	1.00	21	0.476
212	A	10	9	1.00	21	0.429
213	A	10	9	1.00	21	0.429
214	A	10	9	1.00	21	0.429
215	A	11	10	1.00	21	0.476
216	A	4	4	1.00	17	0.235
217	A	4	4	1.00	26	0.154
218	A	3	2	1.00	19	0.105
219	A	4	4	1.00	19	0.210
220	A	3	3	0.91	17	0.176
221	A	2	2	1.00	19	0.105
222	A	2	2	1.00	19	0.105
223	A	7	7	1.00	21	0.333
224	A	6	6	1.00	21	0.286
225	A	6	6	1.00	21	0.286
226	A	5	5	1.00	19	0.263
227	A	4	4	1.00	11	0.364
228	A	7	6	1.00	21	0.286
229	A	8	7	1.00	21	0.333
230	A	9	7	1.00	21	0.333
231	A	7	7	1.00	21	0.333
232	A	7	6	1.00	21	0.286
233	A	6	5	1.00	19	0.263
234	A	5	5	1.00	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	7	6	1.00	21	0.286
236	A	8	7	1.00	21	0.333
237	A	9	7	1.00	21	0.333
238	A	8	7	1.00	21	0.333
239	A	8	6	1.00	21	0.286
240	A	7	5	1.00	19	0.263
241	A	6	5	1.00	11	0.454
242	A	8	7	1.00	21	0.333
243	A	8	7	1.00	21	0.333
244	A	9	8	1.00	21	0.381
245	A	5	5	1.00	21	0.238
246	A	5	5	1.00	21	0.238
247	A	4	4	1.00	19	0.210
248	A	4	4	1.00	11	0.364
249	A	7	6	1.00	21	0.286
250	A	8	7	1.00	21	0.333
251	A	9	7	1.00	21	0.333
252	A	5	5	1.00	21	0.238
253	A	5	5	0.99	21	0.238
254	A	5	5	1.00	19	0.263
255	A	5	5	1.00	11	0.454
256	A	8	7	1.00	21	0.333
257	A	9	8	1.00	21	0.381
258	A	10	8	1.00	21	0.381
259	A	5	5	1.00	21	0.238
260	A	6	6	0.99	21	0.286
261	A	6	5	1.00	19	0.263
262	A	6	5	1.00	11	0.454
263	A	9	7	1.00	21	0.333
264	A	10	8	1.00	21	0.381
265	A	11	8	1.00	21	0.381
266	A	8	8	1.00	23	0.348
267	A	4	4	1.00	23	0.174
268	A	5	5	1.00	23	0.217
269	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	3	3	1.00	17	0.176
271	A	6	6	1.00	23	0.261
272	A	7	7	1.00	23	0.304
273	A	7	7	1.00	23	0.304
274	A	4	3	1.00	19	0.158
275	A	8	8	1.00	17	0.471
276	A	3	2	1.00	21	0.095
277	A	3	2	1.00	17	0.118
278	A	4	4	1.00	17	0.235
279	A	9	9	1.00	17	0.529
280	A	4	3	1.00	17	0.176
281	A	3	3	1.00	24	0.125
282	A	3	3	1.00	24	0.125
283	A	3	3	1.00	20	0.150
284	A	2	1	1.00	17	0.059
285	A	2	1	1.00	17	0.059
286	A	2	1	1.00	17	0.059
287	A	2	1	1.00	15	0.067
288	A	2	2	1.00	17	0.118
289	A	2	2	1.00	17	0.118
290	A	2	2	1.00	17	0.118
291	A	2	2	1.00	17	0.118
292	A	2	1	1.00	19	0.053
293	A	2	1	1.00	19	0.053
294	A	2	1	1.00	17	0.059
295	A	3	3	1.00	19	0.158
296	A	3	3	1.00	19	0.158
297	A	3	3	1.00	19	0.158
298	A	5	4	1.00	19	0.210
299	A	4	4	1.00	19	0.210
300	A	3	3	1.00	19	0.158
301	A	2	2	1.00	17	0.118
302	A	3	2	1.00	19	0.105
303	A	4	3	1.00	19	0.158
304	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	5	4	1.00	19	0.210
306	A	4	4	1.00	19	0.210
307	A	3	3	1.00	19	0.158
308	A	2	2	1.00	17	0.118
309	A	4	3	1.00	19	0.158
310	A	5	4	1.00	19	0.210
311	A	6	4	1.00	19	0.210
312	A	3	2	1.00	19	0.105
313	A	5	5	1.00	19	0.263
314	A	4	4	0.98	19	0.210
315	A	3	3	0.91	17	0.176
316	A	2	2	1.00	9	0.222
317	A	2	2	1.00	19	0.105
318	A	2	2	1.00	19	0.105
319	A	2	2	1.00	19	0.105
320	A	1	1	1.00	28	0.036
321	A	4	2	1.00	25	0.080
322	A	3	2	1.00	25	0.080
323	A	2	2	1.00	23	0.087
324	A	1	1	1.00	15	0.067
325	A	1	1	1.00	23	0.043
326	A	1	1	1.00	25	0.040
327	A	1	1	1.00	25	0.040
328	A	2	2	0.93	28	0.071
329	A	1	1	1.00	69	0.014
330	A	5	3	1.00	25	0.120
331	A	3	3	1.00	23	0.130
332	A	2	2	1.00	15	0.133
333	A	2	2	1.00	25	0.080
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	6	4	1.08	31	0.129
338	A	4	4	1.08	31	0.129
339	A	2	2	1.07	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	5	5	1.00	31	0.161
341	A	5	5	1.19	31	0.161
342	A	5	4	1.36	31	0.129
343	A	9	8	1.00	31	0.258
344	A	7	7	1.00	31	0.226
345	A	5	5	1.00	28	0.179
346	A	5	5	1.00	31	0.161
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	29	0.172
349	A	4	4	1.00	29	0.138
350	A	3	3	1.00	29	0.103
351	A	2	2	1.00	27	0.074
352	A	2	2	1.00	26	0.077
353	A	3	3	1.00	29	0.103
354	A	2	2	1.00	29	0.069
355	A	3	3	1.00	29	0.103
356	A	2	2	1.00	29	0.069
357	A	5	5	1.00	29	0.172
358	A	8	7	1.00	31	0.226
359	A	4	4	1.00	31	0.129
360	A	6	6	1.00	31	0.194
361	A	2	2	1.00	29	0.069
362	A	4	4	1.00	28	0.143
363	A	3	3	1.00	31	0.097
364	A	4	4	1.00	31	0.129
365	A	3	3	1.00	31	0.097
366	A	2	2	1.00	31	0.065
367	A	5	5	1.00	31	0.161
368	A	8	8	1.00	31	0.258
369	A	4	4	1.00	31	0.129
370	A	7	7	1.00	31	0.226
371	A	2	2	1.00	29	0.069
372	A	4	4	1.00	28	0.143
373	A	3	3	1.00	31	0.097
374	A	2	2	1.00	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	5	1.00	31	0.161
376	A	4	4	1.00	31	0.129
377	A	7	7	1.00	31	0.226
378	A	3	3	1.00	31	0.097
379	A	1	1	1.00	57	0.018
380	A	3	3	1.00	32	0.094
381	A	4	4	1.00	41	0.098
382	A	4	3	1.00	31	0.097
383	A	4	4	1.00	35	0.114
384	A	3	3	1.00	31	0.097
385	A	2	2	1.08	76	0.026

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx^3)(c+dx^3)^4\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7 + \frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

```
[Out] a*c^4*x+1/4*c^3*(4*a*d+b*c)*x^4+2/7*c^2*d*(3*a*d+2*b*c)*x^7+1/5*c*d^2*(2*a*
d+3*b*c)*x^10+1/13*d^3*(a*d+4*b*c)*x^13+1/16*b*d^4*x^16
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)(c+dx^3)^4\,dx = \frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc) + \frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

```
[In] Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ac^4 + c^3(bc+4ad)x^3 + 2c^2d(2bc+3ad)x^6 + 2cd^2(3bc+2ad)x^9 \right. \\ &\qquad \left. + d^3(4bc+ad)x^{12} + bd^4x^{15}\right)\,dx \\ &= ac^4x + \frac{1}{4}c^3(bc+4ad)x^4 + \frac{2}{7}c^2d(2bc+3ad)x^7 \\ &\quad + \frac{1}{5}cd^2(3bc+2ad)x^{10} + \frac{1}{13}d^3(4bc+ad)x^{13} + \frac{1}{16}bd^4x^{16} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\begin{aligned} \int (a+bx^3)\,(c+dx^3)^4\,dx &= ac^4x + \frac{1}{4}c^3(bc+4ad)x^4 + \frac{2}{7}c^2d(2bc+3ad)x^7 \\ &\quad + \frac{1}{5}cd^2(3bc+2ad)x^{10} + \frac{1}{13}d^3(4bc+ad)x^{13} + \frac{1}{16}bd^4x^{16} \end{aligned}$$

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
norman	$a\,c^4x + \left(a\,c^3d + \frac{1}{4}b\,c^4\right)x^4 + \left(\frac{6}{7}a\,c^2d^2 + \frac{4}{7}b\,c^3d\right)x^7 + \left(\frac{2}{5}ac\,d^3 + \frac{3}{5}b\,c^2d^2\right)x^{10} + \left(\frac{1}{13}a\,d^4 + \frac{4}{13}bc\right.$
default	$\frac{b\,d^4x^{16}}{16} + \frac{(a\,d^4+4bc\,d^3)x^{13}}{13} + \frac{(4ac\,d^3+6b\,c^2d^2)x^{10}}{10} + \frac{(6a\,c^2d^2+4b\,c^3d)x^7}{7} + \frac{(4a\,c^3d+b\,c^4)x^4}{4} + a\,c^4x$
gosper	$a\,c^4x + x^4a\,c^3d + \frac{1}{4}x^4b\,c^4 + \frac{6}{7}x^7a\,c^2d^2 + \frac{4}{7}x^7b\,c^3d + \frac{2}{5}x^{10}ac\,d^3 + \frac{3}{5}x^{10}b\,c^2d^2 + \frac{1}{13}x^{13}a\,d^4 + \frac{4}{13}$
risch	$a\,c^4x + x^4a\,c^3d + \frac{1}{4}x^4b\,c^4 + \frac{6}{7}x^7a\,c^2d^2 + \frac{4}{7}x^7b\,c^3d + \frac{2}{5}x^{10}ac\,d^3 + \frac{3}{5}x^{10}b\,c^2d^2 + \frac{1}{13}x^{13}a\,d^4 + \frac{4}{13}$
parallelrisch	$a\,c^4x + x^4a\,c^3d + \frac{1}{4}x^4b\,c^4 + \frac{6}{7}x^7a\,c^2d^2 + \frac{4}{7}x^7b\,c^3d + \frac{2}{5}x^{10}ac\,d^3 + \frac{3}{5}x^{10}b\,c^2d^2 + \frac{1}{13}x^{13}a\,d^4 + \frac{4}{13}$

```
[In] int((b*x^3+a)*(d*x^3+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^4*x+(a*c^3*d+1/4*b*c^4)*x^4+(6/7*a*c^2*d^2+4/7*b*c^3*d)*x^7+(2/5*a*c*d^
3+3/5*b*c^2*d^2)*x^10+(1/13*a*d^4+4/13*b*c*d^3)*x^13+1/16*b*d^4*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx=\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\,\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\,\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\,\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\,\left(bc^4+4\,ac^3d\right)x^4$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*
d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*
c^3*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx=ac^4x+\frac{bd^4x^{16}}{16}+x^{13}\left(\frac{ad^4}{13}+\frac{4bcd^3}{13}\right)+x^{10}\cdot\left(\frac{2acd^3}{5}+\frac{3bc^2d^2}{5}\right)+x^7\cdot\left(\frac{6ac^2d^2}{7}+\frac{4bc^3d}{7}\right)+x^4\left(ac^3d+\frac{bc^4}{4}\right)$$

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a
*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4
*(a*c**3*d + b*c**4/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx=\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\,\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\,\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\,\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\,\left(bc^4+4\,ac^3d\right)x^4$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*
d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*
c^3*d)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx = \frac{1}{16}\,bd^4x^{16} + \frac{4}{13}\,bcd^3x^{13} + \frac{1}{13}\,ad^4x^{13} + \frac{3}{5}\,bc^2d^2x^{10} + \frac{2}{5}\,acd^3x^{10} + \frac{4}{7}\,bc^3dx^7 + \frac{6}{7}\,ac^2d^2x^7 + \frac{1}{4}\,bc^4x^4 + ac^3dx^4 + ac^4x$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
+ 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 +
a*c^3*d*x^4 + a*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx = x^4\left(\frac{b\,c^4}{4}+a\,d\,c^3\right) + x^{13}\left(\frac{a\,d^4}{13}+\frac{4\,b\,c\,d^3}{13}\right) + \frac{b\,d^4\,x^{16}}{16} + a\,c^4\,x + \frac{2\,c^2\,d\,x^7\,(3\,a\,d+2\,b\,c)}{7} + \frac{c\,d^2\,x^{10}\,(2\,a\,d+3\,b\,c)}{5}$$

```
[In] int((a + b*x^3)*(c + d*x^3)^4,x)
```

```
[Out] x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^1
6)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d + 3*
b*c))/5
```

3.2 $\int (a+bx^3)(c+dx^3)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^3)(c+dx^3)^3\,dx = ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7 +\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}$$

```
[Out] a*c^3*x+1/4*c^2*(3*a*d+b*c)*x^4+3/7*c*d*(a*d+b*c)*x^7+1/10*d^2*(a*d+3*b*c)*
x^10+1/13*b*d^3*x^13
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)(c+dx^3)^3\,dx = \frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc) +\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

```
[In] Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^3 + c^2(bc+3ad)x^3 + 3cd(bc+ad)x^6 + d^2(3bc+ad)x^9 + bd^3x^{12}\right)\,dx \\ &= ac^3x + \frac{1}{4}c^2(bc+3ad)x^4 + \frac{3}{7}cd(bc+ad)x^7 + \frac{1}{10}d^2(3bc+ad)x^{10} + \frac{1}{13}bd^3x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx = ac^3x + \frac{1}{4}c^2(bc+3ad)x^4 + \frac{3}{7}cd(bc+ad)x^7 + \frac{1}{10}d^2(3bc+ad)x^{10} + \frac{1}{13}bd^3x^{13}$$

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
norman	$\frac{b\,d^3x^{13}}{13} + \left(\frac{1}{10}a\,d^3 + \frac{3}{10}bc\,d^2\right)x^{10} + \left(\frac{3}{7}ac\,d^2 + \frac{3}{7}b\,c^2d\right)x^7 + \left(\frac{3}{4}a\,c^2d + \frac{1}{4}c^3b\right)x^4 + a\,c^3x$	72
default	$\frac{b\,d^3x^{13}}{13} + \frac{(a\,d^3+3bc\,d^2)x^{10}}{10} + \frac{(3ac\,d^2+3b\,c^2d)x^7}{7} + \frac{(3a\,c^2d+c^3b)x^4}{4} + a\,c^3x$	73
gosper	$\frac{1}{13}b\,d^3x^{13} + \frac{1}{10}x^{10}a\,d^3 + \frac{3}{10}x^{10}bc\,d^2 + \frac{3}{7}x^7ac\,d^2 + \frac{3}{7}x^7b\,c^2d + \frac{3}{4}x^4a\,c^2d + \frac{1}{4}x^4c^3b + a\,c^3x$	75
risch	$\frac{1}{13}b\,d^3x^{13} + \frac{1}{10}x^{10}a\,d^3 + \frac{3}{10}x^{10}bc\,d^2 + \frac{3}{7}x^7ac\,d^2 + \frac{3}{7}x^7b\,c^2d + \frac{3}{4}x^4a\,c^2d + \frac{1}{4}x^4c^3b + a\,c^3x$	75
parallelrisch	$\frac{1}{13}b\,d^3x^{13} + \frac{1}{10}x^{10}a\,d^3 + \frac{3}{10}x^{10}bc\,d^2 + \frac{3}{7}x^7ac\,d^2 + \frac{3}{7}x^7b\,c^2d + \frac{3}{4}x^4a\,c^2d + \frac{1}{4}x^4c^3b + a\,c^3x$	75

```
[In] int((b*x^3+a)*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^3*x^13+(1/10*a*d^3+3/10*b*c*d^2)*x^10+(3/7*a*c*d^2+3/7*b*c^2*d)*x^
7+(3/4*a*c^2*d+1/4*c^3*b)*x^4+a*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^3 dx = \frac{1}{13} bd^3x^{13} + \frac{1}{10} (3 bcd^2 + ad^3)x^{10} + \frac{3}{7} (bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4} (bc^3 + 3 ac^2d)x^4$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int (a+bx^3) (c+dx^3)^3 dx = ac^3x + \frac{bd^3x^{13}}{13} + x^{10}\left(\frac{ad^3}{10} + \frac{3bcd^2}{10}\right) + x^7 \cdot \left(\frac{3acd^2}{7} + \frac{3bc^2d}{7}\right) + x^4 \cdot \left(\frac{3ac^2d}{4} + \frac{bc^3}{4}\right)$$

```
[In] integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a*
c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^3 dx = \frac{1}{13} bd^3x^{13} + \frac{1}{10} (3 bcd^2 + ad^3)x^{10} + \frac{3}{7} (bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4} (bc^3 + 3 ac^2d)x^4$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx=\frac{1}{13}\,bd^3x^{13}+\frac{3}{10}\,bcd^2x^{10}+\frac{1}{10}\,ad^3x^{10}+\frac{3}{7}\,bc^2dx^7+\frac{3}{7}\,acd^2x^7+\frac{1}{4}\,bc^3x^4+\frac{3}{4}\,ac^2dx^4+ac^3x$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 + 3
/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx=x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right)+x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right)+\frac{b\,d^3\,x^{13}}{13}+a\,c^3\,x+\frac{3\,c\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

```
[In] int((a + b*x^3)*(c + d*x^3)^3,x)
```

```
[Out] x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*d
^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

3.3 $\int (a+bx^3)(c+dx^3)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)(c+dx^3)^2\,dx = ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}$$

```
[Out] a*c^2*x+1/4*c*(2*a*d+b*c)*x^4+1/7*d*(a*d+2*b*c)*x^7+1/10*b*d^2*x^10
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)(c+dx^3)^2\,dx = \frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

```
[In] Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^2+c(bc+2ad)x^3+d(2bc+ad)x^6+bd^2x^9\right)dx\\ &= ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)^2\,dx = ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}$$

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^2x^{10}}{10}+\frac{(a\,d^2+2bcd)x^7}{7}+\frac{(2acd+b\,c^2)x^4}{4}+a\,c^2x$	49
norman	$\frac{b\,d^2x^{10}}{10}+\left(\frac{1}{7}a\,d^2+\frac{2}{7}bcd\right)x^7+\left(\frac{1}{2}acd+\frac{1}{4}b\,c^2\right)x^4+a\,c^2x$	49
gosper	$\frac{1}{10}b\,d^2x^{10}+\frac{1}{7}x^7a\,d^2+\frac{2}{7}x^7bcd+\frac{1}{2}x^4acd+\frac{1}{4}x^4b\,c^2+a\,c^2x$	51
risch	$\frac{1}{10}b\,d^2x^{10}+\frac{1}{7}x^7a\,d^2+\frac{2}{7}x^7bcd+\frac{1}{2}x^4acd+\frac{1}{4}x^4b\,c^2+a\,c^2x$	51
parallelrisch	$\frac{1}{10}b\,d^2x^{10}+\frac{1}{7}x^7a\,d^2+\frac{2}{7}x^7bcd+\frac{1}{2}x^4acd+\frac{1}{4}x^4b\,c^2+a\,c^2x$	51

```
[In] int((b*x^3+a)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3)(c+dx^3)^2\,dx = \frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\,(2\,bcd+ad^2)x^7+\frac{1}{4}\,(bc^2+2\,acd)x^4+ac^2x$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx = ac^2x+\frac{bd^2x^{10}}{10}+x^7\left(\frac{ad^2}{7}+\frac{2bcd}{7}\right)+x^4\left(\frac{acd}{2}+\frac{bc^2}{4}\right)$$

```
[In] integrate((b*x**3+a)*(d*x**3+c)**2,x)
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
b*c**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx = \frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\,(2\,bcd+ad^2)x^7+\frac{1}{4}\,(bc^2+2\,acd)x^4+ac^2x$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx = \frac{1}{10}\,bd^2x^{10}+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,ad^2x^7+\frac{1}{4}\,bc^2x^4+\frac{1}{2}\,acdx^4+ac^2x$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*d
*x^4 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx = x^4\left(\frac{b\,c^2}{4}+\frac{a\,d\,c}{2}\right)+x^7\left(\frac{a\,d^2}{7}+\frac{2\,b\,c\,d}{7}\right)+\frac{b\,d^2\,x^{10}}{10}+a\,c^2\,x$$

```
[In] int((a + b*x^3)*(c + d*x^3)^2,x)
[Out] x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)/
10 + a*c^2*x
```

3.4 $\int (a+bx^3)(c+dx^3)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^3)(c+dx^3)\,dx = acx + \frac{1}{4}(bc+ad)x^4 + \frac{1}{7}bdx^7$$

```
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\int (a+bx^3)(c+dx^3)\,dx = \frac{1}{4}x^4(ad+bc) + acx + \frac{1}{7}bdx^7$$

```
[In] Int[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ac+(bc+ad)x^3+bdx^6\right)\,dx \\ &= acx + \frac{1}{4}(bc+ad)x^4 + \frac{1}{7}bdx^7 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right) \left(c+dx^3\right)\, dx = acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7$$

```
[In] Integrate[(a + b*x^3)*(c + d*x^3),x]
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^4}{4}+\frac{bd\,x^7}{7}$	25
norman	$\frac{bd\,x^7}{7}+\left(\frac{ad}{4}+\frac{bc}{4}\right)x^4+acx$	26
gosper	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27
risch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27
parallelrisch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}x^4bc+acx$	27

```
[In] int((b*x^3+a)*(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right) \left(c+dx^3\right)\, dx = \frac{1}{7}\,bdx^7+\frac{1}{4}\,(bc+ad)x^4+acx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx=acx+\frac{bdx^7}{7}+x^4\left(\frac{ad}{4}+\frac{bc}{4}\right)$$

```
[In] integrate((b*x**3+a)*(d*x**3+c),x)
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx=\frac{1}{7}\,bdx^7+\frac{1}{4}\,(bc+ad)x^4+acx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx=\frac{1}{7}\,bdx^7+\frac{1}{4}\,bcx^4+\frac{1}{4}\,adx^4+acx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx=\frac{b\,d\,x^7}{7}+\left(\frac{a\,d}{4}+\frac{b\,c}{4}\right)\,x^4+a\,c\,x$$

```
[In] int((a + b*x^3)*(c + d*x^3),x)
[Out] x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

3.5 $\int \frac{a+bx^3}{c+dx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{a+bx^3}{c+dx^3} dx = \frac{bx}{d} + \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}$$

```
[Out] b*x/d-1/3*(-a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+1/6*(-a*d+b*c)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+1/3*(-a*d+b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+bx^3}{c+dx^3} dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{bx}{d}$$

```
[In] Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
```

```
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^3}\,dx}{d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d} - \frac{(bc-ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{c}d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}d^{4/3}}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3}\,dx$$

$$= \frac{6bc^{2/3}\sqrt[3]{d}x + 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) + (bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\right.}{6c^{2/3}d^{4/3}}$$

```
[In] Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{3}+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3d^{2}}$	42
default	$\frac{bx}{d}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)(ad-bc)}{d}$	110

```
[In] int((b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*x/d+1/3/d^2*sum((a*d-b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.56

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \left[\frac{6\,bc^2dx-3\,\sqrt{\frac{1}{3}}(bc^2d-acd^2)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+(c}{6\,c^2d^2}\right.$$

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)*
log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*
d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^2*d
```

```
)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(c
^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*b*c^2
*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (c^2*
d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t\log\left(\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

```
[In] integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2
*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} - \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d
)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(c/d
)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} + \frac{bx}{d} + \frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)
)/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^
(1/3)))/(c*d)

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{b\,x}{d} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)}{3\,c^{2/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,c^{2/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)}{3\,c^{2/3}\,d^{4/3}}$$

[In] int((a + b*x^3)/(c + d*x^3),x)

[Out] (b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))

3.6 $\int \frac{a+bx^3}{(c+dx^3)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{a+bx^3}{(c+dx^3)^2} \, dx = -\frac{(bc-ad)x}{3cd(c+dx^3)} - \frac{(bc+2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)+1/9*(2*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/
3)/d^(4/3)-1/18*(2*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/
3)/d^(4/3)-1/9*(2*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2)
)/c^(5/3)/d^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {393, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(2ad+bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} - \frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}} + \frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

```
[In] Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d
)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x}{3cd\,(c+dx^3)}+\frac{(bc+2ad)\int\frac{1}{c+dx^3}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)}+\frac{(bc+2ad)\int\frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}d}+\frac{(bc+2ad)\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\int\frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{18c^{5/3}d^{4/3}}+\frac{(bc+2ad)\int\frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{4/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}} + \frac{(bc+2ad)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3c^{5/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{3cd\,(c+dx^3)} - \frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$

$$= \frac{-\frac{6c^{2/3}\sqrt[3]{d}(bc-ad)x}{c+dx^3} - 2\sqrt{3}(bc+2ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) - (bc+2ad)\log\left(c^2\right.}{18c^{5/3}d^{4/3}}$$

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
(18*c^(5/3)*d^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(ad-bc)x}{3dc(d\,x^3+c)} + \frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)} \frac{(2ad+bc)\ln(x-\_R)}{\_R^2}}{9c\,d^2}$	65
default	$\frac{(ad-bc)x}{3dc(d\,x^3+c)} + \frac{(2ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3cd}$	134

```
[In] int((b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(a*d-b*c)/c*x/(d*x^3+c)+1/9/c/d^2*sum((2*a*d+b*c)/_R^2*ln(x-_R),_R=Ro
otOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(bc^3d+2\,ac^2d^2+(bc^2d^2+2\,acd^3)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)}{dx^3+c}\right.}{}\right.$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sq
rt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/
3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/
(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(
c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b
*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c
^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^
2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d + 2
*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x
+ (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2
/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^3*d^3*x^3 +
c^4*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{x(ad-bc)}{3c^2d+3cd^2x^3} + \operatorname{RootSum}\left(729t^3c^5d^4 - 8a^3d^3 - 12a^2bcd^2 - 6ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

```
[In] integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8*
a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*l
og(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x}{3\,(cd^2x^3+c^2d)} + \frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b*c
 + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1/9*
(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d} - \frac{bcx-adx}{3\,(dx^3+c)cd}$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-
c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(x -
 (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{x\,\left(a\,d-b\,c\right)}{3\,c\,d\,\left(d\,x^3+c\right)}$$

```
[In] int((a + b*x^3)/(c + d*x^3)^2,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2)
*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c))/
(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))/(3*
c*d*(c + d*x^3))
```

3.7 $\int \frac{a+bx^3}{\left(c+dx^3\right)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 197

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3} \, dx = -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}-\frac{(bc+5ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}$$

```
[Out] -1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)+1/27*(
5*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(4/3)-1/54*(5*a*d+b*c)*ln(c^(2/3
)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(4/3)-1/27*(5*a*d+b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {393, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3} \, dx = -\frac{(5ad+bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\left(c+dx^3\right)}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

```
[In] Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x)/(c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c +
 d*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(2
7*c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/(54*c^(8/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2}+\frac{(bc+5ad)\int \frac{1}{(c+dx^3)^2}\,dx}{6cd}\\
&= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2}+\frac{(bc+5ad)x}{18c^2d\,(c+dx^3)}+\frac{(bc+5ad)\int \frac{1}{c+dx^3}\,dx}{9c^2d}\\
&= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2}+\frac{(bc+5ad)x}{18c^2d\,(c+dx^3)}+\frac{(bc+5ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{27c^{8/3}d}\\
&\quad+\frac{(bc+5ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{27c^{8/3}d}\\
&= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2}+\frac{(bc+5ad)x}{18c^2d\,(c+dx^3)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}\\
&\quad-\frac{(bc+5ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{54c^{8/3}d^{4/3}}+\frac{(bc+5ad)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{18c^{7/3}d}\\
&= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2}+\frac{(bc+5ad)x}{18c^2d\,(c+dx^3)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}\\
&\quad-\frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}\\
&\quad+\frac{(bc+5ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{9c^{8/3}d^{4/3}}
\end{aligned}
$$

$$= -\frac{(bc-ad)x}{6cd\,(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d\,(c+dx^3)} - \frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx$$

$$= \frac{-\frac{9c^{5/3}\sqrt[3]{d}(bc-ad)x}{(c+dx^3)^2} + \frac{3c^{2/3}\sqrt[3]{d}(bc+5ad)x}{c+dx^3} - 2\sqrt{3}(bc+5ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{54c^{8/3}d^{4/3}}$$

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c
 + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x
)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a
*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(dx^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(5ad+bc)\ln(x-\_R)}{\_R^2}}{27c^2d^2}$	84
default	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(dx^3+c)^2}+\frac{(5ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{9c^2d}$	153

```
[In] int((b*x^3+a)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+1/27/c^2/d^2*s
um((5*a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(156) = 312$.

Time = 0.32 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$$

$$= \left[\frac{3\,(bc^3d^2+5\,ac^2d^3)x^4+3\,\sqrt{\frac{1}{3}}((bc^2d^3+5\,acd^4)x^6+bc^4d+5\,ac^3d^2+2\,(bc^3d^2+5\,ac^2d^3)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\right.}{}\right.$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^
4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^
2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c
*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3
+ c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2
*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3
)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(
c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x
^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(
b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*
a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)
*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*
x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*
d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^
3*x^3 + c^6*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5ad^2+bcd)+x(8acd-2bc^2)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6} + \operatorname{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3, \left(t\mapsto t\log\left(\frac{27tc^3d}{5ad+bc}+x\right)\right)\right)$$

```
[In] integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d**
2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d**3
 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27*_t
*c**3*d/(5*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{(bcd+5\,ad^2)x^4-2\,(bc^2-4\,acd)x}{18\,(c^2d^3x^6+2\,c^3d^2x^3+c^4d)} + \frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+5\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+5\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*d
^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)*log
(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^2\right)^{\frac{2}{3}}c^2} - \frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^2\right)^{\frac{2}{3}}c^2} - \frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d} + \frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\left(dx^3+c\right)^2c^2d}$$

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3) +
 (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*log(a
bs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x +
 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{\frac{x^4\,(5\,a\,d+b\,c)}{18\,c^2}+\frac{x\,(4\,a\,d-b\,c)}{9\,c\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}$$

```
[In] int((a + b*x^3)/(c + d*x^3)^3,x)
```

```
[Out] ((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6 +
 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
 - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

3.8 $\int (a+bx^3)^2 (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{1}{4}ac^2(2bc+3ad)x^4 + \frac{1}{7}c(b^2c^2+6abcd+3a^2d^2)\,x^7 + \frac{1}{10}d(3b^2c^2+6abcd+a^2d^2)\,x^{10} + \frac{1}{13}bd^2(3bc+2ad)x^{13} + \frac{1}{16}b^2d^3x^{16}$$

```
[Out] a^2*c^3*x+1/4*a*c^2*(3*a*d+2*b*c)*x^4+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^7+1/10*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^10+1/13*b*d^2*(2*a*d+3*b*c)*x^13+
1/16*b^2*d^3*x^16
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = \frac{1}{10}dx^{10}(a^2d^2+6abcd+3b^2c^2) + \frac{1}{7}cx^7(3a^2d^2+6abcd+b^2c^2) + a^2c^3x + \frac{1}{4}ac^2x^4(3ad+2bc) + \frac{1}{13}bd^2x^{13}(2ad+3bc) + \frac{1}{16}b^2d^3x^{16}$$

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2c^3+ac^2(2bc+3ad)x^3+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6 \right.\\ &\qquad \left. +d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+bd^2(3bc+2ad)x^{12}+b^2d^3x^{15}\right)\,dx\\ &= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7\\ &\quad +\frac{1}{10}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{10}+\frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{16}b^2d^3x^{16}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^3\,dx &= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7\\ &\quad +\frac{1}{10}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{10}\\ &\quad +\frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{16}b^2d^3x^{16}\end{aligned}$$

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x+\left(\frac{3}{4}a^2c^2d+\frac{1}{2}ab\,c^3\right)x^4+\left(\frac{3}{7}c\,a^2d^2+\frac{6}{7}ab\,c^2d+\frac{1}{7}b^2c^3\right)x^7+\left(\frac{1}{10}a^2d^3+\frac{3}{5}abc\,d^2+\frac{3}{10}b^2c^2d\right.$
default	$\frac{b^2d^3x^{16}}{16}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{10}}{10}+\frac{\left(3c\,a^2d^2+6ab\,c^2d+b^2c^3\right)x^7}{7}+\frac{\left(3a^2c^2d+2ab\,c^3\right)x^4}{4}+a$
gosper	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$
risch	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$
parallelrisch	$a^2c^3x+\frac{3}{4}x^4a^2c^2d+\frac{1}{2}x^4ab\,c^3+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7ab\,c^2d+\frac{1}{7}x^7b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$

```
[In] int((b*x^3+a)^2*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^3*x+(3/4*a^2*c^2*d+1/2*a*b*c^3)*x^4+(3/7*c*a^2*d^2+6/7*a*b*c^2*d+1/7*
b^2*c^3)*x^7+(1/10*a^2*d^3+3/5*a*b*c*d^2+3/10*b^2*c^2*d)*x^10+(2/13*a*b*d^3
+3/13*b^2*c*d^2)*x^13+1/16*b^2*d^3*x^16
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{1}{16}\, b^2 d^3 x^{16} + \frac{1}{13}\left(3\, b^2 c d^2 + 2\, abd^3\right)x^{13} + \frac{1}{10}\left(3\, b^2 c^2 d + 6\, abcd^2 + a^2 d^3\right)x^{10} + \frac{1}{7}\left(b^2 c^3 + 6\, abc^2 d + 3\, a^2 c d^2\right)x^7 + a^2 c^3 x + \frac{1}{4}\left(2\, abc^3 + 3\, a^2 c^2 d\right)x^4$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = a^2 c^3 x + \frac{b^2 d^3 x^{16}}{16} + x^{13} \cdot \left(\frac{2abd^3}{13} + \frac{3b^2 cd^2}{13}\right) + x^{10}\left(\frac{a^2 d^3}{10} + \frac{3abcd^2}{5} + \frac{3b^2 c^2 d}{10}\right) + x^7 \cdot \left(\frac{3a^2 cd^2}{7} + \frac{6abc^2 d}{7} + \frac{b^2 c^3}{7}\right) + x^4 \cdot \left(\frac{3a^2 c^2 d}{4} + \frac{abc^3}{2}\right)$$

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{1}{16}\,b^2d^3x^{16} + \frac{1}{13}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{13} + \frac{1}{10}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{10} + \frac{1}{7}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^7 + a^2c^3x + \frac{1}{4}\left(2\,abc^3 + 3\,a^2c^2d\right)x^4$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{1}{16}\,b^2d^3x^{16} + \frac{3}{13}\,b^2cd^2x^{13} + \frac{2}{13}\,abd^3x^{13} + \frac{3}{10}\,b^2c^2dx^{10} + \frac{3}{5}\,abcd^2x^{10} + \frac{1}{10}\,a^2d^3x^{10} + \frac{1}{7}\,b^2c^3x^7 + \frac{6}{7}\,abc^2dx^7 + \frac{3}{7}\,a^2cd^2x^7 + \frac{1}{2}\,abc^3x^4 + \frac{3}{4}\,a^2c^2dx^4 + a^2c^3x$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2*
d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a*b
*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*
c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = x^7 \left(\frac{3 a^2 c d^2}{7}+\frac{6 a b c^2 d}{7}+\frac{b^2 c^3}{7}\right) + x^{10} \left(\frac{a^2 d^3}{10}+\frac{3 a b c d^2}{5}+\frac{3 b^2 c^2 d}{10}\right) + a^2 c^3 x + \frac{b^2 d^3 x^{16}}{16} + \frac{a c^2 x^4 (3 a d+2 b c)}{4} + \frac{b d^2 x^{13} (2 a d+3 b c)}{13}$$

```
[In] int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (a*
c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

3.9 $\int (a+bx^3)^2 (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

```
[Out] a^2*c^2*x+1/2*a*c*(a*d+b*c)*x^4+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+1/5*b*d
*(a*d+b*c)*x^10+1/13*b^2*d^2*x^13
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = \frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right) + a^2c^2x + \frac{1}{5}bdx^{10}(ad+bc) + \frac{1}{2}acx^4(ad+bc) + \frac{1}{13}b^2d^2x^{13}$$

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2c^2 + 2ac(bc+ad)x^3 + \left(b^2c^2 + 4abcd + a^2d^2\right)x^6 + 2bd(bc+ad)x^9 + b^2d^2x^{12}\right)\,dx \\ &= a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}\left(b^2c^2 + 4abcd + a^2d^2\right)x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2\,dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}\left(b^2c^2 + 4abcd + a^2d^2\right)x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^2d^2x^{13}}{13} + \left(\frac{1}{5}ab\,d^2 + \frac{1}{5}b^2cd\right)x^{10} + \left(\frac{1}{7}a^2d^2 + \frac{4}{7}abcd + \frac{1}{7}b^2c^2\right)x^7 + \left(\frac{1}{2}a^2cd + \frac{1}{2}b\,c^2a\right)x^4 + a^2c^2x$
default	$\frac{b^2d^2x^{13}}{13} + \frac{(2ab\,d^2+2b^2cd)x^{10}}{10} + \frac{(a^2d^2+4abcd+b^2c^2)x^7}{7} + \frac{(2a^2cd+2b\,c^2a)x^4}{4} + a^2c^2x$
gosper	$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}x^{10}ab\,d^2 + \frac{1}{5}x^{10}b^2cd + \frac{1}{7}x^7a^2d^2 + \frac{4}{7}x^7abcd + \frac{1}{7}x^7b^2c^2 + \frac{1}{2}x^4a^2cd + \frac{1}{2}x^4b\,c^2a +$
risch	$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}x^{10}ab\,d^2 + \frac{1}{5}x^{10}b^2cd + \frac{1}{7}x^7a^2d^2 + \frac{4}{7}x^7abcd + \frac{1}{7}x^7b^2c^2 + \frac{1}{2}x^4a^2cd + \frac{1}{2}x^4b\,c^2a +$
parallelrisch	$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}x^{10}ab\,d^2 + \frac{1}{5}x^{10}b^2cd + \frac{1}{7}x^7a^2d^2 + \frac{4}{7}x^7abcd + \frac{1}{7}x^7b^2c^2 + \frac{1}{2}x^4a^2cd + \frac{1}{2}x^4b\,c^2a +$

```
[In] int((b*x^3+a)^2*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^2*x^13+(1/5*a*b*d^2+1/5*b^2*c*d)*x^10+(1/7*a^2*d^2+4/7*a*b*c*d+1
/7*b^2*c^2)*x^7+(1/2*a^2*c*d+1/2*b*c^2*a)*x^4+a^2*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} \left(b^2 cd + abd^2\right) x^{10} + \frac{1}{7} \left(b^2 c^2 + 4 \, abcd + a^2 d^2\right) x^7 + a^2 c^2 x + \frac{1}{2} \left(abc^2 + a^2 cd\right) x^4$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + x^{10} \left(\frac{abd^2}{5} + \frac{b^2 cd}{5}\right) + x^7 \left(\frac{a^2 d^2}{7} + \frac{4abcd}{7} + \frac{b^2 c^2}{7}\right) + x^4 \left(\frac{a^2 cd}{2} + \frac{abc^2}{2}\right)$$

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*(
a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} \left(b^2 cd + abd^2\right) x^{10} + \frac{1}{7} \left(b^2 c^2 + 4 \, abcd + a^2 d^2\right) x^7 + a^2 c^2 x + \frac{1}{2} \left(abc^2 + a^2 cd\right) x^4$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = \frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} b^2 c d x^{10} + \frac{1}{5} a b d^2 x^{10} + \frac{1}{7} b^2 c^2 x^7 + \frac{4}{7} a b c d x^7 + \frac{1}{7} a^2 d^2 x^7 + \frac{1}{2} a b c^2 x^4 + \frac{1}{2} a^2 c d x^4 + a^2 c^2 x$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7 +
 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + a^
2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = x^7 \left(\frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7}\right) + a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + \frac{a c x^4 (a d + b c)}{2} + \frac{b d x^{10} (a d + b c)}{5}$$

```
[In] int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^13
)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

3.10 $\int (a+bx^3)^2 (c+dx^3) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)^2 (c+dx^3) \, dx = a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10}$$

```
[Out] a^2*c*x+1/4*a*(a*d+2*b*c)*x^4+1/7*b*(2*a*d+b*c)*x^7+1/10*b^2*d*x^10
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)^2 (c+dx^3) \, dx = a^2cx + \frac{1}{7}bx^7(2ad+bc) + \frac{1}{4}ax^4(ad+2bc) + \frac{1}{10}b^2dx^{10}$$

```
[In] Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2c + a(2bc+ad)x^3 + b(bc+2ad)x^6 + b^2dx^9\right) dx \\ &= a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx=a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}$$

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^{10}}{10}+\frac{(2abd+b^2c)x^7}{7}+\frac{(a^2d+2abc)x^4}{4}+a^2cx$	49
norman	$\frac{b^2 d\,x^{10}}{10}+\left(\frac{2}{7}abd+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{4}a^2d+\frac{1}{2}abc\right)x^4+a^2cx$	49
gosper	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51
risch	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51
parallelrisch	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+a^2cx$	51

```
[In] int((b*x^3+a)^2*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx=\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,\left(b^2c+2\,abd\right)x^7+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4+a^2cx$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = a^2cx + \frac{b^2dx^{10}}{10} + x^7\cdot\left(\frac{2abd}{7}+\frac{b^2c}{7}\right) + x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

```
[In] integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 +
 a*b*c/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\left(b^2c+2\,abd\right)x^7 + \frac{1}{4}\left(2\,abc+a^2d\right)x^4 + a^2cx$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,b^2cx^7 + \frac{2}{7}\,abdx^7 + \frac{1}{2}\,abcx^4 + \frac{1}{4}\,a^2dx^4 + a^2cx$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d
*x^4 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{b^2\,d\,x^{10}}{10}+a^2\,c\,x$$

```
[In] int((a + b*x^3)^2*(c + d*x^3),x)
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)/
10 + a^2*c*x
```

3.11 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(a+bx^3)^2}{c+dx^3} \, dx = -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d+1/3*(-a*d+b*c)^2*ln(c^(1/3)+d^(1/3)*x)/
c^(2/3)/d^(7/3)-1/6*(-a*d+b*c)^2*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
c^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3
^(1/2))/c^(2/3)/d^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$\int \frac{(a+bx^3)^2}{c+dx^3} \, dx = -\frac{(bc-ad)^2 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^4}{4d}$$

```
[In] Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^3}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^3\right)}\right) dx$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2 \int \frac{1}{c+dx^3}\, dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2 \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\, dx}{3c^{2/3}d^2} + \frac{(bc-ad)^2 \int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\, dx}{3c^{2/3}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\, dx}{6c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\, dx}{2\sqrt[3]{c}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}d^{7/3}}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{-12bc^{2/3}\sqrt[3]{d}(bc-2ad)x+3b^2c^{2/3}d^{4/3}x^4+4\sqrt{3}(bc-ad)^2\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\right.}{12c^{2/3}d^{7/3}}$$

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqrt
[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*(b
*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{b^2x^4}{4d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^2}}{3d^3}$	78
default	$\frac{b(\frac{1}{4}bd\,x^4+2adx-bcx)}{d^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)(a^2d^2-2abcd+b^2c^2)}{d^2}$	140

```
[In] int((b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*c*x+1/3/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2)
/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.92

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$= \left[\frac{3\,b^2c^2d^2x^4 + 6\,\sqrt{\frac{1}{3}}(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3 - 3\,(c^2d)^{\frac{1}{3}}cx - c^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2 + (c^2d)^{\frac{2}{3}}x - (c^2d)^{\frac{1}{3}}c\right)}{dx^3+c}\right.}{} \right.$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*s
qrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3
)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*
d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^
2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d - 2*a*b*
c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3
)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3))
 - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2x^4}{4d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right)$$
$$+\mathrm{RootSum}\left(27t^3c^2d^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto t\right.\right.$$

```
[In] integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d*
*3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*lo
g(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2dx^4-4\,(b^2c-2\,abd)x}{4\,d^2} + \frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
[Out] 1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(c/
d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,\left(-cd^2\right)^{\frac{2}{3}}\,d} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,\left(-cd^2\right)^{\frac{2}{3}}\,d} - \frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^4} + \frac{b^2d^3x^4-4\,b^2cd^2x+8\,abd^3x}{4\,d^4}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/
(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2\,x^4}{4\,d}-x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,c^{2/3}\,d^{7/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^2}{c^{2/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,c^{2/3}\,d^{7/3}}$$

```
[In] int((a + b*x^3)^2/(c + d*x^3),x)
```

```
[Out] (b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*(
a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x -
 c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log(3^
(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*c^(2/3)*d^(7/3))
```

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2} dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}$$

```
[Out] b^2*x/d^2+1/3*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)-2/9*(-a*d+b*c)*(a*d+2*b*c)*ln(
c^(1/3)+d^(1/3)*x)/c^(5/3)/d^(7/3)+1/9*(-a*d+b*c)*(a*d+2*b*c)*ln(c^(2/3)-c^
(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(7/3)+2/9*(-a*d+b*c)*(a*d+2*b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/d^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used

$= \{398, 393, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{2(bc-ad)(ad+2bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} + \frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}} - \frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{x(bc-ad)^2}{3cd^2(c+dx^3)} + \frac{b^2x}{d^2}$$

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{d^2\left(c+dx^3\right)^2}\right)dx\\
&= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(c+dx^3\right)^2}\,dx}{d^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{c+dx^3}\,dx}{3cd^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}d^2}\\
&\quad -\frac{(2(bc-ad)(2bc+ad))\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}d^2}
\end{aligned}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{((bc-ad)(2bc+ad))\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}d^{7/3}} - \frac{((bc-ad)(2bc+ad))\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{4/3}d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}} - \frac{(2(bc-ad)(2bc+ad))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3c^{5/3}d^{7/3}}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$$

$$= \frac{9b^2\sqrt[3]{d}x + \frac{3\sqrt[3]{d}(bc-ad)^2x}{c(c+dx^3)} + \frac{2\sqrt{3}(2b^2c^2-abcd-a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} - \frac{2(2b^2c^2-abcd-a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}} + \frac{(2b^2c^2-abcd}{}}{9d^{7/3}}$$

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3]
*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]
])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/c
^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

method	result	siz
risch	$\frac{b^2 x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3c\,d^2(d\,x^3+c)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^2d^2+abcd-2b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{9d^3c}$	99
default	$\frac{b^2 x}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{3c\left(d\,x^3+c\right)}+\frac{2\left(a^2d^2+abcd-2b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}}{d^2}$	16

[In] int((b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)

[Out] b^2*x/d^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^3+c)+2/9/d^3/c*sum((a^2*d^2+a*b*c*d-2*b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(164) = 328.

Time = 0.34 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.80

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \left[\frac{9\,b^2c^3d^2x^4-3\sqrt{\frac{1}{3}}(2\,b^2c^4d-abc^3d^2-a^2c^2d^3+(2\,b^2c^3d^2-abc^2d^3-a^2cd^4)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2 \cdots}{\cdots}\right.}{\cdots}\right.$$

[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")

[Out] [1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)

```
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (2*b^
2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3
 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)
^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt(1/3)*(2*
b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*
c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^
2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2*d - a^2*c*d
^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (
c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 +
(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^
4*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{3c^2d^2+3cd^3x^3}$$
$$+\mathrm{RootSum}\Big(729t^3c^5d^7-8a^6d^6-24a^5bcd^5+24a^4b^2c^2d^4+88a^3b^3c^3d^3-48a^2b^4c^4d^2-96ab^5c^5d+64b^6c^6,$$

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**3
*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 24*
a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 - 96*
a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2
 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(cd^3x^3+c^2d^2)} + \frac{b^2x}{d^2} - \frac{2\,\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 - 2
/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^
2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*b^2*
c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{2\,\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(-cd^2)^{\frac{2}{3}}\,cd} + \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(dx^3+c)cd^2}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)
*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)
*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2\,x}{d^2} + \frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c\,\left(d^3\,x^3+c\,d^2\right)} + \frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}} + \frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}} - \frac{2\,\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}$$

```
[In] int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3)) +
 (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3))
 + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2*b*c)
)/(9*c^(5/3)*d^(7/3))
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 258

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} - \frac{(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+4*b*c)*x
/c^2/d^2/(d*x^3+c)+1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(1/3)+d^(1/3)*
x)/c^(8/3)/d^(7/3)-1/54*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(2/3)-c^(1/3)*
d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(7/3)-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)
*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {424, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(5a^2d^2+2abcd+2b^2c^2)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} - \frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}} + \frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{x(bc-ad)(5ad+4bc)}{18c^2d^2(c+dx^3)} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x*(a + b*x^3))/(c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c
+ 5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)
*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7
/3)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c
^(8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} + \frac{\int \frac{a(bc+5ad)+2b(2bc+ad)x^3}{(c+dx^3)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{1}{c+dx^3}\,dx}{9c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{27c^{8/3}d^2} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{27c^{8/3}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{54c^{8/3}d^{7/3}} + \frac{(2b^2c^2+2abcd+5a^2d^2)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{18c^{7/3}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}} + \frac{(2b^2c^2+2abcd+5a^2d^2)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{9c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x(a+bx^3)}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\,(c+dx^3)} - \frac{(2b^2c^2+2abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$$

$$= \frac{-\frac{3c^{2/3}\sqrt[3]{d}(bc-ad)x(ad(8c+5dx^3)+bc(4c+7dx^3))}{(c+dx^3)^2} - 2\sqrt{3}(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(2b^2c^2+2a}{54c^{8/3}d^{7/3}}$$

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*(b*c - a*d)*x*(a*d*(8*c + 5*d*x^3) + b*c*(4*c + 7*d*x^
3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTan[(
1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)
*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.89 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{(d\,x^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{27c^2d^3}$
default	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{(d\,x^3+c)^2}+\frac{(5a^2d^2+2abcd+2b^2c^2)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\sqrt{}\right.}{3d}\right.}{9c^2d^2}$

```
[In] int((b*x^3+a)^2/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*b*c*d-b^2*
c^2)/c/d^2*x)/(d*x^3+c)^2+1/27/c^2/d^3*sum((5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)/
_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(217) = 434.

Time = 0.32 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.14

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3)
*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*
d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*
x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*
sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/
3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^
2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^
2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c
) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^
3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^
2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3*(7*b^2*c^4*
d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*c^5*d + 2*a*b
*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5*a^2*c*d^5)*x^
6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*sqrt((c^2*d)^(1/
3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(
1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3
)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
(2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5
*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(
2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^2*c^3*d
^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3)]
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5a^2d^3+2abcd^2-7b^2c^2d)+x(8a^2cd^2-4abc^2d-4b^2c^3)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}$$
$$+\text{RootSum}\Big(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210a^4b^2c^2d^4-128a^3b^3c^3d^3-84a^2b^4c^4d^2-24ab^5c^5d-$$

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*a
*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**4*
x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**5 -
 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2
 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**
2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3)x^4+4\,(b^2c^3+abc^2d-2\,a^2cd^2)x}{18\,(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2)} + \frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*d
 - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)*(
2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2
)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*(2*b^
2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^2} - \frac{7\,b^2c^2dx^4-2\,abcd^2x^4-5\,a^2d^3x^4+4\,b^2c^3x+4\,abc^2dx-8\,a^2cd^2x}{18\,(dx^3+c)^2c^2d^2}$$

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x +
 (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2*a
*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3
)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(abs(x
- (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^
2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2
*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\frac{2\,x\,\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}$$

```
[In] int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*
d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2*d
```

```
^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (log
(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a^2*
d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*c^2 +
2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 252

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}-\frac{(bc-ad)^4\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/4*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^4/b^3+1/7*d^3*(-a*d+4*b*c)*x^7/b^2+1/10*d^4*x^10/b+1/3*(-a*
d+b*c)^4*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/6*(-a*d+b*c)^4*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/3)-1/3*(-a*d+b*c)^4*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

= {398, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4}{\sqrt{3}a^{2/3}b^{13/3}} - \frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} + \frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4} + \frac{d^2x^4(a^2d^2-4abcd+6b^2c^2)}{4b^3} + \frac{d^3x^7(4bc-ad)}{7b^2} + \frac{d^4x^{10}}{10b}$$

```
[In] Int[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{b^3}\right. \\
&\qquad \left.+\frac{d^3(4bc-ad)x^6}{b^2}+\frac{d^4x^9}{b}+\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{b^4\left(a+bx^3\right)}\right)dx \\
&= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} \\
&\quad +\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc-ad)^4\int\frac{1}{a+bx^3}\,dx}{b^4} \\
&= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} \\
&\quad +\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc-ad)^4\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^4} \\
&\quad +\frac{(bc-ad)^4\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^4} \\
&= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} \\
&\quad +\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} \\
&\quad -\frac{(bc-ad)^4\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^4}
\end{aligned}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} - \frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{13/3}}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}-\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00

$$\int\frac{\left(c+dx^3\right)^4}{a+bx^3}\,dx$$

$$= \frac{420\sqrt[3]{b}d\left(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3\right)x+105b^{4/3}d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4+60b^{7/3}d^3(4bc-ad)x}{}$$

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
 - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^
(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

method	result
risch	$\frac{d^4x^{10}}{10b} - \frac{d^4x^7a}{7b^2} + \frac{4d^3x^7c}{7b} - \frac{d^3x^4ac}{b^2} + \frac{3d^2x^4c^2}{2b} + \frac{d^4x^4a^2}{4b^3} - \frac{d^4a^3x}{b^4} + \frac{4d^3a^2cx}{b^3} - \frac{6d^2ac^2x}{b^2} + \frac{4dc^3x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_}}{}$
default	$-\frac{d\left(-\frac{d^3x^{10}b^3}{10}+\frac{((ad-2bc)b^2d^2-2b^3cd^2)x^7}{7}+\frac{(2(ad-2bc)b^2cd-bd(a^2d^2-2abcd+2b^2c^2))x^4}{4}+(ad-2bc)(a^2d^2-2abcd+2b^2c^2)x\right)}{b^4} + \frac{\left(\frac{\ln(}{}\right.}{}$

```
[In] int((d*x^3+c)^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*d^4*x^10/b-1/7*d^4/b^2*x^7*a+4/7*d^3/b*x^7*c-d^3/b^2*x^4*a*c+3/2*d^2/b
*x^4*c^2+1/4*d^4/b^3*x^4*a^2-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^
2*x+4*d/b*c^3*x+1/3/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$= \left[\frac{42\,a^2b^4d^4x^{10} + 60\,(4\,a^2b^4cd^3 - a^3b^3d^4)x^7 + 105\,(6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4)x^4 + 210\,\sqrt{\tfrac{1}{3}}(ab^5c^4 - 4\,a}{}\right.$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*
(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqr
t(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d -
 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1/420*(42*a
^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(6*a^2*b^4*c
^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*b^5*c^4 - 4*
a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^
2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d
 - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{13}-a^{12}d^{12}+12a^{11}bcd^{11}-66a^{10}b^2c^2d^{10}+220a^9b^3c^3d^9-495a^8b^4c^4d^8+792a^7b^5c^5d^7\right.$$
$$+\frac{d^4x^{10}}{10b}$$

```
[In] integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*d
**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 - 6
*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d**12
 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9
- 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d*
*6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9
*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12, Lambda
(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \frac{14\,b^3d^4x^{10}+20\,(4\,b^3cd^3-ab^2d^4)x^7+35\,(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4)x^4+140\,(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2bc}{140\,b^4} + \frac{\sqrt{3}(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*d
^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{(b^{10}c^4-4\,ab^9c^3d+6\,a^2b^8c^2d^2-4\,a^3b^7cd^3+a^4b^6d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{10}} + \frac{14\,b^9d^4x^{10}+80\,b^9cd^3x^7-20\,ab^8d^4x^7+210\,b^9c^2d^2x^4-140\,ab^8cd^3x^4+35\,a^2b^7d^4x^4+560\,b^9c^3dx-840\,a}{140\,b^{10}}$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(
2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^3)
- 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4
*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*
d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*
a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^2*d^2*
x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^7\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)+x^4\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right)+\frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^4}{3\,a^{2/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^4}{a^{2/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^4}{3\,a^{2/3}\,b^{13/3}}$$

```
[In] int((c + d*x^3)^4/(a + b*x^3),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x + a^
(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(13/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 208

$$
\begin{aligned}
\int \frac{(c+dx^3)^3}{a+bx^3} \, dx = {} & \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^4}{4b^2} + \frac{d^3x^7}{7b} \\
& - \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{(bc-ad)^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} \\
& - \frac{(bc-ad)^3 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}
\end{aligned}
$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/4*d^2*(-a*d+3*b*c)*x^4/b^2+1/7*d^3*
x^7/b+1/3*(-a*d+b*c)^3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a*d+b*c
)^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/3)-1/3*(-a*d+b*
c)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used

= {398, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3}{\sqrt{3}a^{2/3}b^{10/3}} - \frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}} + \frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} + \frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3} + \frac{d^2x^4(3bc-ad)}{4b^2} + \frac{d^3x^7}{7b}$$

```
[In] Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 206</u>

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 398</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

<u>Rule 631</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3}+\frac{d^2(3bc-ad)x^3}{b^2}+\frac{d^3x^6}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\,(a+bx^3)}\right)dx$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^3}\,dx}{b^3}$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}+\frac{(bc-ad)^3\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^3}$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^3}$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{10/3}}$$

$$= \frac{d(3b^2c^2 - 3abcd + a^2d^2)\,x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} - \frac{(bc - ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{(bc - ad)^3 \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} - \frac{(bc - ad)^3 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

$$\int \frac{(c + dx^3)^3}{a + bx^3}\,dx$$

$$= \frac{84\sqrt[3]{b}d(3b^2c^2 - 3abcd + a^2d^2)\,x + 21b^{4/3}d^2(3bc - ad)x^4 + 12b^{7/3}d^3x^7 + \frac{28\sqrt{3}(bc - ad)^3 \arctan\left(\frac{-\sqrt[3]{a} + 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + \frac{28}{-}}{84b^{10/3}}$$

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
 a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
 + b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.63

method	result
risch	$\frac{d^3x^7}{7b}-\frac{d^3a\,x^4}{4b^2}+\frac{3d^2c\,x^4}{4b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^4}$
default	$\frac{d\left(\frac{1}{7}b^2d^2x^7-\frac{1}{4}ab\,d^2x^4+\frac{3}{4}b^2cd\,x^4+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b^3}$

```
[In] int((d*x^3+c)^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*a*x^4+3/4*d^2/b*c*x^4+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x+1/3/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/_
R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.37

$$\int\frac{(c+dx^3)^3}{a+bx^3}\,dx$$

$$=\left[\frac{12\,a^2b^3d^3x^7+21\,(3\,a^2b^3cd^2-a^3b^2d^3)x^4-42\,\sqrt{\frac{1}{3}}(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\right.}{}\right.$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sqr
t(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((-a
```

```
^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2
*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)
^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)
^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^2*b^4),
1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 + 84*sqrt
(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt(-(-a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(
a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3
)*x)/(a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{10}+a^9d^9-9a^8bcd^8+36a^7b^2c^2d^7-84a^6b^3c^3d^6+126a^5b^4c^4d^5-126a^4b^5c^5d^4+84a^3b^6\right.$$
$$+\frac{d^3x^7}{7b}$$

```
[In] integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d*
*8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d
**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c**7*
d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*d*
*3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**7/(7*b
)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \frac{4\,b^2d^3x^7+7\,(3\,b^2cd^2-abd^3)x^4+28\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{28\,b^3} + \frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*
(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = -\frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}\,b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}\,b^2} - \frac{(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,b^6d^3x^7+21\,b^6cd^2x^4-7\,ab^5d^3x^4+84\,b^6c^2dx-84\,ab^5cd^2x+28\,a^2b^4d^3x}{28\,b^7}$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b
^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1
/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*d*x -
84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\,\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right) +\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}} -\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}} +\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^3}{a^{2/3}\,b^{10/3}}$$

```
[In] int((c + d*x^3)^3/(a + b*x^3),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
- (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c
)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^3)/
(a^(2/3)*b^(10/3))
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/4*d^2*x^4/b+1/3*(-a*d+b*c)^2*ln(a^(1/3)+b^(1/3)*x)/a
^(2/3)/b^(7/3)-1/6*(-a*d+b*c)^2*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(2/3)/b^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {398, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2}{\sqrt{3}a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^3\right)}\right)dx \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^3}\,dx}{b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^2}+\frac{(bc-ad)^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}} \\
&\quad -\frac{(bc-ad)^2\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}} \\
&\quad -\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}} \\
&\quad +\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{7/3}} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} \\
&\quad +\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{-12a^{2/3}\sqrt[3]{b}d(-2bc+ad)x+3a^{2/3}b^{4/3}d^2x^4+4\sqrt{3}(bc-ad)^2\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{a}\right.}{12a^{2/3}b^{7/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result	siz
risch	$\frac{d^2x^4}{4b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^3}$	78
default	$-\frac{d\left(-\frac{1}{4}bd\,x^4+adx-2bcx\right)}{b^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^2d^2-2abcd+b^2c^2\right)}{b^2}$	14

```
[In] int((d*x^3+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*c*x+1/3/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2)
/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.93

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \left[\frac{3\,a^2b^2d^2x^4 + 6\,\sqrt{\frac{1}{3}}(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\,(a^2b)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^2 + (a^2b)^{\frac{2}{3}}x - (a^2b)^{\frac{1}{3}}\right.}{bx^3+a}\right.}{}\right.$$

[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")

[Out] [1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right) + \operatorname{RootSum}\left(27t^3a^2b^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t\mapsto\right.\right. + \frac{d^2x^4}{4b}$$

[In] integrate((d*x**3+c)**2/(b*x**3+a),x)

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**
5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4
*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a*
*2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{bd^2x^4+4\,(2\,bcd-ad^2)x}{4\,b^2} + \frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/
b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{(b^4c^2-2\,ab^3cd+a^2b^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{b^3d^2x^4+8\,b^3cdx-4\,ab^2d^2x}{4\,b^4}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b^
4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{d^2\,x^4}{4\,b} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right) + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)^2}{3\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^2}{a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^2}{3\,a^{2/3}\,b^{7/3}}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3),x)
```

```
[Out] (d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*(
a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*a^(2/3)*b^(7/3))
```

3.17 $\int \frac{c+dx^3}{a+bx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{c+dx^3}{a+bx^3} dx = \frac{dx}{b} - \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

```
[Out] d*x/b+1/3*(-a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(-a*d+b*c)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3}{a+bx^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)}{\sqrt{3}a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} + \frac{dx}{b}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
```

```
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^3}\,dx}{b} \\
&= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b} + \frac{(bc-ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b} \\
&= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} \\
&\quad + \frac{(bc-ad)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b} \\
&= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} \\
&\quad + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}} \\
&= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} \\
&\quad - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{a+bx^3}\,dx$$

$$= \frac{6a^{2/3}\sqrt[3]{b}dx - 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}\right.}{6a^{2/3}b^{4/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$	42
default	$\frac{dx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{b}$	110

```
[In] int((d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+1/3/b^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \left[\frac{6\,a^{2}bdx-3\,\sqrt{\frac{1}{3}}(ab^{2}c-a^{2}bd)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)\cdots}{6\,a^{2}\cdots}\right.$$

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
[Out] [1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)*
log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^
2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (-a
```

```
^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2), 1/6*
(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b)*arct
an(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b
)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^
2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3a^2b^4+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3, \left(t\mapsto t\log\left(-\frac{3tab}{ad-bc}+x\right)\right)\right) + \frac{dx}{b}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3}{a+bx^3}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{dx}{b} - \frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{d\,x}{b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}$$

```
[In] int((c + d*x^3)/(a + b*x^3),x)
```

```
[Out] (d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {400, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} + \frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)}$$
$$+ \frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}(bc-ad)} + \frac{d^{2/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}(bc-ad)} - \frac{d\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{c}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}(bc-ad)} - \frac{d^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\right.}{}}{-6bc+6ad}$$

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan \cdots}{\cdots}\cdots\right)}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}d^{3}-3a^{4}bc\,d^{2}+3a^{3}b^{2}c^{2}d-a^{2}b^{3}c^{3}\right)\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-b^{5}c^{5}\right)\_R^{3}\cdots\right.\right.\right.}{3}$

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int\frac{1}{(a+bx^{3})\,(c+dx^{3})}\,dx = -\frac{2\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}\cdots\right.}{\cdots}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d} - \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)} - \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
```

```
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 346

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = &-\frac{dx}{3c(bc-ad)(c+dx^3)}-\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}\\&+\frac{d^{2/3}(5bc-2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2}\\&+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2}\\&-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}\\&+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}\end{aligned}$$

```
[Out] -1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)+1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(
-a*d+b*c)^2-1/9*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+
b*c)^2-1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+
b*c)^2+1/18*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^2-1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(2/3)/(-a*d+b*c)^2*3^(1/2)+1/9*d^(2/3)*(-2*a*d+5*b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = -\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2} - \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{dx}{3c(c+dx^3)(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] -1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c
 - 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c -
a*d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*
c - a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a
^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{dx}{3c(bc-ad)\,(c+dx^3)} + \frac{\int \frac{3bc-2ad-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{3c(bc-ad)}\\ &= -\frac{dx}{3c(bc-ad)\,(c+dx^3)} + \frac{b^2\int \frac{1}{a+bx^3}\,dx}{(bc-ad)^2} - \frac{(d(5bc-2ad))\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^2}\end{aligned}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^2 \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)^2} + \frac{b^2 \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)^2}$$
$$- \frac{(d(5bc-2ad)) \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}(bc-ad)^2} - \frac{(d(5bc-2ad)) \int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}$$
$$- \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3} \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)^2}$$
$$+ \frac{b^2 \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}(bc-ad)^2} + \frac{\left(d^{2/3}(5bc-2ad)\right) \int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{18c^{5/3}(bc-ad)^2}$$
$$- \frac{(d(5bc-2ad)) \int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{4/3}(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}$$
$$- \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}$$
$$+ \frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}$$
$$+ \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}(bc-ad)^2}$$
$$- \frac{\left(d^{2/3}(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3c^{5/3}(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)(c+dx^3)} - \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \frac{6a^{2/3}c^{2/3}d(-bc+ad)x - 6\sqrt{3}b^{5/3}c^{5/3}(c+dx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt{3}a^{2/3}d^{2/3}(-5bc+2ad)(c+dx^3)\,\mathrm{ar}}{}$$

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x^
3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] +
6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(
c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)
*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.71

method	result
default	$\frac{d\left(\frac{(ad-bc)x}{3c\left(d\,x^{3}+c\right)}+\frac{(2ad-5bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+1/3*(2*a*d-5*b*c)/c*(1/3/d/(c/d)
^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3)
)+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+(1/3/
b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
*b^2/(a*d-b*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 4.48 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^2}\,dx$$

$$= \frac{6\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}((5\,bcd-2\,ad^2)x^3+5\,bc^2-2\,acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\text{arcta}}{}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a
*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5
```

```
*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3)
 - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*
x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c
^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^
2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*l
og(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abc^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{dx}{3\left(bc^3-ac^2d+(bc^2d-acd^2)x^3\right)} - \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{(5\,bc-2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{(5\,bc-2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^2
*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3)) -
1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1
/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6
*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*b*c*d*
(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 - x*(c/d)
^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*
c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*log(x + (c/d
)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*d^2*(c/d)^(
2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = -\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)} + \frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2} + \frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)} + \frac{\left(5\,bcd-2\,ad^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)} - \frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4-2\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2\right)} - \frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)} - \frac{dx}{3\left(dx^3+c\right)\left(bc^2-acd\right)}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d +
 a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/6*(
-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c
- 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/18*(5*
(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d
)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3 + c)*(b*c^
2 - a*c*d))
```

Mupad [B] (verification not implemented)

Time = 21.57 (sec) , antiderivative size = 2589, normalized size of antiderivative = 7.48

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2} dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
+ (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d
 - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^
2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*c^3*d^2
 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 - 43
74*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^
4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*
c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(a*d + b*c)
*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*(b^5/(a^
2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^
2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/
3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*
d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^
5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 -
 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27
*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3
*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3
)/(c^5*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^
3)/(c^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*
a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)
^3)/(c^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 +
 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*
((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^
6*c^11 + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 145
80*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(
3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (27*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(1/
3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(2
/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d
^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(
1/3))/18 - (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b
*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*a^3*d^5 - 125*b^3*c^3*
```

```
d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 -
 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935
*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*((
(3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*
a*b*c*d))/c + (81*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*
(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*
c)^6))^(2/3))/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a
^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 + (2
*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c
^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162
*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^
2*d^4 - 162*a^7*b*c*d^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i +
 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c -
 (81*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*
d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))
/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/
(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 - (2*b^6*d^5*x*(4
*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c
)^4))*(3^(1/2)*1i + 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*
d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a
^7*b*c*d^5))^(1/3))/2 + (d*x)/(3*c*(c + d*x^3)*(a*d - b*c))
```

3.20 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 320

$$\begin{aligned}\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx =& \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\ &+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4}+\frac{d^4(5bc-2ad)x^7}{7b^3}+\frac{d^5x^{10}}{10b^2} \\ &+\frac{(bc-ad)^5x}{3ab^5\,(a+bx^3)}-\frac{(bc-ad)^4(2bc+13ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}} \\ &+\frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} \\ &-\frac{(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}\end{aligned}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/10*
d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)+1/9*(-a*d+b*c)^4*(13*a*d+2*
b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(16/3)-1/18*(-a*d+b*c)^4*(13*a*d+2*b*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(16/3)-1/9*(-a*d+b*c)
^4*(13*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)
/b^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4(13ad+2bc)}{3\sqrt{3}a^{5/3}b^{16/3}} - \frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}} + \frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} + \frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4} + \frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{3ab^5(a+bx^3)} + \frac{d^4x^7(5bc-2ad)}{7b^3} + \frac{d^5x^{10}}{10b^2}$$

```
[In] Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^3}{b^4} + \frac{d^4(5bc - 2ad)x^6}{b^3} + \frac{d^5x^9}{b^2} + \frac{(bc - ad)^4(bc + 4ad) + 5bd(bc - ad)^4x^3}{b^5\,(a + bx^3)^2} \right) dx$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc - 2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{\int \frac{(bc-ad)^4(bc+4ad)+5bd(bc-ad)^4x^3}{(a+bx^3)^2}\,dx}{b^5}$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc - 2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{(bc-ad)^5x}{3ab^5(a+bx^3)} + \frac{((bc-ad)^4(2bc+13ad))\int \frac{1}{a+bx^3}\,dx}{3ab^5}$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc - 2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{(bc-ad)^5x}{3ab^5(a+bx^3)} + \frac{((bc-ad)^4(2bc+13ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^5} + \frac{((bc-ad)^4(2bc+13ad))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^5}$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc - 2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{(bc-ad)^5x}{3ab^5(a+bx^3)} + \frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} - \frac{((bc-ad)^4(2bc+13ad))\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{16/3}} + \frac{((bc-ad)^4(2bc+13ad))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^5}$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc - 2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{(bc-ad)^5x}{3ab^5(a+bx^3)} + \frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} - \frac{(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}} + \frac{((bc-ad)^4(2bc+13ad))\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{16/3}}$$

$$
\begin{aligned}
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\
&\quad + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc-2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} \\
&\quad + \frac{(bc-ad)^5x}{3ab^5\,(a+bx^3)} - \frac{(bc-ad)^4(2bc+13ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}} \\
&\quad + \frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} \\
&\quad - \frac{(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.98

$$
\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx
$$

$$
= \frac{1260\sqrt[3]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+315b^{4/3}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4+180b^{7/3}d^4
$$

```
[In] Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3
)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/
3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*
d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4
*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126
0*b^(16/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.95

method	result
risch	$\frac{d^5x^{10}}{10b^2}-\frac{2d^5a\,x^7}{7b^3}+\frac{5d^4c\,x^7}{7b^2}+\frac{3d^5a^2x^4}{4b^4}-\frac{5d^4ac\,x^4}{2b^3}+\frac{5d^3c^2x^4}{2b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5d^5}{}$
default	$-\frac{d^2\left(-\frac{1}{10}d^3x^{10}b^3+\frac{2}{7}a\,b^2d^3x^7-\frac{5}{7}b^3c\,d^2x^7-\frac{3}{4}a^2b\,d^3x^4+\frac{5}{2}a\,b^2c\,d^2x^4-\frac{5}{2}b^3c^2d\,x^4+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+\frac{-(}{}$

```
[In] int((d*x^3+c)^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*d^5*x^10/b^2-2/7*d^5/b^3*a*x^7+5/7*d^4/b^2*c*x^7+3/4*d^5/b^4*a^2*x^4-5
/2*d^4/b^3*a*c*x^4+5/2*d^3/b^2*c^2*x^4-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-2
0*d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/3*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^
2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^3+a)+1/9/b^6/a
*sum((13*a^5*d^5-50*a^4*b*c*d^4+70*a^3*b^2*c^2*d^3-40*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d+2*b^5*c^5)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(275) = 550$.

Time = 0.32 (sec) , antiderivative size = 1619, normalized size of antiderivative = 5.06

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10
 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(4
0*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)
*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2
+ 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a
^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 +
 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d -
40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*
```

```
b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*
b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^
2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5
*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^
5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 -
 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d
^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(126*a^3*b^5*d^5*x^13
 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*a^3*b^5*c^2*d^3 - 5
0*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*b^5*c^3*d^2 - 70*a^4*b^
4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)*x^4 + 420*sqrt(1/3)*(2*a^2*b
^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6
*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3
*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt((a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
(a^2*b)^(1/3)/b)/a^2) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*
d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b
^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*
a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*
c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2
*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(
a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*
d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d
^5)*x)/(a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [A] (verification not implemented)

Time = 127.22 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = x^7\left(-\frac{2ad^5}{7b^3}+\frac{5cd^4}{7b^2}\right)+x^4\cdot\left(\frac{3a^2d^5}{4b^4}-\frac{5acd^4}{2b^3}+\frac{5c^2d^3}{2b^2}\right)$$
$$+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)$$
$$+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{3a^2b^5+3ab^6x^3}$$
$$+\text{RootSum}\Big(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^2c^2d^{13}+418280a^{12}b^3c^3d^{12}-874635$$
$$+\frac{d^5x^{10}}{10b^2}$$

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
- 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a
```

```
**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5
+ 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b
**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t**3*a
**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b**2*c*
*2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 12
71886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**
7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d**6 + 7194
*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*b**12*c**1
2*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**15*c**15, La
mbda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b
**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x
))) + d**5*x**10/(10*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{3\,(ab^6x^3 + a^2b^5)}$$
$$+ \frac{14\,b^3d^5x^{10} + 20\,(5\,b^3cd^4 - 2\,ab^2d^5)x^7 + 35\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^4 + 140\,(10\,b^3c^3d^2 - 20\,ab^2c^2}{140\,b^5}$$
$$+ \frac{\sqrt{3}(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 + 2
0*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3
*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^
3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5*c^5 +
5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 +
```

```
13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2/3)) + 1/
9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50
*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}ab^4} - \frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}ab^4} - \frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^5} + \frac{b^5c^5x-5\,ab^4c^4dx+10\,a^2b^3c^3d^2x-10\,a^3b^2c^2d^3x+5\,a^4bcd^4x-a^5d^5x}{3\,(bx^3+a)ab^5} + \frac{14\,b^{18}d^5x^{10}+100\,b^{18}cd^4x^7-40\,ab^{17}d^5x^7+350\,b^{18}c^2d^3x^4-350\,ab^{17}cd^4x^4+105\,a^2b^{16}d^5x^4+1400\,b^{18}c^3 \cdots}{140\,b^{20}}$$

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c
^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*d -
 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*b^5*c
^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d
^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) + 1/3*(b
^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x +
5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*d^5*x^10
+ 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a*b^1
7*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*c^2*
d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)$$
$$-x^7\left(\frac{2\,a\,d^5}{7\,b^3}-\frac{5\,c\,d^4}{7\,b^2}\right)+x^4\left(\frac{a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{2\,b}-\frac{a^2\,d^5}{4\,b^4}+\frac{5\,c^2\,d^3}{2\,b^2}\right)+\frac{d^5\,x^{10}}{10\,b^2}$$
$$-\frac{x\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{3\,a\left(b^6\,x^3+a\,b^5\right)}$$
$$+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$

```
[In] int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3 -
(5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5*x^1
0)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) - (log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*
c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b
^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/
(9*a^(5/3)*b^(16/3))
```

3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 267

$$\begin{aligned}\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx &= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}\\ &\quad+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}-\frac{2(bc-ad)^3(bc+5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}}\\ &\quad+\frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}\\ &\quad-\frac{(bc-ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}\end{aligned}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7*
d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)+2/9*(-a*d+b*c)^3*(5*a*d+b*c)
*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(13/3)-1/9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-2/9*(-a*d+b*c)^3*(5*a*
d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3(5ad+bc)}{3\sqrt{3}a^{5/3}b^{13/3}} - \frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}} + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{3ab^4(a+bx^3)} + \frac{d^3x^4(2bc-ad)}{2b^3} + \frac{d^4x^7}{7b^2}$$

```
[In] Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4}+\frac{2d^3(2bc-ad)x^3}{b^3}+\frac{d^4x^6}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{b^4\left(a+bx^3\right)^2}\right)dx$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{\int\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{\left(a+bx^3\right)^2}dx}{b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}+\frac{(2(bc-ad)^3(bc+5ad))\int\frac{1}{a+bx^3}dx}{3ab^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}+\frac{(2(bc-ad)^3(bc+5ad))\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^4}+\frac{(2(bc-ad)^3(bc+5ad))\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}+\frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{((bc-ad)^3(bc+5ad))\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{13/3}}+\frac{((bc-ad)^3(bc+5ad))\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}+\frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{(bc-ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{(2(bc-ad)^3(bc+5ad))\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{13/3}}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)}-\frac{2(bc-ad)^3(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{(bc-ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$

$$= \frac{126\sqrt[3]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+63b^{4/3}d^3(2bc-ad)x^4+18b^{7/3}d^4x^7+\frac{42\sqrt[3]{b}(bc-ad)^4x}{a(a+bx^3)}+\frac{28\sqrt{3}(bc-ad)^3(bc+}{}}{126b^{13/}}$$

[In] Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]

[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2*b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a + b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.93 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.82

method	result
risch	$\frac{d^4x^7}{7b^2}-\frac{d^4a\,x^4}{2b^3}+\frac{d^3c\,x^4}{b^2}+\frac{3d^4a^2x}{b^4}-\frac{8d^3acx}{b^3}+\frac{6d^2c^2x}{b^2}+\frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{3a\,b^4(b\,x^3+a)}-\frac{2\left(\_R=\mathrm{Root}\right.}{}$ $2\left(5a^4d^4-14a^3bc\,d^3+\right.$
default	$\frac{d^2\left(\frac{1}{7}b^2d^2x^7-\frac{1}{2}ab\,d^2x^4+b^2cd\,x^4+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4}-\frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{3a\left(b\,x^3+a\right)}+}{}$

[In] int((d*x^3+c)^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/7*d^4*x^7/b^2-1/2*d^4/b^3*a*x^4+d^3/b^2*c*x^4+3*d^4/b^4*a^2*x-8*d^3/b^3*a*c*x+6*d^2/b^2*c^2*x+1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c

```
^3*d+b^4*c^4)/a*x/b^4/(b*x^3+a)-2/9/b^5/a*sum((5*a^4*d^4-14*a^3*b*c*d^3+12*
a^2*b^2*c^2*d^2-2*a*b^3*c^3*d-b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(224) = 448$.

Time = 0.31 (sec) , antiderivative size = 1316, normalized size of antiderivative = 4.93

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63
*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/3)
*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5
*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3
*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^
2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a^2*b^3*
c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^
4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b
)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4
+ 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*
c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*
x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^
6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*
b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x
^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*
a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2
*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arcta
n(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)
/a^2) - 14*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d
^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2
*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
(-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 +
14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2
+ 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^
2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^6*x^3 + a^4*b^5)]
```

Sympy [A] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right) + \frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{3a^2b^4+3ab^5x^3} + \text{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c^4 \right. + \frac{d^4x^7}{7b^2}$$

```
[In] integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b*
*3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*
d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum
(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**
10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8
- 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**
7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**1
0*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*
a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)x}{3\,(ab^5x^3+a^2b^4)} + \frac{2\,b^2d^4x^7+7\,(2\,b^2cd^3-abd^4)x^4+14\,(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4)x}{14\,b^4} + \frac{2\,\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4)*
x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*(b^
4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3)) - 1/9
*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(b^4*c^4
 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*log(x +
 (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$
$$= -\frac{2\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3}$$
$$-\frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3}$$
$$-\frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4}$$
$$+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{3\,(bx^3+a)ab^4}$$
$$+\frac{2\,b^{12}d^4x^7+14\,b^{12}cd^3x^4-7\,ab^{11}d^4x^4+84\,b^{12}c^2d^2x-112\,ab^{11}cd^3x+42\,a^2b^{10}d^4x}{14\,b^{14}}$$

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3
 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^
3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*
c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/3*
(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*
d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a
*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)
/b^14
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = x\left(\frac{2\,a\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)+\frac{d^4\,x^7}{7\,b^2}$$
$$+\frac{x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{3\,a\,\left(b^5\,x^3+a\,b^4\right)}$$
$$-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$
$$+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$
$$-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$

```
[In] int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*b^4
 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))/(9*
a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3)) - (2*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d
- b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\begin{aligned}\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx &= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\,(a+bx^3)}\\ &\quad-\frac{(bc-ad)^2(2bc+7ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}}\\ &\quad+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}\\ &\quad-\frac{(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}\end{aligned}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a)
+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(10/3)-1/18
*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(10/3)-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2(7ad+2bc)}{3\sqrt{3}a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{3ab^3(a+bx^3)} + \frac{d^3x^4}{4b^2}$$

```
[In] Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(3bc-2ad)}{b^3} + \frac{d^3x^3}{b^2} + \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{b^3\left(a+bx^3\right)^2}\right) dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{(a+bx^3)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)} + \frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)} + \frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^3} + \frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)} + \frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{10/3}} + \frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)} + \frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{\left((bc-ad)^2(2bc+7ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{10/3}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)} - \frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}d^2(3bc-2ad)x+9b^{4/3}d^3x^4+\frac{12\sqrt[3]{b}(bc-ad)^3x}{a(a+bx^3)}+\frac{4\sqrt{3}(bc-ad)^2(2bc+7ad)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{4(bc-ad)^2(2bc+7\cdots}{\cdots}}{36b^{10/3}}$$

[In] Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]

[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c - a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
risch	$\frac{d^3x^4}{4b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x}{3ab^3(bx^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3)\ln(x\cdots}{\_R^2}}{9b^4a}$
default	$-\frac{d^2\left(-\frac{1}{4}bdx^4+2adx-3bcx\right)}{b^3}+\frac{-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x}{3a(bx^3+a)}+\frac{(7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\cdots\right.\right.}{6b\cdots}\cdots\right.}{3a}}{b^3}$

[In] int((d*x^3+c)^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^3+a)+1/9/b^4/a*sum((7*a^3*d^3-12*a^2*b*c*d^2+3*a*b^2*c^2*d+2*b^3*c^3)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(193) = 386.

Time = 0.29 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.39

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sqr
t(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 +
(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqr
t(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(
b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3
 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b
)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d
- 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*
x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 -
 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^
4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d
^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/
3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2
 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)
*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^
3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{3a^2b^3+3ab^4x^3}$$
$$+\text{RootSum}\Bigg(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a^4b^5$$
$$+\frac{d^3x^4}{4b^2}$$

```
[In] integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*
a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7
+ 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c**5*d
**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**8*d -
 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2*b*c*d
**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{3\,(ab^4x^3+a^2b^3)} + \frac{bd^3x^4+4\,(3\,bcd^2-2\,ad^3)x}{4\,b^3} + \frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2*
b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b^3
*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a*b^2
*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*
a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(bx^3+a)ab^3} + \frac{b^6d^3x^4+12\,b^6cd^2x-8\,ab^5d^3x}{4\,b^8}$$

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1
/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*a*b^
2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3
*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x
)/b^8
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{d^3\,x^4}{4\,b^2} - x\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{3\,a\,(b^4\,x^3+a\,b^3)} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(7\,a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{10/3}}$$

```
[In] int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

```
[Out] (d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*
a^(5/3)*b^(10/3))
```

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}-\frac{2(bc-ad)(bc+2ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}$$

```
[Out] d^2*x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)+2/9*(-a*d+b*c)*(2*a*d+b*c)*ln(
a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/9*(-a*d+b*c)*(2*a*d+b*c)*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-2/9*(-a*d+b*c)*(2*a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used

= {398, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)(2ad+bc)}{3\sqrt{3}a^{5/3}b^{7/3}} - \frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} + \frac{x(bc-ad)^2}{3ab^2(a+bx^3)} + \frac{d^2x}{b^2}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{b^2\left(a+bx^3\right)^2}\right)dx \\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(a+bx^3\right)^2}\,dx}{b^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{a+bx^3}\,dx}{3ab^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^2} \\
&\quad+\frac{(2(bc-ad)(bc+2ad))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^2}
\end{aligned}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{3ab^2(a+bx^3)} + \frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{((bc-ad)(bc+2ad))\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{7/3}} + \frac{((bc-ad)(bc+2ad))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}b^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{3ab^2(a+bx^3)} + \frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{(2(bc-ad)(bc+2ad))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{7/3}}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{3ab^2(a+bx^3)} - \frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$= \frac{9\sqrt[3]{b}d^2x + \frac{3\sqrt[3]{b}(bc-ad)^2x}{a(a+bx^3)} - \frac{2\sqrt{3}(b^2c^2+abcd-2a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2(b^2c^2+abcd-2a^2d^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{(b^2c^2+abcd}{}}{9b^{7/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3]
*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x])/a
^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result	si
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3a\,b^2(b\,x^3+a)}-\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2a^2d^2-abcd-b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^3a}$	10
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{3a\left(b\,x^3+a\right)}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}}{b^2}$	17

```
[In] int((d*x^3+c)^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^3+a)-2/9/b^3/a*sum((
2*a^2*d^2-a*b*c*d-b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(164) = 328.

Time = 0.30 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^2}\,dx$$

$$=\left[\frac{9\,a^3b^2d^2x^4-3\sqrt{\frac{1}{3}}(a^2b^3c^2+a^3b^2cd-2\,a^4bd^2+(ab^4c^2+a^2b^3cd-2\,a^3b^2d^2)x^3)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b\ldots}{\ldots}\right.}{\ldots}\right.$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*
d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/b)
```

```
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a
^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a
*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)
*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b
^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4 + 6*sqrt(
1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*b^3*c*d -
2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2*c^2 + a^2*b
*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*
log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2*c^2 + a^2*b*c
*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*lo
g(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + 4*a^4*b*d^2)*x
)/(a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{3a^2b^2+3ab^3x^3} + \mathrm{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6\right. + \frac{d^2x}{b^2}$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + RootS
um(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c**
2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d
 - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d
- 2*b**2*c**2) + x))) + d**2*x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(ab^3x^3+a^2b^2)} + \frac{d^2x}{b^2} + \frac{2\,\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 + 2
/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^
2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^2*c^
2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2x}{b^2} - \frac{2\,\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(bx^3+a)ab^2}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 + a*
b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)
*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)
*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2\,x}{b^2}+\frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{3\,a\,\left(b^3\,x^3+a\,b^2\right)} - \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}} - \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}} + \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3)) -
 (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3))
 - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d + b*c)
)/(9*a^(5/3)*b^(7/3))
```

3.24 $\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3ab(a+bx^3)} - \frac{(2bc+ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

```
[Out] 1/3*(-a*d+b*c)*x/a/b/(b*x^3+a)+1/9*(a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3
)/b^(4/3)-1/18*(a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3
)/b^(4/3)-1/9*(a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {393, 206, 31, 648, 631, 210, 642}

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(ad+2bc)}{3\sqrt{3}a^{5/3}b^{4/3}} - \frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} + \frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\int \frac{1}{a+bx^3}\,dx}{3ab}\\
&= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b}+\frac{(2bc+ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}\\
&= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}\\
&\quad-\frac{(2bc+ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}}+\frac{(2bc+ad)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b}\\
&= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}\\
&\quad-\frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}\\
&\quad+\frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{4/3}}
\end{aligned}
$$

$$= \frac{(bc-ad)x}{3ab\,(a+bx^3)} - \frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6a^{2/3}\sqrt[3]{b}(-bc+ad)x}{a+bx^3} - 2\sqrt{3}(2bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (2bc+ad)\log\left(\right.}{18a^{5/3}b^{4/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3)
 + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(ad+2bc)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{(ad+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

```
[In] int((d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/b/a*x/(b*x^3+a)+1/9/a/b^2*sum((a*d+2*b*c)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(2\,a^2b^2c+a^3bd+(2\,ab^3c+a^2b^2d)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}\,ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}\,x-(a^2b)^{\frac{1}{3}}\,a\right)\sqrt{\cdots}}{bx^3+a}\cdots\right.}{\cdots}\right.$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2
*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a
^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*
d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((2*b^2*c +
 a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{x(-ad+bc)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4-a^3d^3-6a^2bcd^2-12ab^2c^2d-8b^3c^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - a
**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _t*
log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,bc+ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,bc+ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*b*
c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*(
2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(2\,bc+ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx-adx}{3\,(bx^3+a)ab}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} - \frac{x\,(a\,d-b\,c)}{3\,a\,b\,(b\,x^3+a)}$$

```
[In] int((c + d*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))/(3*
a*b*(a + b*x^3))
```

3.25 $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 346

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \frac{bx}{3a(bc-ad)(a+bx^3)} - \frac{b^{2/3}(2bc-5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)+1/9*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(1/3)+b^(1/3
)*x)/a^(5/3)/(-a*d+b*c)^2+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b
*c)^2-1/18*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(5/3)/(-a*d+b*c)^2-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/c^(2/3)/(-a*d+b*c)^2-1/9*b^(2/3)*(-5*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(2bc-5ad)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} + \frac{bx}{3a(a+bx^3)(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{\int \frac{-2bc+3ad-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{d^2 \int \frac{1}{c+dx^3}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad)) \int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{d^2 \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{d^2 \int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{(b(2bc-5ad)) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}(bc-ad)^2} + \frac{(b(2bc-5ad)) \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - \frac{d^{5/3} \int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}(bc-ad)^2} + \frac{d^2 \int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{c}(bc-ad)^2} - \frac{\left(b^{2/3}(2bc-5ad)\right) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}(bc-ad)^2} + \frac{(b(2bc-5ad)) \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2} + \frac{d^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}(bc-ad)^2} + \frac{\left(b^{2/3}(2bc-5ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)(a+bx^3)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$$

$$= \frac{6a^{2/3}bc^{2/3}(bc-ad)x - 2\sqrt{3}b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 6\sqrt{3}a^{5/3}d^{5/3}(a+bx^3)\operatorname{arct}}{}$$

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*a
*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a^(
5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*b^
(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c -
 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{2}}{(ad-bc)^{2}}-\frac{b\left(\frac{(ad-bc)x}{3a\left(b\,x^{3}+a\right)}+\frac{(5ad-2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\right.\right.}{6}\right.}{}\right.}{(ad-b}$
risch	Expression too large to display

[In] int(1/(b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] (1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x +(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x -1)))*d^2/(a*d-b*c)^2-1/(a*d-b*c)^2*b*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+1/3*(5*a *d-2*b*c)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 4.46 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = -\frac{2\sqrt{3}((2b^2c-5abd)x^3+2abc-5a^2d)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}(abdx^3+a^2d)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}}{}$$

[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")

[Out] -1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(3)

```
*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a
^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 3*
(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \frac{\sqrt{3}(2\,bc-5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{bx}{3\left(a^2bc-a^3d+(ab^2c-a^2bd)x^3\right)} - \frac{(2\,bc-5\,ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{(2\,bc-5\,ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1/3
))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3) - 1/1
8*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(a/b)^(
2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(
c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^
2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b^2*c^2*(a
/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*d*log(x +
(c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*d^2*(c/d)^(
2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = -\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^3-2\,abc^2d+a^2cd^2)} + \frac{(-cd^2)^{\frac{1}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2} + \frac{(-cd^2)^{\frac{1}{3}}\,d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^3-2\,abc^2d+a^2cd^2)} - \frac{(2\,b^2c-5\,abd)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)} + \frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^2c^2-2\sqrt{3}a^3bcd+\sqrt{3}a^4d^2\right)} + \frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)} + \frac{bx}{3\,(bx^3+a)(abc-a^2d)}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/6*(
-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c
- 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) + 1/18*(2*
(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3 + a)*(a*b*
c - a^2*d))
```

Mupad [B] (verification not implemented)

Time = 20.56 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.20

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} \, dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
+ 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(
a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))
/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^
2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^
2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d +
10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 437
4*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2
*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c
^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(27
*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b
*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*
c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(d^5/(27*b^6
*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b
^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3) + (log(((3^(1/2)
*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b
^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d
- b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/2)*(-(b^2*(5
*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(27*a^3*d^3 - 8
*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*
(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^5*d^6*x*(85*a^3*d^
3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3
^(1/2)*1i - 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^2 - 60*a*b^4
*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d + 10935*a^7*b^
4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 4374*a^10*b*c*d
^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*
(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (27*a*b^3*c*d^3*(3^(
1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d
- b*c)^6))^(1/3))/2)*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/
324 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/18 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d
^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3
+ 150*a^2*b^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 437
```

```
4*a^6*b^5*c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9
*b^2*c^2*d^4 - 4374*a^10*b*c*d^5))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(
1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*
c*d))/a + (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(
c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^
4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*
a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/6 + (2*b^5*d^6*x*(85*a^3*d^3 - 4
*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2
)*1i - 1)*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b
^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d
- b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (81*a*b^3*c*d^3*(3^(1/2)*
1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/
(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^
2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))
^(1/3))/6 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^6*c^8 + 27*a^6
*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 +
405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3))/2 - (b*x)/(3*a*(a + b*x^3)*(
a*d - b*c))
```

3.26 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 419

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2} dx = \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{2b^{5/3}(bc-4ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} - \frac{2d^{5/3}(4bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} - \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}$$

```
[Out] 1/3*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)
/(d*x^3+c)+2/9*b^(5/3)*(-4*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c
)^3+2/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^3-1/9
*b^(5/3)*(-4*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a
*d+b*c)^3-1/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^3-2/9*b^(5/3)*(-4*a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/
```

```
3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^3*3^(1/2)-2/9*d^(5/3)*(-a*d+4*b*c
)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^3*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {425, 541, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = -\frac{2b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-4ad)}{3\sqrt{3}a^{5/3}(bc-ad)^3} - \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} - \frac{2d^{5/3}(4bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} + \frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)} + \frac{dx(ad+bc)}{3ac(c+dx^3)(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{(a+bx^3)(c+dx^3)^2}\,dx}{3a(bc-ad)} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} \\
&\quad - \frac{\int \frac{-6\left(b^2c^2-3abcd+a^2d^2\right)-6bd(bc+ad)x^3}{(a+bx^3)(c+dx^3)}\,dx}{9ac(bc-ad)^2} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} \\
&\quad + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^3} + \frac{\left(2d^2(4bc-ad)\right)\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} \\
&\quad + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}(bc-ad)^3} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}(bc-ad)^3} \\
&\quad + \frac{\left(2d^2(4bc-ad)\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}(bc-ad)^3} + \frac{\left(2d^2(4bc-ad)\right)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}(bc-ad)^3}
\end{aligned}
$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} - \frac{\left(b^{5/3}(bc-4ad)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}(bc-ad)^3} + \frac{\left(b^2(bc-4ad)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)^3} - \frac{\left(d^{5/3}(4bc-ad)\right)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}(bc-ad)^3} + \frac{\left(d^2(4bc-ad)\right)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{4/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} - \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3} + \frac{\left(2b^{5/3}(bc-4ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}(bc-ad)^3} + \frac{\left(2d^{5/3}(4bc-ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3c^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)}$$
$$- \frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} - \frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3}$$
$$+ \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3}$$
$$- \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}$$
$$- \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \frac{1}{9}\left(\frac{3b^2x}{a(bc-ad)^2(a+bx^3)} + \frac{3d^2x}{c(bc-ad)^2(c+dx^3)} + \frac{2\sqrt{3}b^{5/3}(bc-4ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(-bc+ad)^3} + \frac{2\sqrt{3}d^{5/3}(-4bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(-bc+4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(-bc+ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}(bc-ad)^3} + \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(-bc+ad)^3} + \frac{d^{5/3}(-4bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}\right)$$

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a*d)
*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*
b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^
```

3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d
)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.68

method	result
default	$\frac{d^2\left(\frac{(ad-bc)x}{3c\left(d\,x^3+c\right)}+\frac{2(ad-4bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^3}+\frac{b^2\left(\frac{(ad-bc)x}{3a\left(b\,x^3+a\right)}+\frac{2(4ad-\ldots$
risch	Expression too large to display

[In] int(1/(b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)

[Out] d^2/(a*d-b*c)^3*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+2/3*(a*d-4*b*c)/c*(1/3/d/(c/d)
^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3)
)+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+b^2/(
a*d-b*c)^3*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+2/3*(4*a*d-b*c)/a*(1/3/b/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3
/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(341) = 682.

Time = 52.87 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.14

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$$
$$= \frac{3(b^3c^2d - a^2bd^3)x^4 + 2\sqrt{3}((b^3c^2d - 4ab^2cd^2)x^6 + ab^2c^3 - 4a^2bc^2d + (b^3c^3 - 3ab^2c^2d - 4a^2bcd^2)x^3)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}}{}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b^3*c^2*d - a^2*b*d^3)*x^4 + 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2)
*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2
)*x^3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*
b)/b) + 2*sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*
d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*arctan
(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - ((b^3*c^2*d - 4*a*b^2
*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*
b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^
2/a^2)^(2/3)) - ((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^
2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d^2*
x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*((b^3*c^2*d - 4*a*b^
2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2
*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*((4*a*b^2*c
*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 3*(b
^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c
^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3
*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(341) = 682$.

Time = 0.30 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.87

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$$
$$= \frac{2\sqrt{3}(b^2c-4\,abd)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+ \frac{2\sqrt{3}(4\,bcd-ad^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$
$$- \frac{(b^2c-4\,abd)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$
$$- \frac{(4\,bcd-ad^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$
$$+ \frac{2\,(b^2c-4\,abd)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$
$$+ \frac{2\,(4\,bcd-ad^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$
$$+ \frac{(b^2cd+abd^2)x^4+(b^2c^2+a^2d^2)x}{3\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^6+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*d^
2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d -
a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c/d)^
(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*c*d^3
*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3) + 3*a^3
*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^
(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/9*(b^2*c -
```

```
4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/
b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2/9*(4*b*c*d
- a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^(2
/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 1/3*((b^2*c*d
+ a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^6 + (a*b^3*c^4
- a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = -\frac{2\,(b^3c-4\,ab^2d)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)} - \frac{2\,(4\,bcd^2-ad^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)} + \frac{2\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^3c^3-3\sqrt{3}a^3b^2c^2d+3\sqrt{3}a^4bcd^2-\sqrt{3}a^5d^3\right)} + \frac{2\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^3c^5-3\sqrt{3}ab^2c^4d+3\sqrt{3}a^2bc^3d^2-\sqrt{3}a^3c^2d^3\right)} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)} + \frac{\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)} + \frac{b^2cdx^4+abd^2x^4+b^2c^2x+a^2d^2x}{3\,(bdx^6+bcx^3+adx^3+ac)(ab^2c^3-2\,a^2bc^2d+a^3cd^2)}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)*(
-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3
 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3
*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x +
```

```
(-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*d + 3*sq
rt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b^2*c - 4*
(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3*c^3
 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d
 - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d
^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

Mupad [B] (verification not implemented)

Time = 28.74 (sec) , antiderivative size = 3637, normalized size of antiderivative = 8.68

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4
*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d + b*
c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d -
 b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b
*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*
a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d
- 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(
a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^
4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49
*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 +
 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561
*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*
b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*
b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^3*d^3*x*(a*d -
b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^
3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9
))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^
4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2
*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d
^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^
6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2
*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*a
^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^1
4 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a
^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^
6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3) + (log(((
```

3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3
+ b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)
1i - 1)(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^
9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d
^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^
2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((
b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d
^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^
2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1
/2)*1i - 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c
^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*
c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^
4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8
))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a
*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27
*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)
^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))
^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^
3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c
^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 + (
16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3
*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*
(a*d - b*c)^8))*(3^(1/2)*1i + 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b
^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c
^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^
4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^
7 - 6561*a^13*b*c*d^8))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1
)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^
4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)
/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^
4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a
^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d -
b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*
d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b
*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i - 1)*(-(8*a^3*d^8 - 512*b^
3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^14 - 729*a^9*c^5
*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a^3*b^6*c^11*d^3
+ 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^6*b^3*c^8*d^6 -
26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2 - (log(((3^(1/2)*1i +
1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*(
(d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6
+ b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 -

```
11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4
*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 + (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c
^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a
*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i + 1)*
(-(8*a^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b
^9*c^14 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 6
1236*a^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61
236*a^6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2
```

3.27 $\int (a-bx^3)(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (a-bx^3)(a+bx^3)^{2/3}\,dx = \frac{7}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a+bx^3)^{5/3} + \frac{7a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

```
[Out] 7/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(b*x^3+a)^(5/3)-7/18*a^2*ln(-b^(1/3)*x+(b*x^
3+a)^(1/3))/b^(1/3)+7/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {396, 201, 245}

$$\int (a-bx^3)(a+bx^3)^{2/3}\,dx = \frac{7a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}} + \frac{7}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a+bx^3)^{5/3}$$

```
[In] Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{6}(7a)\int \left(a+bx^3\right)^{2/3}\,dx\\
&= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{9}\left(7a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}\\
&\quad -\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int (a-bx^3)\,(a+bx^3)^{2/3}\,dx = \frac{3\sqrt[3]{b}(a+bx^3)^{2/3}(4ax-3bx^4)+14\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-14a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a}\right.}{54\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b^(1/3)*(a + b*x^3)^(2/3)*(4*a*x - 3*b*x^4) + 14*Sqrt[3]*a^2*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 14*a^2*Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)] + 7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{-9x^4(bx^3+a)^{\frac{2}{3}}b^{\frac{4}{3}}+12ax(bx^3+a)^{\frac{2}{3}}b^{\frac{1}{3}}-14a^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-14a^2\ln\left(\frac{-b^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+7a^2\ln\left(\right.}{54b^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-9*x^4*(b*x^3+a)^(2/3)*b^(4/3)+12*a*x*(b*x^3+a)^(2/3)*b^(1/3)-14*a^2*
3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-14*a^2*
ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+7*a^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(85) = 170.

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.56

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx = \left[\frac{21\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+\right.\right.}{}\right.$$

$$-\frac{42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+14\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-7\,a^2}{54\,b}$$

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*log((
(-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) -
 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^2*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))
/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx = \frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.88

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right.$$

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*b
```

Giac [F]

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx=\int -\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)\,dx$$

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx=\int \left(b\,x^3+a\right)^{2/3}\left(a-b\,x^3\right)\,dx$$

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3),x)
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=-\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)-2/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+4/9*a*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {396, 245}

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=\frac{4a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3)) + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)])/(3*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{1}{3}(4a)\int\frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int\frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{-3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}+4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-4a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+2a\log\left(b^{2/3}x^2+\right.}{9\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b
^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
] + 2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(9*b^(1/3))
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{4\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a+\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}}x\,b^{\frac{1}{3}}}{4}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+\right.}{x^{2}}\right.}{2}}{9b^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -4/9*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a
+3/4*(b*x^3+a)^(2/3)*x*b^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-1/2*ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.99

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-b\right.\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+3\,(bx^3+a)^{\frac{2}{3}}bx+4\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(}{x}\right.}{9\,b}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 + a)
^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
```

```
 a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt
(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 3*(b*x^
3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.68

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right. \\ -\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

Giac [F]

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(1/3),x)
[Out] int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

3.29 $\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 2*x/(b*x^3+a)^(1/3)+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)-1/3*arctan(1
/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {393, 245}

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\text{integral} = \frac{2x}{\sqrt[3]{a+bx^3}} - \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}} - \frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*
x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b
^(1/3)) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
]/(6*b^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 3.93 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right.}{2}}{3b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/
3)/x)*(b*x^3+a)^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*(b*x^3+a)^(1/3)-1/
2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*(b*x^3+a)
^(1/3)+6*b^(1/3)*x)/(b*x^3+a)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.38

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(b^2x^3+ab)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2\right.\right.}{}\right.$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
- 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a)^(
2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
(b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2*(b*x^
3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)*b^(2/3
)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(
1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(4
/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7
/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{1}{6}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+\cdots}{x^2}\right.}{b^{\frac{4}{3}}}\right. \\ + \frac{x}{(bx^3+a)^{\frac{1}{3}}}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b
^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1
/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{a-b\,x^3}{(b\,x^3+a)^{4/3}}\,dx$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(4/3),x)
[Out] int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

3.30 $\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+3/4*x/a/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {386, 197}

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}+\frac{3}{4}\int\frac{1}{(a+bx^3)^{4/3}}\,dx\\&=\frac{x(a-bx^3)}{4a\,(a+bx^3)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int\frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{2ax+bx^4}{2a\,(a+bx^3)^{4/3}}$$

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
[Out] (2*a*x + b*x^4)/(2*a*(a + b*x^3)^(4/3))
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25
trager	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25
pseudoelliptic	$\frac{x(b\,x^3+2a)}{2(b\,x^3+a)^{\frac{4}{3}}\,a}$	25

```
[In] int((-b*x^3+a)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{(bx^4+2\,ax)(bx^3+a)^{\frac{2}{3}}}{2\,(ab^2x^6+2\,a^2bx^3+a^3)}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
[Out] 1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(37) = 74.

Time = 19.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) - \frac{bx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
[Out] a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}a} - \frac{bx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
+ a)^(4/3)*a)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{x\,(b\,x^3+a)+a\,x}{2\,a\,(b\,x^3+a)^{4/3}}$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(7/3),x)
[Out] (x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

3.31 $\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx=\frac{2x}{7(a+bx^3)^{7/3}}+\frac{5x}{28a(a+bx^3)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

```
[Out] 2/7*x/(b*x^3+a)^(7/3)+5/28*x/a/(b*x^3+a)^(4/3)+15/28*x/a^2/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {393, 198, 197}

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx=\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a(a+bx^3)^{4/3}}+\frac{2x}{7(a+bx^3)^{7/3}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5}{7}\int\frac{1}{\left(a+bx^3\right)^{7/3}}\,dx\\
&= \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15\int\frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a}\\
&= \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int\frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx=\frac{28a^2x+35abx^4+15b^2x^7}{28a^2\left(a+bx^3\right)^{7/3}}$$

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
[Out] (28*a^2*x + 35*a*b*x^4 + 15*b^2*x^7)/(28*a^2*(a + b*x^3)^(7/3))
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(15b^2x^6+35ab\,x^3+28a^2\right)}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^2}$	37
trager	$\frac{x\left(15b^2x^6+35ab\,x^3+28a^2\right)}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^2}$	37
pseudoelliptic	$\frac{x\left(15b^2x^6+35ab\,x^3+28a^2\right)}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^2}$	37

```
[In] int((-b*x^3+a)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(15\,b^2x^7+35\,abx^4+28\,a^2x)(bx^3+a)^{\frac{2}{3}}}{28\,(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5)}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
[Out] 1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(49) = 98$.

Time = 100.31 (sec) , antiderivative size = 709, normalized size of antiderivative = 12.89

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = a\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+\cdots}\right.$$

$$+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\cdots}$$

$$+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\cdots}$$

$$+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\cdots}$$

$$-b\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right.$$

$$\left.+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

```
[Out] a*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 8
1*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x*
*6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a
)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3
/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10
/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/
3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*gamma(1
/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(
1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(
1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)
) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma
(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/
3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1
+ b*x**3/a)**(1/3)*gamma(10/3))) - b*(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 +
 b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*g
amma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*
x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/
3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
- 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx=\frac{15\,x\,\left(b\,x^3+a\right)^2+8\,a^2\,x+5\,a\,x\,\left(b\,x^3+a\right)}{28\,a^2\,\left(b\,x^3+a\right)^{7/3}}$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(10/3),x)
[Out] (15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/3
))
```

3.32 $\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x}{5(a+bx^3)^{10/3}} + \frac{4x}{35a(a+bx^3)^{7/3}} + \frac{6x}{35a^2(a+bx^3)^{4/3}} + \frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x/(b*x^3+a)^(10/3)+4/35*x/a/(b*x^3+a)^(7/3)+6/35*x/a^2/(b*x^3+a)^(4/3)+
18/35*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {393, 198, 197}

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{18x}{35a^3\sqrt[3]{a+bx^3}} + \frac{6x}{35a^2(a+bx^3)^{4/3}} + \frac{4x}{35a(a+bx^3)^{7/3}} + \frac{x}{5(a+bx^3)^{10/3}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4}{5}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{24\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a}\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{35a^2}\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{35a^3x+70a^2bx^4+60ab^2x^7+18b^3x^{10}}{35a^3\left(a+bx^3\right)^{10/3}}$$

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
[Out] (35*a^3*x + 70*a^2*b*x^4 + 60*a*b^2*x^7 + 18*b^3*x^10)/(35*a^3*(a + b*x^3)^
(10/3))
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x\left(18 b^{3} x^{9}+60 a \,b^{2} x^{6}+70 a^{2} b \,x^{3}+35 a^{3}\right)}{35 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{3}}$	48
trager	$\frac{x\left(18 b^{3} x^{9}+60 a \,b^{2} x^{6}+70 a^{2} b \,x^{3}+35 a^{3}\right)}{35 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{3}}$	48
pseudoelliptic	$\frac{x\left(18 b^{3} x^{9}+60 a \,b^{2} x^{6}+70 a^{2} b \,x^{3}+35 a^{3}\right)}{35 \left(b \,x^{3}+a \right)^{\frac{10}{3}} a^{3}}$	48

```
[In] int((-b*x^3+a)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
[Out] 1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x)(bx^3+a)^{\frac{2}{3}}}{35\,(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7)}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
[Out] 1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2/
3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(14\,b^2 - \frac{40\,(bx^3+a)b}{x^3} + \frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3 - \frac{60\,(bx^3+a)b^2}{x^3} + \frac{105\,(bx^3+a)^2b}{x^6} - \frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x}{5\,(b\,x^3+a)^{10/3}} + \frac{18\,x}{35\,a^3\,(b\,x^3+a)^{1/3}} + \frac{6\,x}{35\,a^2\,(b\,x^3+a)^{4/3}} + \frac{4\,x}{35\,a\,(b\,x^3+a)^{7/3}}$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a^
2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

3.33 $\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2x}{13(a+bx^3)^{13/3}}+\frac{11x}{130a(a+bx^3)^{10/3}}+\frac{99x}{910a^2(a+bx^3)^{7/3}}+\frac{297x}{1820a^3(a+bx^3)^{4/3}}+\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}$$

```
[Out] 2/13*x/(b*x^3+a)^(13/3)+11/130*x/a/(b*x^3+a)^(10/3)+99/910*x/a^2/(b*x^3+a)^
(7/3)+297/1820*x/a^3/(b*x^3+a)^(4/3)+891/1820*x/a^4/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {393, 198, 197}

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3(a+bx^3)^{4/3}}+\frac{99x}{910a^2(a+bx^3)^{7/3}}+\frac{11x}{130a(a+bx^3)^{10/3}}+\frac{2x}{13(a+bx^3)^{13/3}}$$

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11}{13}\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx\\
&= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a}\\
&= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{297\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a^2}\\
&= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{891\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{1820a^3}\\
&= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{x(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12})}{1820a^4\,(a+bx^3)^{13/3}}$$

[In] Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]

[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4 *x^12))/(1820*a^4*(a + b*x^3)^(13/3))

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}\,a^4}$	59
trager	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}\,a^4}$	59
pseudoelliptic	$\frac{x(891b^4x^{12}+3861a\,b^3x^9+6435a^2b^2x^6+5005a^3b\,x^3+1820a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}\,a^4}$	59

[In] int((-b*x^3+a)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)

[Out] 1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820* a^4)/(b*x^3+a)^(13/3)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x)(bx^3+a)^{\frac{2}{3}}}{1820\,(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9)}$$

[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")

[Out] 1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4 + 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b^3 *x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)

Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.65

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx=\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^4}$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
[Out] 1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(
b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*
x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 45
5*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx=\int -\frac{bx^3-a}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2\,x}{13\,(b\,x^3+a)^{13/3}}+\frac{891\,x}{1820\,a^4\,(b\,x^3+a)^{1/3}}+\frac{297\,x}{1820\,a^3\,(b\,x^3+a)^{4/3}}+\frac{99\,x}{910\,a^2\,(b\,x^3+a)^{7/3}}+\frac{11\,x}{130\,a\,(b\,x^3+a)^{10/3}}$$

```
[In] int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (297
*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (11
*x)/(130*a*(a + b*x^3)^(10/3))
```

3.34 $\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 483

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\frac{7}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{4\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}} - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{2\sqrt[3]{2}a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2}a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

```
[Out] -7/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(b*x^3+a)^(4/3)-7/5*a^2*x*(1+b*x^3/a)^(2/3)*
hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2/3*2^(1/3)*a^(5/3)*ln
(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+2/3*2^(1/3)*a^(5/3)*
ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)-4/3*2^(1/3)*a^(5/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(5/3)*ln(2*2^(1/3)+(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)-4/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)-2/3*2^(1/3)*a^(5/3)*arctan(1/3*(1+2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {427, 542, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\frac{4\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{5/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{2\sqrt[3]{2}a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2}a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2}a^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{3\sqrt[3]{b}} - \frac{7a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} - \frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

```
[In] Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (-7*a*x*(a + b*x^3)^(1/3))/5 - (x*(a + b*x^3)^(4/3))/5 - (4*2^(1/3)*a^(5/3)
*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)) - (2*2^(1/3)*a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (7*a^2*x*(1 + (b*
x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3
)^(2/3)) - (2*2^(1/3)*a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^
```

```
3)^(1/3)])/(3*b^(1/3)) + (2*2^(1/3)*a^(5/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(
1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3)])/(3*b^(1/3)) - (4*2^(1/3)*a^(5/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3
)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^(5/3)*Log[2*2^(1/3) + (a
^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{\int \frac{\sqrt[3]{a+bx^3}\left(-6a^2b-14ab^2x^3\right)}{a-bx^3}\,dx}{5b} \\
&= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} + \frac{\int \frac{26a^3b^2+54a^2b^3x^3}{\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}}\,dx}{10b^2} \\
&= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} \\
&\quad - \frac{1}{5}\left(27a^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + \left(8a^3\right)\int \frac{1}{\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}}\,dx \\
&= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} + \left(4a^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx \\
&\quad + \left(4a^2\right)\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx - \frac{\left(27a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}} \\
&= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{27a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}} \\
&\quad + \frac{\left(36a^{7/3}\right)\text{Subst}\left(\int \frac{x}{\left(4-ax^3\right)\left(1+2ax^3\right)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} \\
&\quad + \frac{\left(4a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

$$= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{\left(4a^{7/3}\right)\text{Subst}\left(\int\frac{x}{4-ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$+ \frac{\left(8a^{7/3}\right)\text{Subst}\left(\int\frac{x}{1+2ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{\left(2\sqrt[3]{2}a^2\right)\text{Subst}\left(\int\frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{\left(2\sqrt[3]{2}a^2\right)\text{Subst}\left(\int\frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{\left(4\ 2^{2/3}a^2\right)\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{\left(4\ 2^{2/3}a^2\right)\text{Subst}\left(\int\frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}$$

$$-\frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+\frac{\left(\sqrt[3]{2}a^{5/3}\right)\text{Subst}\left(\int\frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+\frac{\left(2\sqrt[3]{2}a^{5/3}\right)\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$-\frac{(2a^2)\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$+\frac{\left(2\ 2^{2/3}a^2\right)\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$
\begin{aligned}
&= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x\left(a+bx^3\right)^{4/3} - \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}} \\
&\quad - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} \\
&\quad + \frac{2\sqrt[3]{2}a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} \\
&\quad - \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} \\
&\quad + \frac{\sqrt[3]{2}a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} \\
&\quad + \frac{\left(2\sqrt[3]{2}a^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} \\
&\quad + \frac{\left(4\sqrt[3]{2}a^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}
\end{aligned}
$$

$$= -\frac{7}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a+bx^3)^{4/3} - \frac{4\sqrt[3]{2}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

$$- \frac{2\sqrt[3]{2}a^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

$$- \frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{2\sqrt[3]{2}a^{5/3}\log\left(1+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{\sqrt[3]{2}a^{5/3}\log\left(2\sqrt[3]{2}+\frac{(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \frac{27abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+4x\left(-8a^2-9abx^3-b^2x^6+\frac{\ldots}{(a-bx^3}\right.}{20(a+b}$$

```
[In] Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
(b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
```

```
4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^
3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))
)))/(20*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{7}{3}}}{-b\,x^{3}+a}dx$$

```
[In] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
[Out] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\int \frac{a^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{b^2x^6\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{2abx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
[Out] -Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*(
a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**(1
/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{7/3}}{a-b\,x^3}\,dx$$

```
[In] int((a + b*x^3)^(7/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(7/3)/(a - b*x^3), x)
```

3.35 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 464

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{2\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3]
,-b*x^3/a)/(b*x^3+a)^(2/3)-1/3*2^(1/3)*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/
3)-2/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)+1/6*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(1/3)-2/3*2^(1/3)*a^(2/3)*
arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/
```

```
3)*3^(1/2)-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {427, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx &= -\frac{2\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} \\
&- \frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} \\
&+ \frac{\sqrt[3]{2}a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}} \\
&- \frac{2\sqrt[3]{2}a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}} \\
&+ \frac{a^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{b}} \\
&- \frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}} - \frac{1}{2}x\sqrt[3]{a+bx^3}
\end{aligned}$$

```
[In] Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] -1/2*(x*(a + b*x^3)^(1/3)) - (2*2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (2^(1/3
)*a^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sq
rt[3]])/(Sqrt[3]*b^(1/3)) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/
```

```
3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (2^(1/3)*a^(2/3)*Log[2^
(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^
(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3
)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) - (2*2^(1/3)*a^(2/
3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3))
+ (a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(
2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}x\sqrt[3]{a+bx^3}-\frac{\int \frac{-3a^2b-5ab^2x^3}{(a-bx^3)(a+bx^3)^{2/3}}\,dx}{2b}\\
&= -\frac{1}{2}x\sqrt[3]{a+bx^3}-\frac{1}{2}(5a)\int \frac{1}{(a+bx^3)^{2/3}}\,dx+\left(4a^2\right)\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx\\
&= -\frac{1}{2}x\sqrt[3]{a+bx^3}+(2a)\int \frac{1}{(a+bx^3)^{2/3}}\,dx\\
&\quad +(2a)\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx-\frac{\left(5a\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2\,(a+bx^3)^{2/3}}\\
&= -\frac{1}{2}x\sqrt[3]{a+bx^3}-\frac{5ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\,(a+bx^3)^{2/3}}\\
&\quad +\frac{\left(18a^{4/3}\right)\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}\\
&\quad +\frac{\left(2a\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}\\
&= -\frac{1}{2}x\sqrt[3]{a+bx^3}-\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\,(a+bx^3)^{2/3}}\\
&\quad +\frac{\left(2a^{4/3}\right)\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}\\
&\quad +\frac{\left(4a^{4/3}\right)\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}
\end{aligned}$$

$$= -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{\left(\sqrt[3]{2}a\right)\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{\left(\sqrt[3]{2}a\right)\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{\left(2\ 2^{2/3}a\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{\left(2\ 2^{2/3}a\right)\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

$$-\frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+\frac{a^{2/3}\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

$$+\frac{\left(\sqrt[3]{2}a^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$-\frac{a\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$+\frac{\left(2^{2/3}a\right)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

$$-\frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+\frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$-\frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+\frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

$$+\frac{\left(\sqrt[3]{2}a^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$+\frac{\left(2\sqrt[3]{2}a^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{2\sqrt[3]{2}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$
$$- \frac{\sqrt[3]{2}a^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$
$$- \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$+ \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$- \frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$
$$+ \frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.47

$$\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx = \frac{x\left(-4(a+bx^3)+5bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{\cdots}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\cdots\right.\right.}\right.}{8\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/
a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
```

```
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] -
 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(8*(a + b*x^3)^
(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{-b\,x^{3}+a}dx$$

```
[In] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
[Out] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
[Out] integral(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx=-\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
[Out] -Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b*
x**3)**(1/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{a-b\,x^3}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(4/3)/(a - b*x^3), x)
```

3.36 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 398

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*ln(1+2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)+1/12*ln(2*2^(1/3)+(a
^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(
1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)*3^(1/2)-1/6*arctan(1/3
*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(1/3)/b
^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)) - Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(
1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)) -
 (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^
(1/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3)
+ (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(1/3)*b^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\left(9\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}}$$

$$- \frac{2^{2/3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{2^{2/3}\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$
$$-\frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$
$$+\frac{\sqrt[3]{2}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{a}\sqrt[3]{b}}$$
$$= -\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$
$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$
$$-\frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.08

$$\int\frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$$
$$=\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\sqrt[3]{\right.}}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/
3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^
(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Log[2
^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/3)*a
^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/3) +
2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x^3)
^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Log
[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a + b*x^3)^
(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^3+a}dx$$

```
[In] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(284) = 568.

Time = 16.54 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx =$$

$$-\frac{1}{18}\sqrt{3}2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}(ab^6x^{16}+33\,a^2b^5x^{13}+110\,a^3b^4x^{10}+110\,a^4b^3x^7+33\,a^5b^2x^4+a^6bx}{}\right.$$

$$-\frac{1}{36}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(ab^3x^8+4\,a^2b^2x^5+a^3bx^2)(bx^3+a)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(b^4x^{12}+32\,ab^3x^9+78\,a^2b^2x^6}{b^4x^{12}-4\,ab^3x^9+6\,a^2b^2x^6}\right.$$

$$+\frac{1}{18}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(-\frac{12\,(bx^3+a)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}(b^2x^6-2\,abx^3+a^2)\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}(bx^4+ax)(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{ab}\right.}{b^2x^6-2\,abx^3+a^2}\right.$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^6
*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2*x
```

```
^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a*b^
5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b*x^3
 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417*a^2*
b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(b^6*x^
18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b^2*x^6
+ 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(2/3)*(a*
b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^(2/3) - 2
^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 + a^4)*(-1/
(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x)*(b*x^3 +
a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3 + a^4)) + 1
/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 + 2^(2/3)*(b^2*
x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 + a*x)*(b*x^3 +
a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int \frac{{\left(b\,x^3+a\right)}^{1/3}}{a-b\,x^3}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

3.37 $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 452

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{4/3}\sqrt[3]{b}}$$

```
[Out] 1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2
/3)-1/12*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b
^(1/3)+1/12*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*ln(1+2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/24*ln(2*2^(1/
```

```
3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/b^(1/3)*3^(1/2)-1/12*arct
an(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
4/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {421, 252, 251, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{12\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
/(2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3)) + (x
*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a
*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)
]/(6*2^(2/3)*a^(4/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
```

```
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^
(2/3)*a^(4/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3
)^(1/3)]/(3*2^(2/3)*a^(4/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x
)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/
(12*2^(2/3)*a^(4/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2a} + \frac{\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx}{2a}$$

$$= \frac{9\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2a^{2/3}\sqrt[3]{b}} + \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2a\,(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} + \frac{\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a\sqrt[3]{b}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{4a\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{2}a\sqrt[3]{b}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}}$$

$$+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}}$$

$$+ \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}}$$

$$- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{4/3}\sqrt[3]{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \frac{4ax\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a}\right.}{(a-bx^3)(a+bx^3)^{2/3}\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\text{AppellF1}\left(\frac{4}{3}\right.\right.\right.}$$

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(a
 + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] +
b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Maple [F]

$$\int \frac{1}{(-b\,x^3+a)\,(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = -\int \frac{1}{-a\,(a+bx^3)^{\frac{2}{3}}+bx^3\,(a+bx^3)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
[Out] -Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(a-b\,x^3)}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)), x)
```

3.38 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 473

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{x}{4a^2(a+bx^3)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{7/3}\sqrt[3]{b}}$$

```
[Out] 1/4*x/a^2/(b*x^3+a)^(2/3)+1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3
],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)-1/24*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)+1/24*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/
(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3
)/b^(1/3)-1/12*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^
```

(7/3)/b^(1/3)+1/48*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(7/3)/b^(1/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(7/3)/b^(1/3)*3^(1/2)

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {425, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{24\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2(a+bx^3)^{2/3}} + \frac{x}{4a^2(a+bx^3)^{2/3}}$$

[In] Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]

[Out] x/(4*a^2*(a + b*x^3)^(2/3)) - ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(7/3)*b^(1/3)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3)*Sqrt[3]*a^(7/3)*b^(1/3)) + (x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a^2*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) + Log[1 + (2^

```
(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1
/3)*x))/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) - Log[1 + (2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(7/3)*b^(1/3)) + Log
[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3)
+ b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2^(2/3)*a^(7/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}}-\frac{\int \frac{-3ab+b^2x^3}{(a-bx^3)(a+bx^3)^{2/3}}\,dx}{4a^2b}\\
&= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+\frac{\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{4a^2}+\frac{\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx}{2a}\\
&= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+\frac{\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{4a^2}+\frac{\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx}{4a^2}+\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{4a^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a^2\left(a+bx^3\right)^{2/3}}\\
&\quad+\frac{9\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{4a^{5/3}\sqrt[3]{b}}+\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{4a^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{4a^{5/3}\sqrt[3]{b}}+\frac{\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

$$= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^2\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^2\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{2}a^2\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{2}a^2\sqrt[3]{b}}$$

$$= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{8a^2\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{4\sqrt[3]{2}a^2\sqrt[3]{b}}$$

$$= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}$$

$$- \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{4\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$= \frac{x}{4a^2\left(a+bx^3\right)^{2/3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}}$$

$$+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$+ \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

$$- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{7/3}\sqrt[3]{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}}\,dx = \frac{x\left(\frac{4}{a^2}-\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{\cdots}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)\cdots\right)}\right)}{16\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^
3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Appell
F1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = -\int \frac{1}{-a^2\left(a+bx^3\right)^{\frac{2}{3}}+b^2x^6\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
[Out] -Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(bx^3-a\right)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(a-b\,x^3)}\,dx$$

```
[In] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)
```

3.39 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 492

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{x}{10a^2(a+bx^3)^{5/3}}+\frac{13x}{40a^3(a+bx^3)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} + \frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{20a^3(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{48\,2^{2/3}a^{10/3}\sqrt[3]{b}}$$

```
[Out] 1/10*x/a^2/(b*x^3+a)^(5/3)+13/40*x/a^3/(b*x^3+a)^(2/3)+9/20*x*(1+b*x^3/a)^(
2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)-1/48*ln(2^(2/
3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/48*ln(1
+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*ln(1+2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/96*ln(2*2^(1/3)+(a^(1/3)+b
```

```
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^
(1/3)/a^(10/3)/b^(1/3)-1/24*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1/3)*3^(1/2)-1/48*arctan(1/3*(1+
2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1
/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {425, 541, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{48\ 2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{9x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{20a^3(a+bx^3)^{2/3}} + \frac{13x}{40a^3(a+bx^3)^{2/3}} + \frac{x}{10a^2(a+bx^3)^{5/3}}$$

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] x/(10*a^2*(a + b*x^3)^(5/3)) + (13*x)/(40*a^3*(a + b*x^3)^(2/3)) - ArcTan[(
1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3
)*Sqrt[3]*a^(10/3)*b^(1/3)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
```

```
a + b*x^3)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]*a^(10/3)*b^(1/3)) + (9*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(20*a^3*(
a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/
(24*2^(2/3)*a^(10/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2
^(2/3)*a^(10/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x
^3)^(1/3)]/(12*2^(2/3)*a^(10/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/
3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/
3)]/(48*2^(2/3)*a^(10/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
```

*c - a*d, 0] && EqQ[b*c + a*d, 0]

Rule 421

Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a*d, 0]

Rule 425

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c - a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 493

Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/ (b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]

Rule 541

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f _.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b* c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 544

Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)* (x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e - c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p , n}, x]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}} - \frac{\int \frac{-9ab+4b^2x^3}{\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}}\,dx}{10a^2b} \\
&= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}} + \frac{13x}{40a^3\left(a+bx^3\right)^{2/3}} + \frac{\int \frac{23a^2b^2-13ab^3x^3}{\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}}\,dx}{40a^4b^2} \\
&= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}} + \frac{13x}{40a^3\left(a+bx^3\right)^{2/3}} + \frac{13\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{40a^3} + \frac{\int \frac{1}{\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}}\,dx}{4a^2} \\
&= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}} + \frac{13x}{40a^3\left(a+bx^3\right)^{2/3}} + \frac{\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{8a^3} \\
&\quad + \frac{\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx}{8a^3} + \frac{\left(13\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{40a^3\left(a+bx^3\right)^{2/3}} \\
&= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}} + \frac{13x}{40a^3\left(a+bx^3\right)^{2/3}} + \frac{13x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{40a^3\left(a+bx^3\right)^{2/3}} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{x}{\left(4-ax^3\right)\left(1+2ax^3\right)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{8a^{8/3}\sqrt[3]{b}} + \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{8a^3\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

$$= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}$$

$$+\frac{\text{Subst}\left(\int\frac{x}{4-ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{8a^{8/3}\sqrt[3]{b}}+\frac{\text{Subst}\left(\int\frac{x}{1+2ax^3}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{4a^{8/3}\sqrt[3]{b}}$$

$$= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^3\sqrt[3]{b}}$$

$$-\frac{\text{Subst}\left(\int\frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^3\sqrt[3]{b}}$$

$$-\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\sqrt[3]{2}a^3\sqrt[3]{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{12\sqrt[3]{2}a^3\sqrt[3]{b}}$$

$$= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}$$

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}-\frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{48\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$

$$-\frac{\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{16a^3\sqrt[3]{b}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{8\sqrt[3]{2}a^3\sqrt[3]{b}}$$

$$= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}$$
$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{48\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{8\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{4\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$

$$= \frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}}$$
$$-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}}+\frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}$$
$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$
$$-\frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{48\ 2^{2/3}a^{10/3}\sqrt[3]{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{x\left(16a^2+52a(a+bx^3)-13bx^3(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\right.\right.}{}$$

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

Maple [F]

$$\int \frac{1}{(-b\,x^3+a)\,(b\,x^3+a)^{\frac{8}{3}}}dx$$

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = -\int \frac{1}{-a^3(a+bx^3)^{\frac{2}{3}} - a^2bx^3(a+bx^3)^{\frac{2}{3}} + ab^2x^6(a+bx^3)^{\frac{2}{3}} + b^3x^9(a+bx^3)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
[Out] -Integral(1/(-a**3*(a + b*x**3)**(2/3) - a**2*b*x**3*(a + b*x**3)**(2/3) +
a*b**2*x**6*(a + b*x**3)**(2/3) + b**3*x**9*(a + b*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(a-b\,x^3)}\,dx$$

```
[In] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)
```

3.40 $\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{8}{27}ax(a+bx^3)^{5/3} - \frac{1}{9}x(a-bx^3)(a+bx^3)^{5/3} + \frac{76a^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{38a^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81\sqrt[3]{b}}$$

```
[Out] 38/81*a^2*x*(b*x^3+a)^(2/3)-8/27*a*x*(b*x^3+a)^(5/3)-1/9*x*(-b*x^3+a)*(b*x^
3+a)^(5/3)-38/81*a^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+76/243*a^3*arct
an(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {427, 396, 201, 245}

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \frac{76a^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81\sqrt[3]{b}} + \frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{8}{27}ax(a+bx^3)^{5/3} - \frac{1}{9}x(a-bx^3)(a+bx^3)^{5/3}$$

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*
x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/(81*b^(1/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{\int \left(a+bx^3\right)^{2/3}\left(10a^2b-16ab^2x^3\right)\,dx}{9b}\\
&= -\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{1}{27}\left(38a^2\right)\int \left(a+bx^3\right)^{2/3}\,dx\\
&= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{1}{81}\left(76a^3\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx
\end{aligned}$$

$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{76a^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{38a^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.20

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx = \frac{3\sqrt[3]{b}\left(a+bx^3\right)^{2/3}\left(5a^2x-24abx^4+9b^2x^7\right)+76\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-76a^3\log\left(\cdots\right.}{243\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b^(1/3)*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + 76*Sqrt[3
]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 76*a^
3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 38*a^3*Log[b^(2/3)*x^2 + b^(1/3)*
x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(1/3))
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{27b^{\frac{7}{3}}\left(b\,x^3+a\right)^{\frac{2}{3}}x^7-72a\,b^{\frac{4}{3}}x^4\left(b\,x^3+a\right)^{\frac{2}{3}}+15a^2x\,b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{2}{3}}-76\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a^3-76\ln\left(\frac{-b^{\frac{1}{3}}x+\cdots}{\cdots}\right.}{243b^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/243*(27*b^(7/3)*(b*x^3+a)^(2/3)*x^7-72*a*b^(4/3)*x^4*(b*x^3+a)^(2/3)+15*a
^2*x*b^(1/3)*(b*x^3+a)^(2/3)-76*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*
x^3+a)^(1/3))/b^(1/3)/x)*a^3-76*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^3+38*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^3)/b^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.03

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3}\,dx = \left[\frac{114\sqrt{\frac{1}{3}}a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\right.\right.}{}\right.$$

$$-\frac{228\sqrt{\frac{1}{3}}a^3b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+76\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-38\,a}{243\,b}$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^
(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a^3*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1/243*(
228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2
*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(((-b)^(1
/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*x^7 - 24*a
*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \left(a-bx^3\right)^2\left(a+bx^3\right)^{2/3}\,dx=\frac{a^{\frac{8}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{5}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*hype
r((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.97

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{27}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a^3\log\left(-b^{\frac{1}{3}}+\frac{(bx}{}\right.}{b^{\frac{7}{3}}}\right.$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
```

```
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

Giac [F]

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \int (bx^3+a)^{\frac{2}{3}} (bx^3-a)^2 \, dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \int (b\,x^3+a)^{2/3}\,(a-b\,x^3)^2 \, dx$$

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

3.41 $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}} \, dx = -\frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)\left(a+bx^3\right)^{2/3} + \frac{17a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

```
[Out] -13/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(-b*x^3+a)*(b*x^3+a)^(2/3)-17/18*a^2*ln(-b
^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+17/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {427, 396, 245}

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}} \, dx = \frac{17a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}} - \frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)\left(a+bx^3\right)^{2/3}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{\int \frac{7a^2b-13ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}\\
&= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{1}{9}\left(17a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\
&= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\\
&\quad+\frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}(a+bx^3)^{2/3}(-16ax+3bx^4) + \frac{17a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27\sqrt[3]{b}} + \frac{17a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-16*a*x + 3*b*x^4))/18 + (17*a^2*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (17*a^2*Log[
-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (17*a^2*Log[b^(2/3)*x^2 +
 b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{9x^4(bx^3+a)^{\frac{2}{3}}b^{\frac{4}{3}}-48ax(bx^3+a)^{\frac{2}{3}}b^{\frac{1}{3}}-34a^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-34a^2\ln\left(\frac{-b^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+17a^2\ln\left(\right.}{54b^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(9*x^4*(b*x^3+a)^(2/3)*b^(4/3)-48*a*x*(b*x^3+a)^(2/3)*b^(1/3)-34*a^2*3
^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-34*a^2*l
n((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+17*a^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.32

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\Big((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}\right.}{} \right.$$

$$-\frac{102\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\Big((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\Big)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+34\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-}{54\,b}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*log(
((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)*a^2*b
*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))
*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(
2/3))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(94) = 188.

Time = 0.27 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.63

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*b^2
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.42 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{10a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

```
[Out] 2*x*(-b*x^3+a)/(b*x^3+a)^(1/3)+7/3*x*(b*x^3+a)^(2/3)+5/3*a*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)-10/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {424, 396, 245}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = -\frac{10a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} + \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{\int \frac{-a^2b+7ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab} \\
&= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{1}{3}(10a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.36

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\left(\frac{3(13ax+bx^4)}{\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{10a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}} - \frac{5a\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}\right)$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] ((3*(13*a*x + b*x^4))/(a + b*x^3)^(1/3) - (10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) + (10*a*Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)])/b^(1/3) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3))/9
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{3b^{\frac{4}{3}}x^4+10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a\left(b\,x^3+a\right)^{\frac{1}{3}}+39xa\,b^{\frac{1}{3}}+10\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)a\left(b\,x^3+a\right)^{\frac{1}{3}}-5\ln\left(\frac{b^{\frac{2}{3}}x}{}\right.}{9b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}}$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(3*b^(4/3)*x^4+10*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/
3))/b^(1/3)/x)*a*(b*x^3+a)^(1/3)+39*x*a*b^(1/3)+10*ln((-b^(1/3)*x+(b*x^3+a)
^(1/3))/x)*a*(b*x^3+a)^(1/3)-5*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b
*x^3+a)^(2/3))/x^2)*a*(b*x^3+a)^(1/3))/b^(1/3)/(b*x^3+a)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(89) = 178.

Time = 0.36 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.65

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left[\frac{15\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}b\right.\right.}{} \right.$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*
x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b
*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*x^3
 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 + a^2)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1/9*(10
*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x
^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3)*(b^(1/3
)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13*a*b*x)*(b
*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(89) = 178.

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,b^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right) + \frac{1}{3}\,ab\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right) + \frac{ax}{(bx^3+a)^{\frac{1}{3}}}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{4/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.43 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx = \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*x*(-b*x^3+a)/(b*x^3+a)^(4/3)-1/2*x/(b*x^3+a)^(1/3)-1/2*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/
2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {424, 393, 245}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} \, dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a-bx^3)}{2\left(a+bx^3\right)^{4/3}}+\frac{\int \frac{2a^2b+4ab^2x^3}{\left(a+bx^3\right)^{4/3}}\,dx}{4ab}\\
&= \frac{x(a-bx^3)}{2\left(a+bx^3\right)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{x(a-bx^3)}{2\left(a+bx^3\right)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}} + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + \log\left(b^{2/3}\right.}{6\sqrt[3]{b}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/
(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*
b^(1/3))
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-6b^{\frac{4}{3}}x^4+(bx^3+a)^{\frac{4}{3}}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+(b\right.}{x}\right.\right.}{6b^{\frac{1}{3}}(bx^3+a)^{\frac{4}{3}}}$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^(1/3)/(b*x^3+a)^(4/3)*(-6*b^(4/3)*x^4+(b*x^3+a)^(4/3)*(-2*3^(1/2)*arc
tan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^
(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1
/3))/x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(87) = 174$.

Time = 0.35 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left[-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4 - 3\sqrt{\frac{1}{3}}(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b\right.}{}\right.$$

$$-\frac{6\,(bx^3+a)^{\frac{2}{3}}b^2x^4 + 6\sqrt{\frac{1}{3}}(b^3x^6+2\,ab^2x^3+a^2b)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(b^2}{6\,(b^3x^6+}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 -
 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2
/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^
3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b*x^3 +
a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt(-(-b)^(
1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(
1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (
b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/
3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6
 + 2*a*b^2*x^3 + a^2*b)]
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}} - \frac{bx^4}{2\,(bx^3+a)^{\frac{4}{3}}} - \frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4}{-}\right.$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)^
(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*sqr
t(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3)
- 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{7/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.44 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx=\frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

```
[Out] 1/7*x*(-b*x^3+a)^2/a/(b*x^3+a)^(7/3)+3/14*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+9/
14*x/a/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {386, 197}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx=\frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{6}{7}\int\frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9}{14}\int\frac{1}{(a+bx^3)^{4/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a\,(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int\frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx=\frac{7a^2x+7abx^4+4b^2x^7}{7a\,(a+bx^3)^{7/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
[Out] (7*a^2*x + 7*a*b*x^4 + 4*b^2*x^7)/(7*a*(a + b*x^3)^(7/3))
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{x\left(4b^2x^6+7ab\,x^3+7a^2\right)}{7\left(b\,x^3+a\right)^{\frac{7}{3}}a}$	37
trager	$\frac{x\left(4b^2x^6+7ab\,x^3+7a^2\right)}{7\left(b\,x^3+a\right)^{\frac{7}{3}}a}$	37
pseudoelliptic	$\frac{x\left(4b^2x^6+7ab\,x^3+7a^2\right)}{7\left(b\,x^3+a\right)^{\frac{7}{3}}a}$	37

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{(4\,b^2x^7+7\,abx^4+7\,a^2x)(bx^3+a)^{\frac{2}{3}}}{7\,(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4)}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
[Out] 1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2*
b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(10/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{14\,(bx^3+a)^{\frac{7}{3}}a} + \frac{b^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
[Out] 1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/((
b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/
x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.58

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{4\,x\,(b\,x^3+a)^2+4\,a^2\,x-a\,x\,(b\,x^3+a)}{7\,a\,(b\,x^3+a)^{7/3}}$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
[Out] (4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

3.45 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}} dx = \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

```
[Out] 1/20*x*(-b*x^3+a)^3/a^2/(b*x^3+a)^(10/3)+19/140*x*(-b*x^3+a)^2/a^2/(b*x^3+a
)^(7/3)+57/280*x*(-b*x^3+a)/a^2/(b*x^3+a)^(4/3)+171/280*x/a^2/(b*x^3+a)^(1/
3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {390, 386, 197}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}} dx = \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx}{20a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{70a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{280a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{140a^3x+245a^2bx^4+230ab^2x^7+69b^3x^{10}}{140a^2\,(a+bx^3)^{10/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (140*a^3*x + 245*a^2*b*x^4 + 230*a*b^2*x^7 + 69*b^3*x^10)/(140*a^2*(a + b*x
^3)^(10/3))
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x\left(69b^3x^9+230a\,b^2x^6+245a^2b\,x^3+140a^3\right)}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^2}$	48
trager	$\frac{x\left(69b^3x^9+230a\,b^2x^6+245a^2b\,x^3+140a^3\right)}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^2}$	48
pseudoelliptic	$\frac{x\left(69b^3x^9+230a\,b^2x^6+245a^2b\,x^3+140a^3\right)}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^2}$	48

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a
^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x)(bx^3+a)^{\frac{2}{3}}}{140\,(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6)}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a)
^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.48

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx=-\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)b^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^2}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)^(
10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x
^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{69\,x}{140\,a^2\,(b\,x^3+a)^{1/3}} - \frac{2\,x}{35\,(b\,x^3+a)^{7/3}} + \frac{23\,x}{140\,a\,(b\,x^3+a)^{4/3}} + \frac{2\,a\,x}{5\,(b\,x^3+a)^{10/3}}$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
[Out] (69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)/
(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

3.46 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}+\frac{8x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}}+\frac{141x}{910a^2(a+bx^3)^{4/3}}+\frac{423x}{910a^3\sqrt[3]{a+bx^3}}$$

```
[Out] 2/13*x*(-b*x^3+a)/(b*x^3+a)^(13/3)+8/65*x/(b*x^3+a)^(10/3)+47/455*x/a/(b*x^
3+a)^(7/3)+141/910*x/a^2/(b*x^3+a)^(4/3)+423/910*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 393, 198, 197}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{47x}{455a(a+bx^3)^{7/3}}+\frac{8x}{65(a+bx^3)^{10/3}}+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x(a-bx^3)}{13\left(a+bx^3\right)^{13/3}}+\frac{\int \frac{11a^2b-5ab^2x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}\\
&= \frac{2x(a-bx^3)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47}{65}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx\\
&= \frac{2x(a-bx^3)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{282\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a}\\
&= \frac{2x(a-bx^3)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{423\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{910a^2}
\end{aligned}$$

$$= \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{423x}{910a^3\sqrt[3]{a+bx^3}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{910a^4x+2275a^3bx^4+3055a^2b^2x^7+1833ab^3x^{10}+423b^4x^{13}}{910a^3(a+bx^3)^{13/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
[Out] (910*a^4*x + 2275*a^3*b*x^4 + 3055*a^2*b^2*x^7 + 1833*a*b^3*x^10 + 423*b^4*
x^13)/(910*a^3*(a + b*x^3)^(13/3))
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x(423b^4x^{12}+1833a\,b^3x^9+3055a^2b^2x^6+2275a^3b\,x^3+910a^4)}{910(b\,x^3+a)^{\frac{13}{3}}a^3}$	59
trager	$\frac{x(423b^4x^{12}+1833a\,b^3x^9+3055a^2b^2x^6+2275a^3b\,x^3+910a^4)}{910(b\,x^3+a)^{\frac{13}{3}}a^3}$	59
pseudoelliptic	$\frac{x(423b^4x^{12}+1833a\,b^3x^9+3055a^2b^2x^6+2275a^3b\,x^3+910a^4)}{910(b\,x^3+a)^{\frac{13}{3}}a^3}$	59

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
[Out] 1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a^
4)/(b*x^3+a)^(13/3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(423\,b^4x^{13} + 1833\,ab^3x^{10} + 3055\,a^2b^2x^7 + 2275\,a^3bx^4 + 910\,a^4x)(bx^3+a)^{\frac{2}{3}}}{910\,(a^3b^5x^{15} + 5\,a^4b^4x^{12} + 10\,a^5b^3x^9 + 10\,a^6b^2x^6 + 5\,a^7bx^3 + a^8)}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4 +
 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3*x
^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.10

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = {} & \frac{\left(35\,b^2 - \frac{91\,(bx^3+a)b}{x^3} + \frac{65\,(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} \\ & + \frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^3} \\ & + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}\end{aligned}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*x
^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) + 1/
455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*
x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^3)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{423\,x}{910\,a^3\,(b\,x^3+a)^{1/3}} - \frac{2\,x}{65\,(b\,x^3+a)^{10/3}} + \frac{141\,x}{910\,a^2\,(b\,x^3+a)^{4/3}} + \frac{47\,x}{455\,a\,(b\,x^3+a)^{7/3}} + \frac{4\,a\,x}{13\,(b\,x^3+a)^{13/3}}$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141*
x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*x)
/(13*(a + b*x^3)^(13/3))
```

3.47 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}} \, dx = \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} + \frac{81x}{182a^4\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x*(-b*x^3+a)/(b*x^3+a)^(16/3)+11/104*x/(b*x^3+a)^(13/3)+1/13*x/a/(b*x^3
+a)^(10/3)+9/91*x/a^2/(b*x^3+a)^(7/3)+27/182*x/a^3/(b*x^3+a)^(4/3)+81/182*x
/a^4/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 393, 198, 197}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}} \, dx = \frac{81x}{182a^4\sqrt[3]{a+bx^3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}}+\frac{\int \frac{14a^2b-8ab^2x^3}{(a+bx^3)^{16/3}}\,dx}{16ab} \\
&= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{10}{13}\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx \\
&= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{9\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{13a} \\
&= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}} \\
&\quad+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{54\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{91a^2}
\end{aligned}
$$

$$= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}} + \frac{11x}{104\left(a+bx^3\right)^{13/3}} + \frac{x}{13a\left(a+bx^3\right)^{10/3}} + \frac{9x}{91a^2\left(a+bx^3\right)^{7/3}} + \frac{27x}{182a^3\left(a+bx^3\right)^{4/3}} + \frac{81\int\frac{1}{(a+bx^3)^{4/3}}\,dx}{182a^3}$$

$$= \frac{x(a-bx^3)}{8\left(a+bx^3\right)^{16/3}} + \frac{11x}{104\left(a+bx^3\right)^{13/3}} + \frac{x}{13a\left(a+bx^3\right)^{10/3}} + \frac{9x}{91a^2\left(a+bx^3\right)^{7/3}} + \frac{27x}{182a^3\left(a+bx^3\right)^{4/3}} + \frac{81x}{182a^4\sqrt[3]{a+bx^3}}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{364a^5x+1183a^4bx^4+2080a^3b^2x^7+1872a^2b^3x^{10}+864ab^4x^{13}+162b^5x^{16}}{364a^4\left(a+bx^3\right)^{16/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (364*a^5*x + 1183*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1872*a^2*b^3*x^10 + 864*a*
b^4*x^13 + 162*b^5*x^16)/(364*a^4*(a + b*x^3)^(16/3))
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x\left(162b^5x^{15}+864a\,b^4x^{12}+1872a^2b^3x^9+2080a^3b^2x^6+1183a^4b\,x^3+364a^5\right)}{364(b\,x^3+a)^{\frac{16}{3}}\,a^4}$	70
trager	$\frac{x\left(162b^5x^{15}+864a\,b^4x^{12}+1872a^2b^3x^9+2080a^3b^2x^6+1183a^4b\,x^3+364a^5\right)}{364(b\,x^3+a)^{\frac{16}{3}}\,a^4}$	70
pseudoelliptic	$\frac{x\left(162b^5x^{15}+864a\,b^4x^{12}+1872a^2b^3x^9+2080a^3b^2x^6+1183a^4b\,x^3+364a^5\right)}{364(b\,x^3+a)^{\frac{16}{3}}\,a^4}$	70

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+1183
*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(162\,b^5x^{16}+864\,ab^4x^{13}+1872\,a^2b^3x^{10}+2080\,a^3b^2x^7+1183\,a^4bx^4+364\,a^5x)(bx^3+a}{364\,(a^4b^6x^{18}+6\,a^5b^5x^{15}+15\,a^6b^4x^{12}+20\,a^7b^3x^9+15\,a^8b^2x^6+6\,a^9bx^3+a^{10})}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^7
 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^5*
x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 + a^
10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(94) = 188.

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.20

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = &-\frac{\left(455\,b^3-\frac{1680\,(bx^3+a)b^2}{x^3}+\frac{2184\,(bx^3+a)^2b}{x^6}-\frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4}\\ &-\frac{\left(455\,b^4-\frac{2240\,(bx^3+a)b^3}{x^3}+\frac{4368\,(bx^3+a)^2b^2}{x^6}-\frac{4160\,(bx^3+a)^3b}{x^9}+\frac{1820\,(bx^3+a)^4}{x^{12}}\right)bx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^4}\\ &-\frac{\left(91\,b^5-\frac{560\,(bx^3+a)b^4}{x^3}+\frac{1456\,(bx^3+a)^2b^3}{x^6}-\frac{2080\,(bx^3+a)^3b^2}{x^9}+\frac{1820\,(bx^3+a)^4b}{x^{12}}-\frac{1456\,(bx^3+a)^5}{x^{15}}\right)x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^4}\end{aligned}$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4 -
```

```
2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1456*
(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*x^
3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*x^1
6/((b*x^3 + a)^(16/3)*a^4)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{81\,x}{182\,a^4\,(b\,x^3+a)^{1/3}} - \frac{x}{52\,(b\,x^3+a)^{13/3}} + \frac{27\,x}{182\,a^3\,(b\,x^3+a)^{4/3}} + \frac{9\,x}{91\,a^2\,(b\,x^3+a)^{7/3}} + \frac{x}{13\,a\,(b\,x^3+a)^{10/3}} + \frac{a\,x}{4\,(b\,x^3+a)^{16/3}}$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
[Out] (81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a +
b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

3.48 $\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx = -\frac{9}{44} a x (a+bx^3)^{7/3} - \frac{1}{11} x (a-bx^3) (a+bx^3)^{7/3} + \frac{57 a^3 x \sqrt[3]{a+bx^3} \, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44 \sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -9/44*a*x*(b*x^3+a)^(7/3)-1/11*x*(-b*x^3+a)*(b*x^3+a)^(7/3)+57/44*a^3*x*(b*
x^3+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {427, 396, 252, 251}

$$\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx = \frac{57 a^3 x \sqrt[3]{a+bx^3} \, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44 \sqrt[3]{\frac{bx^3}{a}+1}} - \frac{9}{44} a x (a+bx^3)^{7/3} - \frac{1}{11} x (a-bx^3) (a+bx^3)^{7/3}$$

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (-9*a*x*(a + b*x^3)^(7/3))/44 - (x*(a - b*x^3)*(a + b*x^3)^(7/3))/11 + (57*
a^3*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4
4*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\int\left(a+bx^3\right)^{4/3}\left(12a^2b-18ab^2x^3\right)dx}{11b}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{1}{44}\left(57a^2\right)\int\left(a+bx^3\right)^{4/3}dx\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\left(57a^3\sqrt[3]{a+bx^3}\right)\int\left(1+\frac{bx^3}{a}\right)^{4/3}dx}{44\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx = \frac{x\left(106a^4+53a^3bx^3-78a^2b^2x^6-5ab^3x^9+20b^4x^{12}+114a^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\right.}{220\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 + 1
14*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]
))/(220*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}dx$$

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int (a-bx^3)^2 (a+bx^3)^{4/3} \, dx = \int (bx^3+a)^{\frac{4}{3}} (bx^3-a)^2 \, dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.79

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = \frac{a^{\frac{10}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{7}{3}} b x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{a^{\frac{4}{3}} b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a} b^3 x^{10} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
```

```
[Out] a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hyper
((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)
*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

Maxima [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(bx^3-a\right)^2 \, dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Giac [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(bx^3-a\right)^2 \, dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx=\int {\left(b\,x^3+a\right)}^{4/3}\,{\left(a-b\,x^3\right)}^2 \,d x$$

```
[In] int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)
[Out] int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)
```

3.49 $\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -3/8*a*x*(b*x^3+a)^(4/3)-1/8*x*(-b*x^3+a)*(b*x^3+a)^(4/3)+3/2*a^2*x*(b*x^3+
a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {427, 396, 252, 251}

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\frac{3a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(4/3))/8 - (x*(a - b*x^3)*(a + b*x^3)^(4/3))/8 + (3*a^2
*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(2*(1
 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\int\sqrt[3]{a+bx^3}\left(9a^2b-15ab^2x^3\right)\,dx}{8b}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{1}{2}\left(3a^2\right)\int\sqrt[3]{a+bx^3}\,dx\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\left(3a^2\sqrt[3]{a+bx^3}\right)\int\sqrt[3]{1+\frac{bx^3}{a}}\,dx}{2\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \frac{x\left(2a^3-a^2bx^3-2ab^2x^6+b^3x^9+6a^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{8(a+bx^3)^{2/3}}$$

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
[Out] (x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \left(-b\,x^3+a\right)^2 \left(b\,x^3+a\right)^{\frac{1}{3}} dx$$

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{4}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*hype
r((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int (a-bx^3)^2\sqrt[3]{a+bx^3}\,dx=\int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Giac [F]

$$\int (a-bx^3)^2\sqrt[3]{a+bx^3}\,dx=\int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^3)^2\sqrt[3]{a+bx^3}\,dx=\int (b\,x^3+a)^{1/3}\,(a-b\,x^3)^2\,dx$$

```
[In] int((a + b*x^3)^(1/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(a - b*x^3)^2, x)
```

3.50 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = -\frac{6}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3} + \frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}$$

```
[Out] -6/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(-b*x^3+a)*(b*x^3+a)^(1/3)+12/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {427, 396, 252, 251}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} - \frac{6}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-6*a*x*(a + b*x^3)^(1/3))/5 - (x*(a - b*x^3)*(a + b*x^3)^(1/3))/5 + (12*a^
2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
5*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\int \frac{6a^2b-12ab^2x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{5b}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{1}{5}\left(12a^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\left(12a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{-7a^2x-6abx^4+b^2x^7+12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
[Out] (-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeome
tric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt[3]{a}bx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{2/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.51 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}$$

```
[Out] x*(-b*x^3+a)/(b*x^3+a)^(2/3)+3/4*b*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/
3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 12, 372, 371}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{\int \frac{6ab^2x^3}{(a+bx^3)^{2/3}}\,dx}{2ab}\\
&= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+(3b)\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx\\
&= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{\left(3b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}\\
&= \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{5ax+bx^4-3ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/
3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

Maxima [F]

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{5/3}}\,dx = \int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{5/3}}\,dx = \int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{5/3}}\,dx = \int \frac{(a - b\,x^3)^2}{(b\,x^3 + a)^{5/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.52 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}} + \frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

```
[Out] 2/5*x*(-b*x^3+a)/(b*x^3+a)^(5/3)+3/5*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/
3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 21, 252, 251}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}} + \frac{2x(a-bx^3)}{5\,(a+bx^3)^{5/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}+\frac{\int \frac{3a^2b+3ab^2x^3}{(a+bx^3)^{5/3}}\,dx}{5ab}\\
&= \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}+\frac{3}{5}\int \frac{1}{(a+bx^3)^{2/3}}\,dx\\
&= \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5(a+bx^3)^{2/3}}\\
&= \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{2x(a-bx^3)+3x(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{5/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
[Out] (2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{8}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(8/3), x)
```

Maxima [F]

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{8/3}}\,dx = \int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{8/3}}\,dx = \int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a - bx^3)^2}{(a + bx^3)^{8/3}}\,dx = \int \frac{(a - b\,x^3)^2}{(b\,x^3 + a)^{8/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.53 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} \, dx = \frac{x(a-bx^3)}{4(a+bx^3)^{8/3}} + \frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/(b*x^3+a)^(8/3)+3/4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 8/
3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 12, 252, 251}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} \, dx = \frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}} + \frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a-bx^3)}{4\left(a+bx^3\right)^{8/3}}+\frac{\int \frac{6a^2b}{(a+bx^3)^{8/3}}\,dx}{8ab}\\
&= \frac{x(a-bx^3)}{4\left(a+bx^3\right)^{8/3}}+\frac{1}{4}(3a)\int \frac{1}{\left(a+bx^3\right)^{8/3}}\,dx\\
&= \frac{x(a-bx^3)}{4\left(a+bx^3\right)^{8/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{4a\left(a+bx^3\right)^{2/3}}\\
&= \frac{x(a-bx^3)}{4\left(a+bx^3\right)^{8/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \frac{7a^2x+5abx^4+3b^2x^7+3x(a+bx^3)^2\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10a\,(a+bx^3)^{8/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
[Out] (7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{11}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3*
x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{11/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)
```

3.54 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}} dx = \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}} + \frac{3x}{22(a+bx^3)^{8/3}} + \frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{22a^2(a+bx^3)^{2/3}}$$

```
[Out] 2/11*x*(-b*x^3+a)/(b*x^3+a)^(11/3)+3/22*x/(b*x^3+a)^(8/3)+15/22*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 8/3],[4/3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {424, 393, 252, 251}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}} dx = \frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{22a^2(a+bx^3)^{2/3}} + \frac{3x}{22(a+bx^3)^{8/3}} + \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*x)/(22*(a + b*x^3)^(8/3)) +
(15*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(22*a^2*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x(a-bx^3)}{11\left(a+bx^3\right)^{11/3}}+\frac{\int \frac{9a^2b-3ab^2x^3}{(a+bx^3)^{11/3}}\,dx}{11ab}\\
&= \frac{2x(a-bx^3)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{15}{22}\int\frac{1}{\left(a+bx^3\right)^{8/3}}\,dx\\
&= \frac{2x(a-bx^3)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{\left(15\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{22a^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{2x(a-bx^3)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \frac{x\left(16a^3+23a^2bx^3+21ab^2x^6+6b^3x^9+6(a+bx^3)^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\right.\right.}{22a^2\left(a+bx^3\right)^{11/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
[Out] (x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^2*
(a + b*x^3)^(11/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4*
x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{14/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(14/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(14/3), x)
```

3.55 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}} dx = \frac{x(a-bx^3)}{7(a+bx^3)^{14/3}} + \frac{9x}{77(a+bx^3)^{11/3}} + \frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77a^3(a+bx^3)^{2/3}}$$

```
[Out] 1/7*x*(-b*x^3+a)/(b*x^3+a)^(14/3)+9/77*x/(b*x^3+a)^(11/3)+57/77*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 11/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {424, 393, 252, 251}

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}} dx = \frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77a^3(a+bx^3)^{2/3}} + \frac{9x}{77(a+bx^3)^{11/3}} + \frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (9*x)/(77*(a + b*x^3)^(11/3)) + (5
7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)/a)])/
(77*a^3*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a-bx^3)}{7\left(a+bx^3\right)^{14/3}}+\frac{\int \frac{12a^2b-6ab^2x^3}{\left(a+bx^3\right)^{14/3}}\,dx}{14ab}\\
&= \frac{x(a-bx^3)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57}{77}\int \frac{1}{\left(a+bx^3\right)^{11/3}}\,dx\\
&= \frac{x(a-bx^3)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{\left(57\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{11/3}}\,dx}{77a^3\left(a+bx^3\right)^{2/3}}\\
&= \frac{x(a-bx^3)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \frac{x\left(2282a^4+4879a^3bx^3+6270a^2b^2x^6+3591ab^3x^9+798b^4x^{12}+798(a+bx^3)^4\left(1+\frac{bx^3}{a}\right.\right.}{3080a^3\,(a+bx^3)^{14/3}}$$

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(2282*a^4 + 4879*a^3*b*x^3 + 6270*a^2*b^2*x^6 + 3591*a*b^3*x^9 + 798*b^4
*x^12 + 798*(a + b*x^3)^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)]))/(3080*a^3*(a + b*x^3)^(14/3))
```

Maple [F]

$$\int \frac{(-b\,x^3+a)^2}{(b\,x^3+a)^{\frac{17}{3}}}dx$$

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^6*x^18 + 6*a*b^5*
x^15 + 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^
6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(17/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{17}{3}}}\,dx$$

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{17/3}}\,dx$$

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(17/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(17/3), x)
```

3.56 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx = \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{5a^2(9bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}} - \frac{5a^2(9bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{162b^{4/3}}$$

```
[Out] 5/162*a*(-a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b+1/54*(-a*d+9*b*c)*x*(b*x^3+a)^(5/3
)/b+1/9*d*x*(b*x^3+a)^(8/3)/b-5/162*a^2*(-a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(4/3)+5/243*a^2*(-a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

= {396, 201, 245}

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx = \frac{5a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(9bc-ad)}{81\sqrt{3}b^{4/3}} - \frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{162b^{4/3}} + \frac{x(a+bx^3)^{5/3}(9bc-ad)}{54b} + \frac{5ax(a+bx^3)^{2/3}(9bc-ad)}{162b} + \frac{dx(a+bx^3)^{8/3}}{9b}$$

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule Rubi steps

$$\text{integral} = \frac{dx(a+bx^3)^{8/3}}{9b} - \frac{(-9bc+ad)\int (a+bx^3)^{5/3}\,dx}{9b}$$
$$= \frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b} + \frac{dx(a+bx^3)^{8/3}}{9b} + \frac{(5a(9bc-ad))\int (a+bx^3)^{2/3}\,dx}{54b}$$

$$= \frac{5a(9bc-ad)x(a+bx^3)^{2/3}}{162b} + \frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b} + \frac{dx(a+bx^3)^{8/3}}{9b} + \frac{(5a^2(9bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{81b}$$

$$= \frac{5a(9bc-ad)x(a+bx^3)^{2/3}}{162b} + \frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b} + \frac{dx(a+bx^3)^{8/3}}{9b} + \frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}} - \frac{5a^2(9bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{162b^{4/3}}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int (a+bx^3)^{5/3}(c+dx^3)\,dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}(10a^2d+9b^2x^3(3c+2dx^3)+ab(72c+33dx^3)) - 10\sqrt{3}a^2(-9bc+ad)\arctan\left(\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(10*a^2*d + 9*b^2*x^3*(3*c + 2*d*x^3) + a*b*
(72*c + 33*d*x^3)) - 10*Sqrt[3]*a^2*(-9*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*
x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 10*a^2*(-9*b*c + a*d)*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] - 5*a^2*(-9*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x
*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(4/3))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{5\left(-\frac{216\left(\frac{11dx^3}{24}+c\right)x\left(bx^3+a\right)^{\frac{2}{3}}ab^{\frac{4}{3}}}{5}-\frac{81x^4\left(bx^3+a\right)^{\frac{2}{3}}\left(\frac{2dx^3}{3}+c\right)b^{\frac{7}{3}}}{5}+a^2\left(-6\left(bx^3+a\right)^{\frac{2}{3}}dxb^{\frac{1}{3}}+(ad-9bc)\left(-2\sqrt{3}\arctan\left(\right.\right.\right.\right.}{486b^{\frac{4}{3}}}$

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -5/486*(-216/5*(11/24*d*x^3+c)*x*(b*x^3+a)^(2/3)*a*b^(4/3)-81/5*x^4*(b*x^3+
a)^(2/3)*(2/3*d*x^3+c)*b^(7/3)+a^2*(-6*(b*x^3+a)^(2/3)*d*x*b^(1/3)+(a*d-9*b
```

```
*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)
+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(
1/3)*x+(b*x^3+a)^(1/3))/x))))/b^(4/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.77

$$\int (a+bx^3)^{5/3}(c+dx^3)\,dx = \left[-\frac{15\sqrt{\frac{1}{3}}(9a^2b^2c-a^3bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2\right.\right.}{}\right.$$

$$-\frac{10(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5(9a^2bc-a^3d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{30\sqrt{\frac{1}{3}}(9a}{}}{48}$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
- 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^
(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(
9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a
^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*
(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^2*b*c -
 a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^
3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3)*(b^(1/3)
*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7 + 3*(9*b^3
*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)\,dx = \frac{a^{\frac{5}{3}} cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{3}} dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} bdx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

```
[Out] a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hyper
((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b
*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.33

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx =$$

$$-\frac{1}{54}\left(\frac{10\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{486}\left(\frac{10\,\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{4}{3}}}\right.$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 1
0*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3)*
a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x^8
)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/
x^9))*d
```

Giac [F]

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{5/3}\left(d\,x^3+c\right)\,dx$$

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

3.57 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)\, dx = \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{a(6bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

```
[Out] 1/18*(-a*d+6*b*c)*x*(b*x^3+a)^(2/3)/b+1/6*d*x*(b*x^3+a)^(5/3)/b-1/18*a*(-a*
d+6*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)+1/27*a*(-a*d+6*b*c)*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 201, 245}

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)\, dx = \frac{a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(6bc-ad)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{5/3}}{6b} - \frac{(-6bc+ad)\int (a+bx^3)^{2/3}\,dx}{6b} \\
&= \frac{(6bc-ad)x(a+bx^3)^{2/3}}{18b} + \frac{dx(a+bx^3)^{5/3}}{6b} + \frac{(a(6bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b} \\
&= \frac{(6bc-ad)x(a+bx^3)^{2/3}}{18b} + \frac{dx(a+bx^3)^{5/3}}{6b} \\
&\quad + \frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)\,dx = \frac{3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}\left(6bc+2ad+3bdx^3\right)-2\sqrt{3}a(-6bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2a(-6 \ldots}{54b \ldots}$$

[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]

[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*b*c + 2*a*d + 3*b*d*x^3) - 2*Sqrt[3]*a*(-6*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*a*(-6*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-6*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(4/3))

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{3\left(b\,x^3+a\right)^{\frac{2}{3}} x \left(\frac{d\,x^3}{2}+c\right) b^{\frac{4}{3}}+\left(\left(b\,x^3+a\right)^{\frac{2}{3}} d x\, b^{\frac{1}{3}}+\frac{(ad-6bc)\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\ldots\right.}{3}\ldots\right.}{9b^{\frac{4}{3}}}$

[In] int((b*x^3+a)^(2/3)*(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/9/b^(4/3)*(3*(b*x^3+a)^(2/3)*x*(1/2*d*x^3+c)*b^(4/3)+((b*x^3+a)^(2/3)*d*x*b^(1/3)+1/3*(a*d-6*b*c)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))*a)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.01

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx = \left[-\frac{3\sqrt{\frac{1}{3}}(6\,ab^2c-a^2bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2\right.\right.}{} \right.$$

$$-\frac{2\,(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{6\sqrt{\frac{1}{3}}(6\,ab^2c-a^2bd}{}}{54\,b^2}$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*
(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3
)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*a*b
*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c - a^
2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*b^2*d*x
^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int (a+bx^3)^{2/3} (c+dx^3) \, dx = \frac{a^{\frac{2}{3}} cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{2}{3}} dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(114) = 228.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.28

$$\int (a+bx^3)^{2/3} (c+dx^3) \, dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{54}\left(\frac{2\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
```

```
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*d
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)dx$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)\,dx$$

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

3.58 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx(a+bx^3)^{2/3}}{3b} + \frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

```
[Out] 1/3*d*x*(b*x^3+a)^(2/3)/b-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b
^(4/3)+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {396, 245}

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}} + \frac{dx(a+bx^3)^{2/3}}{3b}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^3)^{2/3}}{3b} - \frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}$$

$$= \frac{dx(a+bx^3)^{2/3}}{3b} + \frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{6\sqrt[3]{b}dx(a+bx^3)^{2/3} + 2\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2(-3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (6*b^(1/3)*d*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*
b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*b*c + a*d)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + (3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
+ b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 4.05 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$\frac{6\left(b\,x^{3}+a\right)^{\frac{2}{3}} d x\,b^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a d-6\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) b c+2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x\right.}{x}\right.}{}$

```
[In] int((d*x^3+c)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(6*(b*x^3+a)^(2/3)*d*x*b^(1/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x
+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a*d-6*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+
2*(b*x^3+a)^(1/3))/b^(1/3)/x)*b*c+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d-
6*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*b*c-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d+3*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)*b*c)/b^(4/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx-3\sqrt{\frac{1}{3}}\left(3\,b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx\right.\right.}{}\right.$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)
*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(
2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))
/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c
- a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*d)*b^(2
```

```
/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.20

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right) + \frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

Giac [F]

$$\int \frac{c + dx^3}{\sqrt[3]{a + bx^3}}\,dx = \int \frac{dx^3 + c}{(bx^3 + a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^3}{\sqrt[3]{a + bx^3}}\,dx = \int \frac{d\,x^3 + c}{(b\,x^3 + a)^{1/3}}\,dx$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

3.59 $\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx=\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

```
[Out] (-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/
3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {393, 245}

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx=\frac{d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b}$$

$$= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} - \frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{b}(bc-ad)x}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + d\log}{6b^{4/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((6*b^(1/3)*(b*c - a*d)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*d*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*Log[-(b^(1/3)*x) +
(a + b*x^3)^(1/3)] + d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)])/(6*b^(4/3))
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.63

method	result
pseudoelliptic	$\frac{6b^{\frac{4}{3}}cx-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)ad\left(bx^{3}+a\right)^{\frac{1}{3}}-6adx\,b^{\frac{1}{3}}-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(bx^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b \cdots}{\cdots}\right.}{6b^{\frac{4}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a}$

```
[In] int((d*x^3+c)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*b^(4/3)*c*x-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3
))/b^(1/3)/x)*a*d*(b*x^3+a)^(1/3)-6*a*d*x*b^(1/3)-2*ln((-b^(1/3)*x+(b*x^3+a
)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(
b*x^3+a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3))/b^(4/3)/(b*x^3+a)^(1/3)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.93

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-\cdots\right.\right.}{\cdots}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-6\,(bx^3+a)^{\frac{2}{3}}(b^2c-abd)x+2\,(\cdots}{6\,(ab^3x^3+\cdots}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3
 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) +
2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*
```

```
sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*
sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2*(a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

```
[Out] c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma
(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = -\frac{1}{6}\,d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(-b\right.}{} \right. + \frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*
b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(
1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{4/3}}\,dx$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

3.60 $\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx=\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a\,(a+bx^3)^{4/3}}$$

```
[Out] 3/4*c*x/a^2/(b*x^3+a)^(1/3)+1/4*x*(d*x^3+c)/a/(b*x^3+a)^(4/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {386, 197}

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx=\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a\,(a+bx^3)^{4/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}} + \frac{(3c)\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{4a}$$
$$= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{x(4ac+3bcx^3+adx^3)}{4a^2(a+bx^3)^{4/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(a d\,x^{3}+3 b c\,x^{3}+4 a c\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}} a^{2}}$	34
trager	$\frac{x\left(a d\,x^{3}+3 b c\,x^{3}+4 a c\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}} a^{2}}$	34
pseudoelliptic	$\frac{x\left(a d\,x^{3}+3 b c\,x^{3}+4 a c\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}} a^{2}}$	34

```
[In] int((d*x^3+c)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
[Out] 1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{((3\,bc+ad)x^4+4\,acx)(bx^3+a)^{\frac{2}{3}}}{4\,(a^2b^2x^6+2\,a^3bx^3+a^4)}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(41) = 82.

Time = 20.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

```
[Out] c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)cx^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2}+\frac{dx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b*
x^3 + a)^(4/3)*a)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{4\,a\,c\,x+a\,d\,x^4+3\,b\,c\,x^4}{4\,a^2\,(b\,x^3+a)^{4/3}}$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(7/3),x)
[Out] (4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

3.61 $\int \frac{c+dx^3}{(a+bx^3)^{10/3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}} dx = \frac{(bc-ad)x}{7ab(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b(a+bx^3)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^(7/3)+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^(4/
3)+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 198, 197}

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}} dx = \frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}} + \frac{x(ad+6bc)}{28a^2b(a+bx^3)^{4/3}} + \frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{7ab} \\
&= \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\,(a+bx^3)^{4/3}} + \frac{(3(6bc+ad))\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{28a^2b} \\
&= \frac{(bc-ad)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\,(a+bx^3)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{28a^2cx+42abcx^4+7a^2dx^4+18b^2cx^7+3abdx^7}{28a^3\,(a+bx^3)^{7/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
[Out] (28*a^2*c*x + 42*a*b*c*x^4 + 7*a^2*d*x^4 + 18*b^2*c*x^7 + 3*a*b*d*x^7)/(28*
a^3*(a + b*x^3)^(7/3))
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{2}+\frac{3x^{3}\left(\frac{d\,x^{3}}{14}+c\right)ba}{2}+\frac{9b^{2}c\,x^{6}}{14}\right)}{\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	52
gosper	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57
trager	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57

```
[In] int((d*x^3+c)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

```
[Out] x/(b*x^3+a)^(7/3)*((1/4*d*x^3+c)*a^2+3/2*x^3*(1/14*d*x^3+c)*b*a+9/14*b^2*c*
x^6)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(3\,(6\,b^2c+abd)x^7+7\,(6\,abc+a^2d)x^4+28\,a^2cx)(bx^3+a)^{\frac{2}{3}}}{28\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x^
3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(83) = 166$.

Time = 100.01 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = c\left(\frac{28a^5x\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+2\ldots}\right.$$

$$+\frac{70a^4bx^4\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\ldots\right.}$$

$$+\frac{60a^3b^2x^7\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\ldots\right.}$$

$$+\frac{18a^2b^3x^{10}\Gamma\left(\frac{1}{3}\right)}{27a^{\frac{25}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{22}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+81a^{\frac{19}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+27a^{\frac{16}{3}}b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\ldots\right.}$$

$$+d\left(\frac{7ax^4\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right.$$

$$\left.+\frac{3bx^7\Gamma\left(\frac{4}{3}\right)}{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+18a^{\frac{10}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)+9a^{\frac{7}{3}}b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{10}{3}\right)}\right)$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

```
[Out] c*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 8
1*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x*
*6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a
)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3
/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10
/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/
3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*gamma(1
/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(
1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(
1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)
) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma
(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/
3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1
```

```
+ b*x**3/a)**(1/3)*gamma(10/3))) + d*(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 +
 b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*g
amma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*
x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/
3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)dx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)cx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a^3
)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{3\,a\,d\,x\,(b\,x^3+a)^2-4\,a^3\,d\,x+18\,b\,c\,x\,(b\,x^3+a)^2+a^2\,d\,x\,(b\,x^3+a)+4\,a^2\,b\,c\,x+6\,a\,b}{28\,a^3\,b\,(b\,x^3+a)^{7/3}}$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3))
```

3.62 $\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\,(a+bx^3)^{7/3}} + \frac{3(9bc+ad)x}{140a^3b\,(a+bx^3)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/10*(-a*d+b*c)*x/a/b/(b*x^3+a)^(10/3)+1/70*(a*d+9*b*c)*x/a^2/b/(b*x^3+a)^(
7/3)+3/140*(a*d+9*b*c)*x/a^3/b/(b*x^3+a)^(4/3)+9/140*(a*d+9*b*c)*x/a^4/b/(b
*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 198, 197}

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\,(a+bx^3)^{4/3}} + \frac{x(ad+9bc)}{70a^2b\,(a+bx^3)^{7/3}}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{10ab}\\
&= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{(3(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a^2b}\\
&= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{(9(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{140a^3b}\\
&= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{x\left(81b^3cx^9+35a^3\left(4c+dx^3\right)+9ab^2x^6\left(30c+dx^3\right)+15a^2bx^3\left(21c+2dx^3\right)\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*a
^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{3}+\frac{9x^{3}b\left(\frac{2d\,x^{3}}{21}+c\right)a^{2}}{4}+\frac{27x^{6}b^{2}\left(\frac{d\,x^{3}}{30}+c\right)a}{14}+\frac{81b^{3}c\,x^{9}}{140}\right)}{\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	71
gosper	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}x^{3}bc+140c\,a^{3}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81
trager	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}x^{3}bc+140c\,a^{3}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81

```
[In] int((d*x^3+c)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] x/(b*x^3+a)^(10/3)*((1/4*d*x^3+c)*a^3+9/4*x^3*b*(2/21*d*x^3+c)*a^2+27/14*x^
6*b^2*(1/30*d*x^3+c)*a+81/140*b^3*c*x^9)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{\left(9\left(9\,b^3c+ab^2d\right)x^{10}+30\left(9\,ab^2c+a^2bd\right)x^7+140\,a^3cx+35\left(9\,a^2bc+a^3d\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3*
c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b
^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\text{Timed out}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{\left(14\,b^2 - \frac{40\,(bx^3+a)b}{x^3} + \frac{35\,(bx^3+a)^2}{x^6}\right)dx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3 - \frac{60\,(bx^3+a)b^2}{x^3} + \frac{105\,(bx^3+a)^2b}{x^6} - \frac{140\,(bx^3+a)^3}{x^9}\right)cx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^4}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
[Out] 1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^3
 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a
)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x\left(\frac{c}{10\,a} - \frac{d}{10\,b}\right)}{(b\,x^3+a)^{10/3}} + \frac{x\,(a\,d+9\,b\,c)}{70\,a^2\,b\,(b\,x^3+a)^{7/3}} + \frac{x\,(3\,a\,d+27\,b\,c)}{140\,a^3\,b\,(b\,x^3+a)^{4/3}} + \frac{x\,(9\,a\,d+81\,b\,c)}{140\,a^4\,b\,(b\,x^3+a)^{1/3}}$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(13/3),x)
[Out] (x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b*
(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) + (
x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

3.63 $\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\,(a+bx^3)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\,(a+bx^3)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\,(a+bx^3)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\,(a+bx^3)^{4/3}} + \frac{81(12bc+ad)x}{1820a^5b\sqrt[3]{a+bx^3}}$$

```
[Out] 1/13*(-a*d+b*c)*x/a/b/(b*x^3+a)^(13/3)+1/130*(a*d+12*b*c)*x/a^2/b/(b*x^3+a)
^(10/3)+9/910*(a*d+12*b*c)*x/a^3/b/(b*x^3+a)^(7/3)+27/1820*(a*d+12*b*c)*x/a
^4/b/(b*x^3+a)^(4/3)+81/1820*(a*d+12*b*c)*x/a^5/b/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 198, 197}

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}} + \frac{27x(ad+12bc)}{1820a^4b\,(a+bx^3)^{4/3}} + \frac{9x(ad+12bc)}{910a^3b\,(a+bx^3)^{7/3}} + \frac{x(ad+12bc)}{130a^2b\,(a+bx^3)^{10/3}} + \frac{x(bc-ad)}{13ab\,(a+bx^3)^{13/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)\int\frac{1}{(a+bx^3)^{13/3}}\,dx}{13ab}\\
&= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{(9(12bc+ad))\int\frac{1}{(a+bx^3)^{10/3}}\,dx}{130a^2b}\\
&= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}}\\
&\quad+\frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{(27(12bc+ad))\int\frac{1}{(a+bx^3)^{7/3}}\,dx}{455a^3b}\\
&= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{(81(12bc+ad))\int\frac{1}{(a+bx^3)^{4/3}}\,dx}{1820a^4b}\\
&= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{81(12bc+ad)x}{1820a^5b\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{x\left(972b^4cx^{12}+455a^4\left(4c+dx^3\right)+351a^2b^2x^6\left(20c+dx^3\right)+81ab^3x^9\left(52c+dx^3\right)+195a^3b\right.}{1820a^5\left(a+bx^3\right)^{13/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3)
 + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a^5
*(a + b*x^3)^(13/3))
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.60

method	result	si
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{4}+3\left(\frac{3d\,x^{3}}{28}+c\right)x^{3}b\,a^{3}+\frac{27x^{6}b^{2}\left(\frac{d\,x^{3}}{20}+c\right)a^{2}}{7}+\frac{81x^{9}\left(\frac{d\,x^{3}}{52}+c\right)b^{3}a}{35}+\frac{243b^{4}c\,x^{12}}{455}\right)}{\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	90
gosper	$\frac{x\left(81a\,b^{3}d\,x^{12}+972b^{4}c\,x^{12}+351a^{2}b^{2}d\,x^{9}+4212a\,b^{3}c\,x^{9}+585a^{3}bd\,x^{6}+7020a^{2}b^{2}c\,x^{6}+455a^{4}d\,x^{3}+5460a^{3}bc\,x^{3}+1820a^{4}c\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	10
trager	$\frac{x\left(81a\,b^{3}d\,x^{12}+972b^{4}c\,x^{12}+351a^{2}b^{2}d\,x^{9}+4212a\,b^{3}c\,x^{9}+585a^{3}bd\,x^{6}+7020a^{2}b^{2}c\,x^{6}+455a^{4}d\,x^{3}+5460a^{3}bc\,x^{3}+1820a^{4}c\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$	10

```
[In] int((d*x^3+c)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*((1/4*d*x^3+c)*a^4+3*(3/28*d*x^3+c)*x^3*b*a^3+27/7*x^6*b^2*(1/20*d*x^3+c)
*a^2+81/35*x^9*(1/52*d*x^3+c)*b^3*a+243/455*b^4*c*x^12)/(b*x^3+a)^(13/3)/a^
5
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{\left(81\left(12\,b^4c+ab^3d\right)x^{13}+351\left(12\,ab^3c+a^2b^2d\right)x^{10}+585\left(12\,a^2b^2c+a^3bd\right)x^7+1820\,a^4cx\right.}{1820\left(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9bx\right.}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d)*
x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10
*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = -\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)dx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)cx^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
[Out] -1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b
*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 4
55*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{(b\,x^3+a)^{13/3}}+\frac{x\,(a\,d+12\,b\,c)}{130\,a^2\,b\,(b\,x^3+a)^{10/3}} + \frac{x\,(9\,a\,d+108\,b\,c)}{910\,a^3\,b\,(b\,x^3+a)^{7/3}}+\frac{x\,(27\,a\,d+324\,b\,c)}{1820\,a^4\,b\,(b\,x^3+a)^{4/3}}+\frac{x\,(81\,a\,d+972\,b\,c)}{1820\,a^5\,b\,(b\,x^3+a)^{1/3}}$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2*
b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3))
 + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d + 972
*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

3.64 $\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx = \frac{dx(a+bx^3)^{10/3}}{11b} + \frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/11*d*x*(b*x^3+a)^(10/3)/b+1/11*a^2*(-a*d+11*b*c)*x*(b*x^3+a)^(1/3)*hyperg
eom([-7/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 252, 251}

$$\int (a+bx^3)^{7/3} \left(c+dx^3\right) \, dx = \frac{a^2x\sqrt[3]{a+bx^3}(11bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{10/3}}{11b}$$

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*H
ypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx\left(a+bx^3\right)^{10/3}}{11b}-\frac{(-11bc+ad)\int\left(a+bx^3\right)^{7/3}\,dx}{11b}\\
&= \frac{dx\left(a+bx^3\right)^{10/3}}{11b}-\frac{\left(a^2(-11bc+ad)\sqrt[3]{a+bx^3}\right)\int\left(1+\frac{bx^3}{a}\right)^{7/3}\,dx}{11b\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{dx\left(a+bx^3\right)^{10/3}}{11b}+\frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^3-\frac{a^2(-11bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{11b}$$

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometric
2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right) dx$$

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right) dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.12

$$\int (a+bx^3)^{7/3}(c+dx^3)\,dx = \frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}b^2dx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

```
[Out] a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*hyp
er((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**10*gamm
a(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/
3))
```

Maxima [F]

$$\int (a+bx^3)^{7/3}(c+dx^3)\,dx = \int (bx^3+a)^{\frac{7}{3}}(dx^3+c)\,dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int (a+bx^3)^{7/3} (c+dx^3) \, dx = \int (bx^3+a)^{\frac{7}{3}} (dx^3+c) \, dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{7/3} (c+dx^3) \, dx = \int (b\,x^3+a)^{7/3} \, (d\,x^3+c) \, dx$$

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3), x)
```

3.65 $\int (a+bx^3)^{4/3} (c+dx^3) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int (a+bx^3)^{4/3} (c+dx^3) \, dx = \frac{dx(a+bx^3)^{7/3}}{8b} + \frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/8*d*x*(b*x^3+a)^(7/3)/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-
4/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 252, 251}

$$\int (a+bx^3)^{4/3} (c+dx^3) \, dx = \frac{ax\sqrt[3]{a+bx^3}(8bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{7/3}}{8b}$$

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperg
eometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}-\frac{(-8bc+ad)\int\left(a+bx^3\right)^{4/3}\,dx}{8b}\\
&= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}-\frac{\left(a(-8bc+ad)\sqrt[3]{a+bx^3}\right)\int\left(1+\frac{bx^3}{a}\right)^{4/3}\,dx}{8b\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}+\frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^2-\frac{a(-8bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{8b}$$

[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]

[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)dx$$

[In] int((b*x^3+a)^(4/3)*(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)*(d*x^3+c),x)

Fricas [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\,dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(dx^3+c\right)dx$$

[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")

[Out] integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx = \frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{4}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
[Out] a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hyper
((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b
*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Maxima [F]

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx = \int (bx^3+a)^{\frac{4}{3}}(dx^3+c)\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx = \int (bx^3+a)^{\frac{4}{3}}(dx^3+c)\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{4/3}\left(d\,x^3+c\right)\,dx$$

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3), x)
```

3.66 $\int \sqrt[3]{a+bx^3}(c+dx^3)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\, dx=\frac{dx(a+bx^3)^{4/3}}{5b}+\frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1/
3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 252, 251}

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\, dx=\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}}{5b}$$

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hypergeo
metric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{4/3}}{5b} - \frac{(-5bc+ad)\int \sqrt[3]{a+bx^3}\,dx}{5b} \\
&= \frac{dx(a+bx^3)^{4/3}}{5b} - \frac{\left((-5bc+ad)\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{5b\sqrt[3]{1+\frac{bx^3}{a}}} \\
&= \frac{dx(a+bx^3)^{4/3}}{5b} + \frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$
\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)+\frac{(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}
$$

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)dx$$

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\,(c+dx^3)\ dx = \int (bx^3+a)^{\frac{1}{3}}(dx^3+c)\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\,(c+dx^3)\ dx = \int (bx^3+a)^{\frac{1}{3}}(dx^3+c)\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\,(c+dx^3)\ dx = \int {\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)\ dx$$

```
[In] int((a + b*x^3)^(1/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

3.67 $\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

```
[Out] 1/2*d*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d+2*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([1
/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 252, 251}

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} + \frac{dx\sqrt[3]{a+bx^3}}{2b}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hype
rgeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{(-2bc+ad)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2b}\\ &= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{\left((-2bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2b\,(a+bx^3)^{2/3}}\\ &= \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{dx(a+bx^3)+(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}d x$$

```
[In] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^{3}+c}{(bx^{3}+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^{3}+c}{(bx^{3}+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{c + dx^3}{(a + bx^3)^{2/3}}\,dx = \int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^3}{(a + bx^3)^{2/3}}\,dx = \int \frac{d\,x^3 + c}{(b\,x^3 + a)^{2/3}}\,dx$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(2/3), x)
```

3.68 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^3+a)^(2/3)+1/2*(a*d+b*c)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 252, 251}

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^(2
/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{(bc+ad)\int\frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}\\
&= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\left((bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2ab\left(a+bx^3\right)^{2/3}}\\
&= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int\frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\frac{-adx+(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
[Out] (-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3,
 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}d x$$

```
[In] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^{3}+c}{{\left(bx^{3}+a\right)}^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
[Out] c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^{3}+c}{{\left(bx^{3}+a\right)}^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{5/3}}\,dx$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(5/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(5/3), x)
```

3.69 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx = \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}} + \frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/5*(-a*d+b*c)*x/a/b/(b*x^3+a)^(5/3)+1/5*(a*d+4*b*c)*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 5/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 252, 251}

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}} + \frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] ((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x^3
)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{5ab\,(a+bx^3)^{5/3}} + \frac{(4bc+ad)\int \frac{1}{(a+bx^3)^{5/3}}\,dx}{5ab} \\
&= \frac{(bc-ad)x}{5ab\,(a+bx^3)^{5/3}} + \frac{\left((4bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{5a^2b\,(a+bx^3)^{2/3}} \\
&= \frac{(bc-ad)x}{5ab\,(a+bx^3)^{5/3}} + \frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\,(a+bx^3)^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(-d+\frac{(4bc+ad)(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{a^2}\right)}{4b\,(a+bx^3)^{5/3}}$$

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
[Out] (x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}d x$$

```
[In] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{c+dx^{3}}{\left(a+bx^{3}\right)^{8/3}}\,dx = \int \frac{dx^{3}+c}{\left(bx^{3}+a\right)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 35.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^{3}}{\left(a+bx^{3}\right)^{8/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{8/3}}\,dx$$

```
[In] int((c + d*x^3)/(a + b*x^3)^(8/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(8/3), x)
```

3.70 $\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 262

$$\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx = \frac{5a(27b^2c^2-6abcd+a^2d^2)\, x(a+bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2-6abcd+a^2d^2)\, x(a+bx^3)^{5/3}}{162b^2} + \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} + \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{243\sqrt{3}b^{7/3}} - \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{486b^{7/3}}$$

```
[Out] 5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^2
-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/108*d*(-4*a*d+15*b*c)*x*(b*x
^3+a)^(8/3)/b^2+1/12*d*x*(b*x^3+a)^(8/3)*(d*x^3+c)/b-5/486*a^2*(a^2*d^2-6*a
*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+5/729*a^2*(a^2*d^
2-6*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 396, 201, 245}

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{5a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(a^2d^2-6abcd+27b^2c^2)}{243\sqrt{3}b^{7/3}} + \frac{x(a+bx^3)^{5/3}(a^2d^2-6abcd+27b^2c^2)}{162b^2} + \frac{5ax(a+bx^3)^{2/3}(a^2d^2-6abcd+27b^2c^2)}{486b^2} - \frac{5a^2(a^2d^2-6abcd+27b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{486b^{7/3}} + \frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{108b^2} + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} + \frac{\int (a+bx^3)^{5/3}(c(12bc-ad)+d(15bc-4ad)x^3)\,dx}{12b} \\
&= \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} \\
&\quad + \frac{(27b^2c^2-6abcd+a^2d^2)\int (a+bx^3)^{5/3}\,dx}{27b^2} \\
&= \frac{(27b^2c^2-6abcd+a^2d^2)x(a+bx^3)^{5/3}}{162b^2} + \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} \\
&\quad + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} + \frac{(5a(27b^2c^2-6abcd+a^2d^2))\int (a+bx^3)^{2/3}\,dx}{162b^2} \\
&= \frac{5a(27b^2c^2-6abcd+a^2d^2)x(a+bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2-6abcd+a^2d^2)x(a+bx^3)^{5/3}}{162b^2} \\
&\quad + \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} \\
&\quad + \frac{(5a^2(27b^2c^2-6abcd+a^2d^2))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{243b^2} \\
&= \frac{5a(27b^2c^2-6abcd+a^2d^2)x(a+bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2-6abcd+a^2d^2)x(a+bx^3)^{5/3}}{162b^2} \\
&\quad + \frac{d(15bc-4ad)x(a+bx^3)^{8/3}}{108b^2} + \frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b} \\
&\quad + \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{243\sqrt{3}b^{7/3}} \\
&\quad - \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{486b^{7/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(-20a^3d^2+15a^2bd(8c+dx^3)+27b^3x^3(6c^2+8cdx^3+3d^2x^6)+18ab^2(24c^2+\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 27
*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3 +
7*d^2*x^6)) + 20*Sqrt[3]*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 20*a^2*(27*b^2*c^2 - 6
*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 10*a^2*(27*b^2*
c^2 - 6*a*b*c*d + a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) +
(a + b*x^3)^(2/3)])/(2916*b^(7/3))
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{x^4\left(\frac{1}{2}d^2x^6+\frac{4}{3}cdx^3+c^2\right)\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{10}{3}}}{6}+\frac{5\left(36xd\left(\frac{dx^3}{8}+c\right)\left(bx^3+a\right)^{\frac{2}{3}}ab^{\frac{4}{3}}+\frac{648x\left(\frac{7}{24}d^2x^6+\frac{11}{12}cdx^3+c^2\right)\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{7}{3}}}{5}+\left(-6\left(bx^3+a\cdots\right.\right.\right.}{\cdots}$

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/1458/b^(7/3)*(243/5*x^4*(1/2*d^2*x^6+4/3*c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^(
10/3)+(36*x*d*(1/8*d*x^3+c)*(b*x^3+a)^(2/3)*a*b^(4/3)+648/5*x*(7/24*d^2*x^6
+11/12*c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^(7/3)+(-6*(b*x^3+a)^(2/3)*a*d^2*x*b^(
1/3)+(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x
)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^
(1/3)*x+(b*x^3+a)^(1/3))/x))*(a^2*d^2-6*a*b*c*d+27*b^2*c^2))*a)*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.74

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2 dx = \left[\frac{30\sqrt{\frac{1}{3}}(27\,a^2b^3c^2-6\,a^3b^2cd+a^4bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)\right.\right.}{}\right.$$

$$-\frac{60\sqrt{\frac{1}{3}}(27\,a^2b^3c^2-6\,a^3b^2cd+a^4bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+20\,(27\,a^2b^2c^2}{}$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2*c^2
- 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c
*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4
+ 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^
3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt(
-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(
-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2 - 6*a^3*b*c*
d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c*d + 7*a*b^3
*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 4*(108*a*b
^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{a^{\frac{5}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{5}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{5}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{2}{3}}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(227) = 454$.

Time = 0.28 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.56

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx =$$

$$-\frac{1}{54}\left(\frac{10\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{243}\left(\frac{10\,\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{2916}\left(\frac{20\,\sqrt{3}a^4\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{10\,a^4\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{20\,a^4\log\left(-b^{\frac{1}{3}}\right.}{b^{\frac{7}{3}}}\right.$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) +
 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3
)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x
^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*
b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^
```

```
(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36*(b*x^3 + a)^(
5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^3 + a)^(11/3)*a
^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 - 4*(b*x^3
+ a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

Giac [F]

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2dx$$

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx=\int {\left(b\,x^3+a\right)}^{5/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

3.71 $\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx = \frac{(27b^2c^2-9abcd+2a^2d^2)\,x(a+bx^3)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x(a+bx^3)^{5/3}}{27b^2} + \frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b} + \frac{2a(27b^2c^2-9abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{7/3}} - \frac{a(27b^2c^2-9abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81b^{7/3}}$$

```
[Out] 1/81*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+2/27*d*(-a*d+3*
b*c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d*x*(b*x^3+a)^(5/3)*(d*x^3+c)/b-1/81*a*(2*a^
2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2/243*a*
(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 201, 245}

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{2a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(2a^2d^2-9abcd+27b^2c^2\right)}{81\sqrt{3}b^{7/3}} + \frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2} - \frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81b^{7/3}} + \frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{27b^2} + \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}+\frac{\int \left(a+bx^3\right)^{2/3}\left(c(9bc-ad)+4d(3bc-ad)x^3\right)\,dx}{9b}\\
&= \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}\\
&\quad -\frac{\left(4ad(3bc-ad)-6bc(9bc-ad)\right)\int \left(a+bx^3\right)^{2/3}\,dx}{54b^2}\\
&= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}\\
&\quad +\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}+\frac{\left(2a\left(27b^2c^2-9abcd+2a^2d^2\right)\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{81b^2}\\
&= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}\\
&\quad +\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}+\frac{2a\left(27b^2c^2-9abcd+2a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{7/3}}\\
&\quad -\frac{a\left(27b^2c^2-9abcd+2a^2d^2\right)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81b^{7/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(-4a^2d^2+3abd(6c+dx^3)+9b^2(3c^2+3cdx^3+d^2x^6)\right)+2\sqrt{3}a(27b^2c^2-9abc}{}$$

[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]

[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2*(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + 2*Sqrt[3]*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(7/3))

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{4\left(-\frac{27x\left(b\,x^3+a\right)^{\frac{2}{3}}da\left(\frac{d\,x^3}{6}+c\right)b^{\frac{4}{3}}}{2}-\frac{81x\left(\frac{1}{3}d^2x^6+cd\,x^3+c^2\right)\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{7}{3}}}{4}+\left(3\left(b\,x^3+a\right)^{\frac{2}{3}}a\,d^2x\,b^{\frac{1}{3}}+\left(a^2d^2-\frac{9}{2}abcd+\frac{27}{2}b^2c^2\right)\right.\right.}{243b^{\frac{7}{3}}}$

[In] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)

[Out] -4/243*(-27/2*x*(b*x^3+a)^(2/3)*d*a*(1/6*d*x^3+c)*b^(4/3)-81/4*x*(1/3*d^2*x^6+c*d*x^3+c^2)*(b*x^3+a)^(2/3)*b^(7/3)+(3*(b*x^3+a)^(2/3)*a*d^2*x*b^(1/3)+(a^2*d^2-9/2*a*b*c*d+27/2*b^2*c^2)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))*a)/b^(7/3)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.89

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(27\,ab^3c^2-9\,a^2b^2cd+2\,a^3bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(27\,ab^3c^2-9\,a^2b^2cd+2\,a^3bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(27\,ab^2c^2-9}{}$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)^
(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-
b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x
)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9*a^2*
b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2
*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))
/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt
(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a^2*b*c*d + 2
*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2*d^2)*x^4
 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.60

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \frac{a^{\frac{2}{3}} c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{2}{3}} c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(188) = 376.

Time = 0.28 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.52

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{27}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+\right.}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^3\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
```

```
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)^2\,dx$$

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

3.72 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

```
[Out] 1/18*d*(-4*a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d*x*(b*x^3+a)^(2/3)*(d*x^3+
c)/b-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^
(7/3)+1/27*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{427, 396, 245\}$

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2a^2d^2-6abcd+9b^2c^2)}{9\sqrt{3}b^{7/3}} - \frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{7/3}} + \frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{\int \frac{c(6bc-ad)+d(9bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b^2}$$

$$= \frac{d(9bc-4ad)x(a+bx^3)^{2/3}}{18b^2} + \frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{3\sqrt[3]{b}dx(a+bx^3)^{2/3}(-4ad+3b(4c+dx^3)) + 2\sqrt{3}(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2}{}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + 2*Sqrt[3]*(
9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
2*(a + b*x^3)^(1/3))] - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*
x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(7/3))
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(-9x\left(\frac{dx^3}{4}+c\right)d\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{4}{3}}+3\left(bx^3+a\right)^{\frac{2}{3}}ad^2xb^{\frac{1}{3}}+\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.\right.\right.}{27b^{\frac{7}{3}}}$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*(-9*x*(1/4*d*x^3+c)*d*(b*x^3+a)^(2/3)*b^(4/3)+3*(b*x^3+a)^(2/3)*a*d^2
*x*b^(1/3)+(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3
)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-3*a*b*c*d+9/2*b^2*c^2))/b^(7/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.17

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(9\,b^3c^2 - 6\,ab^2cd + 2\,a^2bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3 - (bx^3+a\right.\right.}{} \right.$$

$$- \frac{6\sqrt{\frac{1}{3}}(9\,b^3c^2 - 6\,ab^2cd + 2\,a^2bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x - 2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) + 2\,(9\,b^2c^2 - 6\,abc}{}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)/
b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt
((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*s
qrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*s
qrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(148) = 296.

Time = 0.28 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{7}{3}}}\right.$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*d^2
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

3.73 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2}+\frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}}-\frac{d(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{7/3}}$$

```
[Out] -1/3*d*(-4*a*d+3*b*c)*x*(b*x^3+a)^(2/3)/a/b^2+(-a*d+b*c)*x*(d*x^3+c)/a/b/(b
*x^3+a)^(1/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2
/9*d*(-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {424, 396, 245}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-2ad)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{7/3}} - \frac{dx(a+bx^3)^{2/3}(3bc-4ad)}{3ab^2} + \frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] -1/3*(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(a*b^2) + ((b*c - a*d)*x*(c +
d*x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{\int \frac{acd-d(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2} + \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{(2d(3bc-2ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2}$$

$$= -\frac{d(3bc-4ad)x(a+bx^3)^{2/3}}{3ab^2} + \frac{(bc-ad)x(c+dx^3)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{7/3}}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{b}x(3b^2c^2+4a^2d^2+abd(-6c+dx^3))}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}d(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2d(-3bc$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] ((3*b^(1/3)*x*(3*b^2*c^2 + 4*a^2*d^2 + a*b*d*(-6*c + d*x^3)))/(a*(a + b*x^3
)^(1/3)) + 2*Sqrt[3]*d*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*
x + 2*(a + b*x^3)^(1/3))] + 2*d*(-3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)] + d*(3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{2\left(d\left(ad-\frac{3bc}{2}\right)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)}{x}\right.\right.\right.}{9b^{\frac{7}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a}$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(d*(a*d-3/2*b*c)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)
^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*a*(b*x^3+a)^(1/3)-6*x*(-3/2*d
*(-1/6*d*x^3+c)*a*b^(4/3)+a^2*d^2*b^(1/3)+3/4*b^(7/3)*c^2))/b^(7/3)/(b*x^3+
a)^(1/3)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(134) = 268$.

Time = 0.33 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.10

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(3\,a^2b^2cd-2\,a^3bd^2+(3\,ab^3cd-2\,a^2b^2d^2)x^3)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b\right.}{}\right.$$

$$-\frac{2\,(3\,a^2bcd-2\,a^3d^2+(3\,ab^2cd-2\,a^2bd^2)x^3)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(3\,a^2bcd-2\,a^3d^2+(3\,ab^2cd-2\,a^2b}{}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 -
3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*
d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/
3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(a*b^2*d
^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b
^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*
b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a^2*b*c*d
- 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (
b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(3*a^2*b
^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a*b^2*d^2*x^4 +
 (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*x^3 +
 a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(134) = 268.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,d^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{(bx^3 \cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{1}{3}\,cd\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}} \cdots\right.}{\cdots}\right.$$

$$+\frac{c^2x}{(bx^3+a)^{\frac{1}{3}}a}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
[Out] 1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{4/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

3.74 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}}-\frac{d^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{7/3}}$$

```
[Out] 1/4*(-a*d+b*c)*(4*a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(1/3)+1/4*(-a*d+b*c)*x*(d*
x^3+c)/a/b/(b*x^3+a)^(4/3)-1/2*d^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+1
/3*d^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {424, 393, 245}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}+\frac{d^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} -\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{7/3}}+\frac{x(c+dx^3)(bc-ad)}{4ab\,(a+bx^3)^{4/3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}} + \frac{\int \frac{c(3bc+ad)+4ad^2x^3}{(a+bx^3)^{4/3}}\,dx}{4ab} \\
&= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b^2} \\
&= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{4ab\,(a+bx^3)^{4/3}} \\
&\quad + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{7/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{\frac{3\sqrt[3]{b}(bc-ad)x(4a^2d+3b^2cx^3+ab(4c+5dx^3))}{a^2(a+bx^3)^{4/3}} + 4\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 4d^2\log\left(-\sqrt[3]{b}x\right.}{12b^{7/3}}$$

[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]

[Out] ((3*b^(1/3)*(b*c - a*d)*x*(4*a^2*d + 3*b^2*c*x^3 + a*b*(4*c + 5*d*x^3)))/(a^2*(a + b*x^3)^(4/3)) + 4*Sqrt[3]*d^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*d^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*d^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(12*b^(7/3))

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{-3x\left(\frac{dx^3}{2}+c\right)cab^{\frac{7}{3}}-\frac{9b^{\frac{10}{3}}c^2x^4}{4}+d^2\left(\frac{15b^{\frac{4}{3}}x^4}{4}+3xab^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{4}{3}}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+2\ln\left(\frac{-b^{\frac{1}{3}}x+(bx\right.}{x}\right.}{2}\right.}{3b^{\frac{7}{3}}(bx^3+a)^{\frac{4}{3}}a^2}$

[In] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)

[Out] -1/3*(-3*x*(1/2*d*x^3+c)*c*a*b^(7/3)-9/4*b^(10/3)*c^2*x^4+d^2*(15/4*b^(4/3)*x^4+3*x*a*b^(1/3)+1/2*(b*x^3+a)^(4/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))*a^2)/b^(7/3)/(b*x^3+a)^(4/3)/a^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(128) = 256$.

Time = 0.40 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.73

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(a^2b^3d^2x^6+2\,a^3b^2d^2x^3+a^4bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{}\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}(a^2b^3d^2x^6+2\,a^3b^2d^2x^3+a^4bd^2)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+4\,(a^2b^2d^2x^6}{}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt((
-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/
3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^
4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 + 2*a*b^3
*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2*b^3*d^2*
x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*
((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 +
a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*
b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)c^2x^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{cdx^4}{2\,(bx^3+a)^{\frac{4}{3}}a} - \frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4\,\mathrm{l}}{}\right.$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4/
((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4
/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d^2
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{7/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

3.75 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

```
[Out] 9/14*c^2*x/a^3/(b*x^3+a)^(1/3)+3/14*c*x*(d*x^3+c)/a^2/(b*x^3+a)^(4/3)+1/7*x
*(d*x^3+c)^2/a/(b*x^3+a)^(7/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {386, 197}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{(6c)\int\frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx}{7a}\\
&= \frac{3cx(c+dx^3)}{14a^2\,(a+bx^3)^{4/3}}+\frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}}+\frac{(9c^2)\int\frac{1}{(a+bx^3)^{4/3}}\,dx}{14a^2}\\
&= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx(c+dx^3)}{14a^2\,(a+bx^3)^{4/3}}+\frac{x(c+dx^3)^2}{7a\,(a+bx^3)^{7/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int\frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx=\frac{9b^2c^2x^7+3abcx^4(7c+dx^3)+a^2(14c^2x+7cdx^4+2d^2x^7)}{14a^3\,(a+bx^3)^{7/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
[Out] (9*b^2*c^2*x^7 + 3*a*b*c*x^4*(7*c + d*x^3) + a^2*(14*c^2*x + 7*c*d*x^4 + 2*
d^2*x^7))/(14*a^3*(a + b*x^3)^(7/3))
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{(2a^2d^2+3abcd+9b^2c^2)x^7+7(a^2cd+3b\,c^2a)x^4+14a^2c^2x}{14(b\,x^3+a)^{\frac{7}{3}}a^3}$	71
gosper	$\frac{x(2a^2d^2x^6+3abcd\,x^6+9b^2c^2x^6+7a^2cd\,x^3+21ab\,c^2x^3+14a^2c^2)}{14(b\,x^3+a)^{\frac{7}{3}}a^3}$	76
trager	$\frac{x(2a^2d^2x^6+3abcd\,x^6+9b^2c^2x^6+7a^2cd\,x^3+21ab\,c^2x^3+14a^2c^2)}{14(b\,x^3+a)^{\frac{7}{3}}a^3}$	76

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*((2*a^2*d^2+3*a*b*c*d+9*b^2*c^2)*x^7+7*(a^2*c*d+3*a*b*c^2)*x^4+14*a^2*
c^2*x)/(b*x^3+a)^(7/3)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{((9\,b^2c^2+3\,abcd+2\,a^2d^2)x^7+14\,a^2c^2x+7\,(3\,abc^2+a^2cd)x^4)(bx^3+a)^{\frac{2}{3}}}{14\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^2
 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x
^3 + a^6)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(10/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)cdx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{d^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)c^2x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x
^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 +
a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{2\,a^4\,d^2\,x+2\,a^2\,d^2\,x\,{\left(b\,x^3+a\right)}^2+9\,b^2\,c^2\,x\,{\left(b\,x^3+a\right)}^2+2\,a^2\,b^2\,c^2\,x-4\,a^3\,d^2\,x\,\left(b\,x^3+a\right.}{14\,a^3\,b^2\,(b\,x^3}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

```
[Out] (2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a^
2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3*b
*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^2*(
a + b*x^3)^(7/3))
```

3.76 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\,(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\,(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)\,(a+bx^3)^{10/3}}$$

```
[Out] 9/140*c^2*(-10*a*d+9*b*c)*x/a^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+3/140*c*(-10*a*d
+9*b*c)*x*(d*x^3+c)/a^3/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/70*(-10*a*d+9*b*c)*x*(
d*x^3+c)^2/a^2/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/10*b*x*(d*x^3+c)^3/a/(-a*d+b*c)
/(b*x^3+a)^(10/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 386, 197}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}(bc-ad)} + \frac{3cx(c+dx^3)(9bc-10ad)}{140a^3(a+bx^3)^{4/3}(bc-ad)} + \frac{x(c+dx^3)^2(9bc-10ad)}{70a^2(a+bx^3)^{7/3}(bc-ad)} + \frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
```

```
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^3)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}}+\frac{(9bc-10ad)\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx}{10a(bc-ad)}\\
&= \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}}+\frac{bx(c+dx^3)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}}+\frac{(3c(9bc-10ad))\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx}{35a^2(bc-ad)}\\
&= \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}}+\frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}}\\
&\quad+\frac{bx(c+dx^3)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}}+\frac{(9c^2(9bc-10ad))\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{140a^3(bc-ad)}\\
&= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}}\\
&\quad+\frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}}+\frac{bx(c+dx^3)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x(81b^3c^2x^9+18ab^2cx^6(15c+dx^3)+10a^3(14c^2+7cdx^3+2d^2x^6)+3a^2bx^3(105c^2+20c}{140a^4\,(a+bx^3)^{10/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*d
*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140*a
^4*(a + b*x^3)^(10/3))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}d^2x^6+\frac{1}{2}cd\,x^3+c^2\right)a^3+\frac{9x^3b\left(\frac{2}{105}d^2x^6+\frac{4}{21}cd\,x^3+c^2\right)a^2}{4}+\frac{27x^6\left(\frac{d\,x^3}{15}+c\right)b^2ca}{14}+\frac{81b^3c^2x^9}{140}\right)}{(b\,x^3+a)^{\frac{10}{3}}a^4}$	96
gosper	$\frac{x\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140a^3c^2\right)}{140(b\,x^3+a)^{\frac{10}{3}}a^4}$	115
trager	$\frac{x\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140a^3c^2\right)}{140(b\,x^3+a)^{\frac{10}{3}}a^4}$	115

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^3+9/4*x^3*b*(2/105*d^2*x^6+4/21*c*d*x^3+
c^2)*a^2+27/14*x^6*(1/15*d*x^3+c)*b^2*c*a+81/140*b^3*c^2*x^9)/(b*x^3+a)^(10
/3)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(3\,(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2)x^{10}+10\,(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2)x^7+140\,a^3c^2x+3}{140\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2 +
 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c*d
```

```
)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*
a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)d^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2}+\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)cdx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)c^2x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^4}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 + a)
^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b
/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{(b\,x^3+a)^{10/3}}-\frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{(b\,x^3+a)^{7/3}}+\frac{x\,(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2)}{140\,a^3\,b^2\,(b\,x^3+a)^{4/3}}+\frac{x\,(6\,a^2\,d^2+18\,a\,b\,c\,d+81\,b^2\,c^2)}{140\,a^4\,b^2\,(b\,x^3+a)^{1/3}}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (x
*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x^3
)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x^3)
^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b*x^3
)^(1/3))
```

3.77 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2(a+bx^3)^{7/3}} + \frac{3(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^4b^2(a+bx^3)^{4/3}} + \frac{9(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{13ab(a+bx^3)^{13/3}}$$

```
[Out] 2/65*(-a*d+b*c)*(a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(10/3)+1/455*(2*a^2*d^2+9*a
*b*c*d+54*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(7/3)+3/910*(2*a^2*d^2+9*a*b*c*d+54*
b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(4/3)+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x
/a^5/b^2/(b*x^3+a)^(1/3)+1/13*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(13/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 393, 198, 197}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{2x(bc-ad)(ad+3bc)}{65a^2b^2(a+bx^3)^{10/3}} + \frac{9x(2a^2d^2+9abcd+54b^2c^2)}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{3x(2a^2d^2+9abcd+54b^2c^2)}{910a^4b^2(a+bx^3)^{4/3}} + \frac{x(2a^2d^2+9abcd+54b^2c^2)}{455a^3b^2(a+bx^3)^{7/3}} + \frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
```

```
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}}+\frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{(a+bx^3)^{13/3}}\,dx}{13ab}\\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}}+\frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}}+\frac{(54b^2c^2+9abcd+2a^2d^2)\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{65a^2b^2}\\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\,(a+bx^3)^{10/3}}+\frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2\,(a+bx^3)^{7/3}}\\
&\quad+\frac{(bc-ad)x(c+dx^3)}{13ab\,(a+bx^3)^{13/3}}+\frac{(6(54b^2c^2+9abcd+2a^2d^2))\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{455a^3b^2}
\end{aligned}
$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{(bc-ad)x(c+dx^3)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(9\left(54b^2c^2+9abcd+2a^2d^2\right)\right)\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{910a^4b^2}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{9\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{13ab\left(a+bx^3\right)^{13/3}}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.65

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{x(486b^4c^2x^{12}+81ab^3cx^9(26c+dx^3)+65a^4(14c^2+7cdx^3+2d^2x^6)+39a^3bx^3(70c^2+ \cdots}{910a^5\left(a+bx^3\right)^{13/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*c
*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*a^
2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/3))
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}d^2x^6+\frac{1}{2}cd\,x^3+c^2\right)a^4+3x^3b\left(\frac{1}{35}d^2x^6+\frac{3}{14}cd\,x^3+c^2\right)a^3+\frac{27x^6b^2\left(\frac{1}{195}d^2x^6+\frac{1}{10}cd\,x^3+c^2\right)a^2}{7}+\frac{81x^9b^3c\left(\frac{d\,x^3}{26}+c\right)a}{35}+\frac{243b^4c^2 \cdots}{455 \cdots}\right.}{\left(b\,x^3+a\right)^{\frac{13}{3}}a^5}$
gosper	$\frac{x\left(18a^2b^2d^2x^{12}+81a\,b^3cd\,x^{12}+486b^4c^2x^{12}+78a^3b\,d^2x^9+351a^2b^2cd\,x^9+2106a\,b^3c^2x^9+130a^4d^2x^6+585a^3bcd\,x^6+3510a^2b^2 \cdots\right.}{910\left(b\,x^3+a\right)^{\frac{13}{3}}a^5}$
trager	$\frac{x\left(18a^2b^2d^2x^{12}+81a\,b^3cd\,x^{12}+486b^4c^2x^{12}+78a^3b\,d^2x^9+351a^2b^2cd\,x^9+2106a\,b^3c^2x^9+130a^4d^2x^6+585a^3bcd\,x^6+3510a^2b^2 \cdots\right.}{910\left(b\,x^3+a\right)^{\frac{13}{3}}a^5}$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^4+3*x^3*b*(1/35*d^2*x^6+3/14*c*d*x^3+c^2
)*a^3+27/7*x^6*b^2*(1/195*d^2*x^6+1/10*c*d*x^3+c^2)*a^2+81/35*x^9*b^3*c*(1/
26*d*x^3+c)*a+243/455*b^4*c^2*x^12)/(b*x^3+a)^(13/3)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(9\,(54\,b^4c^2+9\,ab^3cd+2\,a^2b^2d^2)x^{13}+39\,(54\,ab^3c^2+9\,a^2b^2cd+2\,a^3bd^2)x^{10}+65\,(54\,a^2b}{910\,(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+}$$

[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")

[Out] 1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^2 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d + 2*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx &= \frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)d^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} \\ &\quad-\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)cdx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^4} \\ &\quad+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)c^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}\end{aligned}$$

[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*x
^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4) +
1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(
b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13/3)*a
^5)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{(b\,x^3+a)^{13/3}} - \frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{(b\,x^3+a)^{10/3}} + \frac{x\,(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2)}{455\,a^3\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(6\,a^2\,d^2+27\,a\,b\,c\,d+162\,b^2\,c^2)}{910\,a^4\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(18\,a^2\,d^2+81\,a\,b\,c\,d+486\,b^2\,c^2)}{910\,a^5\,b^2\,(b\,x^3+a)^{1/3}}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3) -
 (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a +
 b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(a +
 b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^2*(a
 + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^5*b^2
*(a + b*x^3)^(1/3))
```

3.78 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 253

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx &= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}} \\ &+ \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2+6abcd+a^2d^2)\,x}{7280a^5b^2(a+bx^3)^{4/3}} \\ &+ \frac{81(45b^2c^2+6abcd+a^2d^2)\,x}{7280a^6b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}}\end{aligned}$$

```
[Out] 1/208*(-a*d+b*c)*(4*a*d+15*b*c)*x/a^2/b^2/(b*x^3+a)^(13/3)+1/520*(a^2*d^2+6
*a*b*c*d+45*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(10/3)+9/3640*(a^2*d^2+6*a*b*c*d+4
5*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(7/3)+27/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)
*x/a^5/b^2/(b*x^3+a)^(4/3)+81/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^6/b^2
/(b*x^3+a)^(1/3)+1/16*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(16/3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 393, 198, 197}

$$\begin{aligned}\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx &= \frac{x(bc-ad)(4ad+15bc)}{208a^2b^2(a+bx^3)^{13/3}} + \frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}} \\ &+ \frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}} \\ &+ \frac{x(a^2d^2+6abcd+45b^2c^2)}{520a^3b^2(a+bx^3)^{10/3}} + \frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}\end{aligned}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^3)}{16ab\,(a+bx^3)^{16/3}} + \frac{\int \frac{c(15bc+ad)+4d(3bc+ad)x^3}{(a+bx^3)^{16/3}}\,dx}{16ab}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\,(a+bx^3)^{13/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab\,(a+bx^3)^{16/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\int \frac{1}{(a+bx^3)^{13/3}}\,dx}{52a^2b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{(9(45b^2c^2+6abcd+a^2d^2))\int\frac{1}{(a+bx^3)^{10/3}}\,dx}{520a^3b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{(27(45b^2c^2+6abcd+a^2d^2))\int\frac{1}{(a+bx^3)^{7/3}}\,dx}{1820a^4b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2+6abcd+a^2d^2)\,x}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{(81(45b^2c^2+6abcd+a^2d^2))\int\frac{1}{(a+bx^3)^{4/3}}\,dx}{7280a^5b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\,x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\,x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2+6abcd+a^2d^2)\,x}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{81(45b^2c^2+6abcd+a^2d^2)\,x}{7280a^6b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}}$$

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(3645b^5c^2x^{15}+486ab^4cx^{12}(40c+dx^3)+81a^2b^3x^9(520c^2+32cdx^3+d^2x^6)+520a^5(14}{7280}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(5
20*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6) +
 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175*c^
2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}d^2x^6+\frac{1}{2}cd\,x^3+c^2\right)a^5+\frac{15x^3b\left(\frac{6}{175}d^2x^6+\frac{8}{35}cd\,x^3+c^2\right)a^4}{4}+\frac{45x^6\left(\frac{3}{325}d^2x^6+\frac{3}{25}cd\,x^3+c^2\right)b^2a^3}{7}+\frac{81x^9b^3\left(\frac{1}{520}d^2x^6+\frac{4}{65}cd\,x^3+\right.}{14}}{(b\,x^3+a)^{\frac{16}{3}}a^6}$
gosper	$\frac{x(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2cd}{7280(b\,x^3+a)^{\frac{16}{3}}a}$
trager	$\frac{x(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2cd}{7280(b\,x^3+a)^{\frac{16}{3}}a}$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^5+15/4*x^3*b*(6/175*d^2*x^6+8/35*c*d*x^3
+c^2)*a^4+45/7*x^6*(3/325*d^2*x^6+3/25*c*d*x^3+c^2)*b^2*a^3+81/14*x^9*b^3*(
1/520*d^2*x^6+4/65*c*d*x^3+c^2)*a^2+243/91*x^12*b^4*(1/40*d*x^3+c)*c*a+729/
1456*b^5*c^2*x^15)/(b*x^3+a)^(16/3)/a^6
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(81\,(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2)x^{16}+432\,(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2)x^{13}+936\,(45\,a}{7280\,(a^6b^6x^{18}+6\,a^7b^5x}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c^
2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c*d
 + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d +
a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^6*
b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10*b^2*
x^6 + 6*a^11*b*x^3 + a^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = -\frac{\left(455\,b^3 - \frac{1680\,(bx^3+a)b^2}{x^3} + \frac{2184\,(bx^3+a)^2 b}{x^6} - \frac{1040\,(bx^3+a)^3}{x^9}\right)d^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4} + \frac{\left(455\,b^4 - \frac{2240\,(bx^3+a)b^3}{x^3} + \frac{4368\,(bx^3+a)^2 b^2}{x^6} - \frac{4160\,(bx^3+a)^3 b}{x^9} + \frac{1820\,(bx^3+a)^4}{x^{12}}\right)cdx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^5} - \frac{\left(91\,b^5 - \frac{560\,(bx^3+a)b^4}{x^3} + \frac{1456\,(bx^3+a)^2 b^3}{x^6} - \frac{2080\,(bx^3+a)^3 b^2}{x^9} + \frac{1820\,(bx^3+a)^4 b}{x^{12}} - \frac{1456\,(bx^3+a)^5}{x^{15}}\right)c^2x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^6}$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1/145
6*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*
x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*c
^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x\left(\frac{c^2}{16\,a}+\frac{a\left(\frac{d^2}{16\,b}-\frac{c\,d}{8\,a}\right)}{b}\right)}{(b\,x^3+a)^{16/3}} - \frac{x\left(\frac{d^2}{13\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{(b\,x^3+a)^{13/3}} + \frac{x\,(a^2\,d^2+6\,a\,b\,c\,d+45\,b^2\,c^2)}{520\,a^3\,b^2\,(b\,x^3+a)^{10/3}} + \frac{x\,(9\,a^2\,d^2+54\,a\,b\,c\,d+405\,b^2\,c^2)}{3640\,a^4\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(27\,a^2\,d^2+162\,a\,b\,c\,d+1215\,b^2\,c^2)}{7280\,a^5\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(81\,a^2\,d^2+486\,a\,b\,c\,d+3645\,b^2\,c^2)}{7280\,a^6\,b^2\,(b\,x^3+a)^{1/3}}$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (x
*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a + b*
x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b*x^
3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(a +
b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^5*b^2
*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(7280*a
^6*b^2*(a + b*x^3)^(1/3))
```

3.79 $\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} + \frac{a^2(77b^2c^2-14abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/154*d*(-4*a*d+17*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d*x*(b*x^3+a)^(10/3)*(d
*x^3+c)/b+1/77*a^2*(2*a^2*d^2-14*a*b*c*d+77*b^2*c^2)*x*(b*x^3+a)^(1/3)*hype
rgeom([-7/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 252, 251}

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{a^2x\sqrt[3]{a+bx^3}(2a^2d^2-14abcd+77b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{10/3}(17bc-4ad)}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b}$$

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/
3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a +
b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(77*b^2*(1 +
(b*x^3)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} + \frac{\int (a+bx^3)^{7/3}\left(c(14bc-ad)+d(17bc-4ad)x^3\right)\,dx}{14b} \\
&= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} \\
&\quad - \frac{(ad(17bc-4ad)-11bc(14bc-ad))\int (a+bx^3)^{7/3}\,dx}{154b^2}
\end{aligned}
$$

$$= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} - \frac{\left(a^2(ad(17bc-4ad)-11bc(14bc-ad))\sqrt[3]{a+bx^3}\right)\int\left(1+\frac{bx^3}{a}\right)^{7/3}dx}{154b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{d(17bc-4ad)x(a+bx^3)^{10/3}}{154b^2} + \frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b} + \frac{a^2(77b^2c^2-14abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] (warning: unable to verify)

Time = 12.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.31

$$\int (a+bx^3)^{7/3}(c+dx^3)^2\,dx = \frac{ax\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\operatorname{Gamma}\left(-\frac{7}{3}\right)\operatorname{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)-\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*H
ypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3)/a
)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}, {1
, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

Maple [F]

$$\int \left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)^2dx$$

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)^2 \, dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.10

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \frac{a^{\frac{7}{3}} c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{7}{3}} c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{7}{3}} d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2a^{\frac{4}{3}} b c^2 x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4a^{\frac{4}{3}} b c d x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2a^{\frac{4}{3}} b d^2 x^{10} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{\sqrt[3]{a} b^2 c^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2\sqrt[3]{a} b^2 c d x^{10} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{\sqrt[3]{a} b^2 d^2 x^{13} \Gamma\left(\frac{13}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{13}{3} \\ \frac{16}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
```

```
2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3)*b*d**
2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10
/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2*c*d*x**10
*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13/3), (16/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)^2 dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)^2 dx$$

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^2 \,dx$$

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3)^2, x)
```

3.80 $\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b} + \frac{a(44b^2c^2-11abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/44*d*(-2*a*d+7*b*c)*x*(b*x^3+a)^(7/3)/b^2+1/11*d*x*(b*x^3+a)^(7/3)*(d*x^3
+c)/b+1/44*a*(2*a^2*d^2-11*a*b*c*d+44*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom(
[-4/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 252, 251}

$$\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx = \frac{ax\sqrt[3]{a+bx^3}(2a^2d^2-11abcd+44b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{7/3}(7bc-2ad)}{44b^2} + \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b}$$

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(7*b*c - 2*a*d)*x*(a + b*x^3)^(7/3))/(44*b^2) + (d*x*(a + b*x^3)^(7/3)*(
c + d*x^3))/(11*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(44*b^2*(1 + (b*x^3
)/a)^(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b}+\frac{\int (a+bx^3)^{4/3}\left(c(11bc-ad)+2d(7bc-2ad)x^3\right)\,dx}{11b}\\
&= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2}+\frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b}\\
&\quad-\frac{(2ad(7bc-2ad)-8bc(11bc-ad))\int (a+bx^3)^{4/3}\,dx}{88b^2}
\end{aligned}$$

$$= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b} - \frac{\left(a(2ad(7bc-2ad)-8bc(11bc-ad))\sqrt[3]{a+bx^3}\right)\int\left(1+\frac{bx^3}{a}\right)^{4/3}dx}{88b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{d(7bc-2ad)x(a+bx^3)^{7/3}}{44b^2} + \frac{dx(a+bx^3)^{7/3}(c+dx^3)}{11b} + \frac{a(44b^2c^2-11abcd+2a^2d^2)\,x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] (warning: unable to verify)

Time = 12.00 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int (a+bx^3)^{4/3}(c+dx^3)^2\,dx = \frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{4}{3}\right)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right) - \right.}{}$$

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-4/3]*Hyp
ergeometric2F1[-4/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[-1/3]*Hypergeometric2F1[-1/3, 4/3, 13/3, -((b*x^3)/a)]
 - 9*b*x^3*(c + d*x^3)^2*Gamma[-1/3]*HypergeometricPFQ[{-1/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-4/3])
```

Maple [F]

$$\int \left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)^2 dx$$

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.03

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}bc^2x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2\sqrt[3]{a}bcdx^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}bd^2x^{10}\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int {\left(b\,x^3+a\right)}^{4/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

3.81 $\int \sqrt[3]{a+bx^3}(c+dx^3)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2}+\frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\left(10b^2c^2-4abcd+a^2d^2\right)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/40*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d*x*(b*x^3+a)^(4/3)*(d*x^3
+c)/b+1/10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3
, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 252, 251}

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{x\sqrt[3]{a+bx^3}(a^2d^2-4abcd+10b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}\left(11bc-4ad\right)}{40b^2}+\frac{dx(a+bx^3)^{4/3}\left(c+dx^3\right)}{8b}$$

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*
(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3
)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(10*b^2*(1 + (b*x^3)/a)^
(1/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{\int \sqrt[3]{a+bx^3}\left(c(8bc-ad)+d(11bc-4ad)x^3\right)\,dx}{8b} \\
&= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} \\
&\quad + \frac{(10b^2c^2-4abcd+a^2d^2)\int \sqrt[3]{a+bx^3}\,dx}{10b^2}
\end{aligned}$$

$$= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{\left((10b^2c^2-4abcd+a^2d^2)\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{(10b^2c^2-4abcd+a^2d^2)\,x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] (verified)

Time = 9.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$$

$$= \frac{x\sqrt[3]{a+bx^3}\Big(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{1}{3}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)-3bx^3(11c^2}{280a\sqrt[3]{}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hyp
ergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)] -
 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1, 13/3
}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

Maple [F]

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)^2dx$$

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^2\,dx$$

[In] int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)

[Out] int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)

3.82 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} + \frac{(5b^2c^2-5abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2\,(a+bx^3)^{2/3}}$$

```
[Out] 2/5*d*(-a*d+2*b*c)*x*(b*x^3+a)^(1/3)/b^2+1/5*d*x*(b*x^3+a)^(1/3)*(d*x^3+c)/
b+1/5*(2*a^2*d^2-5*a*b*c*d+5*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2
/3],[4/3],-b*x^3/a)/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 252, 251}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2\,(a+bx^3)^{2/3}} + \frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*(a + b*x^3)^(1/3))/(5*b^2) + (d*x*(a + b*x^3)^(1/3)*(c
 + d*x^3))/(5*b) + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(
```

```
2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*b^2*(a + b*x^3)^(2/
3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} + \frac{\int \frac{c(5bc-ad)+4d(2bc-ad)x^3}{(a+bx^3)^{2/3}}\,dx}{5b} \\
&= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} \\
&\quad - \frac{(4ad(2bc-ad)-2bc(5bc-ad))\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{10b^2}
\end{aligned}
$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} - \frac{\left((4ad(2bc-ad)-2bc(5bc-ad))\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{10b^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b} + \frac{\left(5b^2c^2-5abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 15.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{x\left(-d\left(a+bx^3\right)\left(2ad-b\left(5c+dx^3\right)\right)+\left(5b^2c^2-5abcd+2a^2d^2\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeor}\right.}{5b^2\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(-(d*(a + b*x^3)*(2*a*d - b*(5*c + d*x^3))) + (5*b^2*c^2 - 5*a*b*c*d + 2
*a^2*d^2)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)]))/(5*b^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx$$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
[Out] c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{2/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(2/3), x)
```

3.83 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x(c+dx^3)}{2ab(a+bx^3)^{2/3}} + \frac{(b^2c^2+2abcd-2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab^2(a+bx^3)^{2/3}}$$

```
[Out] -1/2*d*(-2*a*d+b*c)*x*(b*x^3+a)^(1/3)/a/b^2+1/2*(-a*d+b*c)*x*(d*x^3+c)/a/b/
(b*x^3+a)^(2/3)+1/2*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 396, 252, 251}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(-2a^2d^2+2abcd+b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab^2(a+bx^3)^{2/3}} - \frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{2ab^2}+\frac{x(c+dx^3)(bc-ad)}{2ab(a+bx^3)^{2/3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] -1/2*(d*(b*c - 2*a*d)*x*(a + b*x^3)^(1/3))/(a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(2*a*b*(a + b*x^3)^(2/3)) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 +
```

```
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*
(a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^3)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}\\
&= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x(c+dx^3)}{2ab\left(a+bx^3\right)^{2/3}}+\\
&\quad -\frac{(-2ad(bc-2ad)-2bc(bc+ad))\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{4ab^2}
\end{aligned}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x(c+dx^3)}{2ab\left(a+bx^3\right)^{2/3}} + -\frac{\left((-2ad(bc-2ad)-2bc(bc+ad))\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}dx}{4ab^2\left(a+bx^3\right)^{2/3}}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x(c+dx^3)}{2ab\left(a+bx^3\right)^{2/3}} + \frac{(b^2c^2+2abcd-2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] (warning: unable to verify)

Time = 12.87 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17

$$\int\frac{(c+dx^3)^2}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(4a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{10}{3},-\frac{b}{}\right.\right.}{84}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x^3
 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*(c
+ d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/(84*
a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

Maple [F]

$$\int\frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{5}{3}}}dx$$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{5/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(5/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(5/3), x)
```

3.84 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5(a+bx^3)^{2/3}} + \frac{(bc-ad)x(c+dx^3)}{5ab(a+bx^3)^{5/3}} + \frac{(2b^2c^2+abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b^2(a+bx^3)^{2/3}}$$

```
[Out] 2/5*(c^2/a^2-d^2/b^2)*x/(b*x^3+a)^(2/3)+1/5*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x
^3+a)^(5/3)+1/5*(2*a^2*d^2+a*b*c*d+2*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom
([1/3, 2/3],[4/3],-b*x^3/a)/a^2/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 393, 252, 251}

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2+abcd+2b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b^2(a+bx^3)^{2/3}} + \frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5(a+bx^3)^{2/3}} + \frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*(c^2/a^2 - d^2/b^2)*x)/(5*(a + b*x^3)^(2/3)) + ((b*c - a*d)*x*(c + d*x^3
))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*a^2*b^2*(
a + b*x^3)^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(bc-ad)x(c+dx^3)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\int \frac{c(4bc+ad)+d(bc+4ad)x^3}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\ &= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x(c+dx^3)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left(2b^2c^2+abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{5a^2b^2}\end{aligned}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x(c+dx^3)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left(\left(2b^2c^2+abcd+2a^2d^2\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}dx}{5a^2b^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x(c+dx^3)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left(2b^2c^2+abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 14.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int\frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(5a(14c^2+7cdx^3+2d^2x^6)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{10}{3},-\right.\right.}{}$$

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(5*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 8/3, 10/3, -((b*x^3)/a)] - 2*b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 11/3, 13/3, -((b*x^3)/a)] - 6*b*x^3*
(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 11/3}, {1, 13/3}, -((b*x^3)/a)]))/
(63*a^3*(a + b*x^3)^(2/3)*Gamma[8/3])
```

Maple [F]

$$\int\frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{8}{3}}}dx$$

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(8/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{8/3}}\,dx$$

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(8/3),x)
[Out] int((c + d*x^3)^2/(a + b*x^3)^(8/3), x)
```

3.85 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} \, dx = \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

```
[Out] 1/10*x*(b*x^3+a)^3/c/(d*x^3+c)^(10/3)+9/70*a*x*(b*x^3+a)^2/c^2/(d*x^3+c)^(7
/3)+27/140*a^2*x*(b*x^3+a)/c^3/(d*x^3+c)^(4/3)+81/140*a^3*x/c^4/(d*x^3+c)^(
1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {386, 197}

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} \, dx = \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}}$$

```
[In] Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{(9a)\int \frac{(a+bx^3)^2}{(c+dx^3)^{10/3}}\,dx}{10c} \\
&= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{(27a^2)\int \frac{a+bx^3}{(c+dx^3)^{7/3}}\,dx}{35c^2} \\
&= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{(81a^3)\int \frac{1}{(c+dx^3)^{4/3}}\,dx}{140c^3} \\
&= \frac{x(a+bx^3)^3}{10c\,(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2\,(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3\,(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x(14b^3c^3x^9 + 6ab^2c^2x^6(10c+3dx^3) + 3a^2bcx^3(35c^2+30cdx^3+9d^2x^6) + a^3(140c^3+3}{140c^4\,(c+dx^3)^{10/3}}$$

```
[In] Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c
^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6
 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{10}b^{3}x^{9}+\frac{3}{7}a\,b^{2}x^{6}+\frac{3}{4}a^{2}b\,x^{3}+a^{3}\right)c^{3}+\frac{9x^{3}\left(\frac{2}{35}b^{2}x^{6}+\frac{2}{7}ab\,x^{3}+a^{2}\right)da\,c^{2}}{4}+\frac{27x^{6}\left(\frac{b\,x^{3}}{10}+a\right)d^{2}a^{2}c}{14}+\frac{81a^{3}d^{3}x^{9}}{140}\right)}{\left(d\,x^{3}+c\right)^{\frac{10}{3}}c^{4}}$
gosper	$\frac{x\left(81a^{3}d^{3}x^{9}+27a^{2}bc\,d^{2}x^{9}+18a\,b^{2}c^{2}d\,x^{9}+14b^{3}c^{3}x^{9}+270a^{3}c\,d^{2}x^{6}+90a^{2}b\,c^{2}d\,x^{6}+60a\,b^{2}c^{3}x^{6}+315a^{3}c^{2}d\,x^{3}+105a^{2}b\,c^{3}x^{3}+140\right.}{140\left(d\,x^{3}+c\right)^{\frac{10}{3}}c^{4}}$
trager	$\frac{x\left(81a^{3}d^{3}x^{9}+27a^{2}bc\,d^{2}x^{9}+18a\,b^{2}c^{2}d\,x^{9}+14b^{3}c^{3}x^{9}+270a^{3}c\,d^{2}x^{6}+90a^{2}b\,c^{2}d\,x^{6}+60a\,b^{2}c^{3}x^{6}+315a^{3}c^{2}d\,x^{3}+105a^{2}b\,c^{3}x^{3}+140\right.}{140\left(d\,x^{3}+c\right)^{\frac{10}{3}}c^{4}}$

```
[In] int((b*x^3+a)^3/(d*x^3+c)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] x/(d*x^3+c)^(10/3)*((1/10*b^3*x^9+3/7*a*b^2*x^6+3/4*a^2*b*x^3+a^3)*c^3+9/4*
x^3*(2/35*b^2*x^6+2/7*a*b*x^3+a^2)*d*a*c^2+27/14*x^6*(1/10*b*x^3+a)*d^2*a^2
*c+81/140*a^3*d^3*x^9)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{((14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3)x^{10}+30\,(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2)x^7+}{140\,(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7dx^3}$$

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 + 3
0*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(a^
2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*x^9
 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{b^3x^{10}}{10\,(dx^3+c)^{\frac{10}{3}}c} - \frac{3\,ab^2\left(7\,d-\frac{10\,(dx^3+c)}{x^3}\right)x^{10}}{70\,(dx^3+c)^{\frac{10}{3}}c^2} + \frac{3\left(14\,d^2-\frac{40\,(dx^3+c)d}{x^3}+\frac{35\,(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^3} - \frac{\left(14\,d^3-\frac{60\,(dx^3+c)d^2}{x^3}+\frac{105\,(dx^3+c)^2d}{x^6}-\frac{140\,(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^4}$$

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^3
)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 + 35
*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3 - 6
0*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^
3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

Giac [F]

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \int \frac{(bx^3+a)^3}{(dx^3+c)^{\frac{13}{3}}}\,dx$$

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.49

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x\left(\frac{a^3}{10\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{(d\,x^3+c)^{10/3}} - \frac{x\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{(d\,x^3+c)^{4/3}} + \frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{70\,c^2\,d^3}\right)}{(d\,x^3+c)^{7/3}} + \frac{x\,(81\,a^3\,d^3+27\,a^2\,b\,c\,d^2+18\,a\,b^2\,c^2\,d+14\,b^3\,c^3)}{140\,c^4\,d^3\,(d\,x^3+c)^{1/3}}$$

```
[In] int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

```
[Out] (x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10*
c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 + 6
*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c*(b
^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*a^3*d
^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c + d*x^3
)^(1/3))
```

3.86 $\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 331

$$
\begin{aligned}
\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx &= -\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d} \\
&+ \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^3} \\
&- \frac{(bc-ad)^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^3} - \frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3} \\
&+ \frac{(bc-ad)^{8/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3} \\
&- \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^3}
\end{aligned}
$$

```
[Out] -1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^(2/3)/d^2+1/6*b*x*(b*x^3+a)^(5/3)/d-1/6
*(-a*d+b*c)^(8/3)*ln(d*x^3+c)/c^(2/3)/d^3+1/2*(-a*d+b*c)^(8/3)*ln((-a*d+b*c
)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^3-1/18*b^(2/3)*(20*a^2*d^2-24*
a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/27*b^(2/3)*(20*a^2*
d^2-24*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2
))/d^3*3^(1/2)-1/3*(-a*d+b*c)^(8/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {427, 542, 544, 245, 384}

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(20a^2d^2-24abcd+9b^2c^2)}{9\sqrt{3}d^3} - \frac{b^{2/3}(20a^2d^2-24abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18d^3} - \frac{(bc-ad)^{8/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^3} - \frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3} + \frac{(bc-ad)^{8/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3} - \frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{18d^2} + \frac{bx(a+bx^3)^{5/3}}{6d}$$

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] -1/18*(b*(6*b*c - 11*a*d)*x*(a + b*x^3)^(2/3))/d^2 + (b*x*(a + b*x^3)^(5/3)
)/(6*d) + (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^3) - ((b*c - a*d)^(8/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^3) - ((b*c - a*d)^(8/3)*Log[c + d*x^3])/(6*c^(2/3)*d^3) +
 ((b*c - a*d)^(8/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^3) - (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(18*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(a+bx^3)^{5/3}}{6d}+\frac{\int \frac{(a+bx^3)^{2/3}(-a(bc-6ad)-b(6bc-11ad)x^3)}{c+dx^3}\,dx}{6d}\\
&= -\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d}\\
&\quad+\frac{\int \frac{2a(3b^2c^2-7abcd+9a^2d^2)+2b(9b^2c^2-24abcd+20a^2d^2)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{18d^2}\\
&= -\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d}-\frac{(bc-ad)^3\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^3}\\
&\quad+\frac{(b(9b^2c^2-24abcd+20a^2d^2))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9d^3}
\end{aligned}$$

$$= -\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2} + \frac{bx(a+bx^3)^{5/3}}{6d} + \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^3} - \frac{(bc-ad)^{8/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^3} - \frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3} + \frac{(bc-ad)^{8/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3} - \frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.83 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{3b\sqrt[3]{bc-ad}(9b^2c^2-24abcd+20a^2d^2)\,x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2}{}$$

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1
/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a*d)
^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b*c -
 a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*(3*b^
2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7*a*b*c*
d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a
*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)
] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 9*a^3*d
^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3
```

```
) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c*d^2*(b*c - a*
d)^(1/3)*(a + b*x^3)^(1/3))
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{\frac{(ad-bc)^3 \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}-\frac{\left(\frac{20a^2d^2b^{\frac{2}{3}}}{9}+b^{\frac{5}{3}}c\left(bc-\frac{8ad}{3}\right)\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\right.}{}\right.}{2}}{}$

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/((a*d-b*c)/c)^(1/3)*(1/2*(a*d-b*c)^3*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-1/2*(20/9*a^2*d^2*b^
(2/3)+b^(5/3)*c*(b*c-8/3*a*d))*c*((a*d-b*c)/c)^(1/3)*ln((b^(2/3)*x^2+b^(1/3
)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(a*d-b*c)^3*ln((((a*d-b*c)/c)^(1/
3)*x+(b*x^3+a)^(1/3))/x)+(20/9*a^2*d^2*b^(2/3)+b^(5/3)*c*(b*c-8/3*a*d))*3^(
1/2)*c*((a*d-b*c)/c)^(1/3)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))
/b^(1/3)/x)+(20/9*a^2*d^2*b^(2/3)+b^(5/3)*c*(b*c-8/3*a*d))*c*((a*d-b*c)/c)^
(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-7/3*x*b*d*(-3/7*b*c+d*(3/14*b*x^3+
a))*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)-3^(1/2)*arctan(1/3*3^(1/2)*(((a*d
-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*(a*d-b*c)^3)/c/d
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(273) = 546$.

Time = 6.48 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx =$$

$$-\frac{18\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\cdots}{}$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) +
```

```
2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/3*(s
qrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*
x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*
d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(
-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (
b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^2*d^2*x^4 - 2*
(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

Sympy [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx=\int \frac{(a+bx^3)^{\frac{8}{3}}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(8/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

3.87 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 273

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx = \frac{bx(a+bx^3)^{2/3}}{3d} - \frac{b^{2/3}(3bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} + \frac{(bc-ad)^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^2} + \frac{(bc-ad)^{5/3}\log(c+dx^3)}{6c^{2/3}d^2} - \frac{(bc-ad)^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2} + \frac{b^{2/3}(3bc-5ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^2}$$

```
[Out] 1/3*b*x*(b*x^3+a)^(2/3)/d+1/6*(-a*d+b*c)^(5/3)*ln(d*x^3+c)/c^(2/3)/d^2-1/2*
(-a*d+b*c)^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^2
+1/6*b^(2/3)*(-5*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^2-1/9*b^(2/3)*
(-5*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1
/2)+1/3*(-a*d+b*c)^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+
a)^(1/3))*3^(1/2))/c^(2/3)/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {427, 544, 245, 384}

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-5ad)}{3\sqrt{3}d^2} + \frac{(bc-ad)^{5/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^2} + \frac{b^{2/3}(3bc-5ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6d^2} + \frac{(bc-ad)^{5/3}\log(c+dx^3)}{6c^{2/3}d^2} - \frac{(bc-ad)^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2} + \frac{bx(a+bx^3)^{2/3}}{3d}$$

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (b*x*(a + b*x^3)^(2/3))/(3*d) - (b^(2/3)*(3*b*c - 5*a*d)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) + ((b*c - a*d)^(5/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^2) + ((b*c - a*d)^(5/3)*Log[c + d*x^3])/(6*c^(2/3)*d^2) -
 ((b*c - a*d)^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^2) + (b^(2/3)*(3*b*c - 5*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)])/(6*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = \frac{bx(a+bx^3)^{2/3}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3d}$$

$$= \frac{bx(a+bx^3)^{2/3}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^2}$$

$$= \frac{bx(a+bx^3)^{2/3}}{3d} - \frac{b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} + \frac{(bc-ad)^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^2} + \frac{(bc-ad)^{5/3}\log\left(c+dx^3\right)}{6c^{2/3}d^2} - \frac{(bc-ad)^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2} + \frac{b^{2/3}(3bc-5ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.51 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{3b\sqrt[3]{bc-ad}(-3bc+5ad)x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2\sqrt[3]{c}\left(6abc^{2/\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(-3*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[
4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1/3)*(6*a*b*c^(2/3)*(b
*c - a*d)^(1/3)*x + 6*b^2*c^(2/3)*(b*c - a*d)^(1/3)*x^4 + 2*Sqrt[3]*a*(-(b*
c) + 3*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*(b*c - 3*a*d)*(a + b*x^3)^(1/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*b*c*(a + b*x^3)^(1/3)*Lo
g[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d
)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a^2*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)]))/(36*c*d*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.47

method	result
pseudoelliptic	$-\frac{-\frac{5\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c\left(a d\, b^{\frac{2}{3}}-\frac{3 b^{\frac{5}{3}} c}{5}\right) \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{3}-2(ad-bc)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+\frac{10\sqrt{3}\,(\cdots}{\cdots}}{\cdots}$

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-5/3*((a*d-b*c)/c)^(1/3)*c*(a*d*b^(2/3)-3/5*b^(5/3)*c)*ln((b^(2/3)*x^
2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*(a*d-b*c)^2*ln((((a*d-b
*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+10/3*3^(1/2)*((a*d-b*c)/c)^(1/3)*c*(a*d*
b^(2/3)-3/5*b^(5/3)*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(
1/3)/x)+10/3*((a*d-b*c)/c)^(1/3)*c*(a*d*b^(2/3)-3/5*b^(5/3)*c)*ln((-b^(1/3)
*x+(b*x^3+a)^(1/3))/x)-2*(b*x^3+a)^(2/3)*x*b*c*((a*d-b*c)/c)^(1/3)*d+(-2*ar
ctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1
```

```
/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c)^2)/((a*d-b*c)/c)^(1/3)/c/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(220) = 440$.

Time = 0.76 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{6\,(bx^3+a)^{\frac{2}{3}}bdx + 6\,\sqrt{3}(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c}{}\right.}{3\,(bc-ad)x}\right.}{}$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*
x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c - 5*a*
d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3*b*c - 5
*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
+ a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*
x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a
)^(2/3)*(b*c - a*d))/x^2))/d^2
```

Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx=\int \frac{(b\,x^3+a)^{5/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

3.88 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {399, 245, 384}

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d}$$

$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.46 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{\cdots}}\right.}{c^{2/3}}}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1
/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)
^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*
I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c - a*d)
^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(2
/3))/(12*d)
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{2}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right) c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+(ad-bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\sqrt{3}\,b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\right.}{3b}\right.}{}$

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)/c)^(1/3)*(1/2*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1/3)+(a*d-b*c)*ln((((a*d-b*c)/c)
^(1/3)*x+(b*x^3+a)^(1/3))/x)-3^(1/2)*b^(2/3)*arctan(1/3*3^(1/2)*(b^(1/3)*x+
2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c*((a*d-b*c)/c)^(1/3)-b^(2/3)*ln((-b^(1/3)*x+
(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b*c)/c
)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b
*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2
))*(a*d-b*c))/d/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.

Time = 0.30 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}} c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}bx}{}\right.}{}$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
```

```
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]

$$\int \frac{(a + bx^3)^{2/3}}{c + dx^3}\, dx = \int \frac{(a + bx^3)^{\frac{2}{3}}}{c + dx^3}\, dx$$

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a + bx^3)^{2/3}}{c + dx^3}\, dx = \int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c}\, dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a + bx^3)^{2/3}}{c + dx^3}\, dx = \int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c}\, dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^3)^{2/3}}{c + dx^3}\, dx = \int \frac{(b\,x^3 + a)^{2/3}}{d\,x^3 + c}\, dx$$

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.89 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {384}

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$
$$= \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right) + \left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{\right.\right.\right.}{12c^{2/}}$$

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3
])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(12*c^
(2/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+(b}{x^{2}}\right.}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3/((a*d-b*c)/c)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3
+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+
a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2))/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.90 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {390, 384}

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*(-I
 + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/
3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(4/3)
))/12
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-}{}\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(ad-bc)ca}$

[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/c/a

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\text{Timed out}$$

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{1}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.91 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^3)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}} + \frac{d^2\log(c+dx^3)}{6c^{2/3}(bc-ad)^{7/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/4*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^3+a)^(1/3)+1/6*d^2*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(7/3)-1/2*d^2*ln((
-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(7/3)+1/3*d^2
*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/
3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{bx(3bc-7ad)}{4a^2\sqrt[3]{a+bx^3}(bc-ad)^2} + \frac{d^2\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}} + \frac{d^2\log(c+dx^3)}{6c^{2/3}(bc-ad)^{7/3}} - \frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}} + \frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + (b*(3*b*c - 7*a*d)*x)/(4*a^2*(b
*c - a*d)^2*(a + b*x^3)^(1/3)) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(7/3)) +
 (d^2*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(7/3)) - (d^2*Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{4a(bc-ad)\left(a+bx^3\right)^{4/3}} - \frac{\int \frac{-3bc+4ad-3bdx^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{\int \frac{4a^2d^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{4a^2(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}}$$

$$+ \frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{3bx(-8a^2d+3b^2cx^3+ab(4c-7dx^3))}{a^2(bc-ad)^2(a+bx^3)^{4/3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{7/3}} + \frac{2(1+i\sqrt{3})d^2\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{7/3}} - \frac{i(-i+\sqrt{3})d^2\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{7/3}}\right)$$

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] ((3*b*x*(-8*a^2*d + 3*b^2*c*x^3 + a*b*(4*c - 7*d*x^3)))/(a^2*(b*c - a*d)^2*
(a + b*x^3)^(4/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^
(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)
^(1/3))]])/(c^(2/3)*(b*c - a*d)^(7/3)) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c -
 a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*(b*c -
 a*d)^(7/3)) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(7/3)))/12
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)a^2d^2\left(bx^3+a\right)^{\frac{4}{3}}+12xbc\left(a^2d-\frac{\left(-\frac{7dx^3}{4}+c\right)ba}{2}-\frac{3b^2cx^3}{8}\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+\left(-2\arctan\left(\frac{\sqrt{3}}{\cdots}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{4}{3}}(\cdots}$

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
a^2*d^2*(b*x^3+a)^(4/3)+12*x*b*c*(a^2*d-1/2*(-7/4*d*x^3+c)*b*a-3/8*b^2*c*x^
3)*((a*d-b*c)/c)^(1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x
^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a
*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d^2*(b*x^3+a)^(4/
3)*a^2)/(b*x^3+a)^(4/3)/(a*d-b*c)^2/c/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

3.92 $\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^3)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2(a+bx^3)^{4/3}} + \frac{b(18b^2c^2-57abcd+67a^2d^2)\,x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{10/3}} - \frac{d^3\log(c+dx^3)}{6c^{2/3}(bc-ad)^{10/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^3+a)^(4/3)+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d+b
*c)^3/(b*x^3+a)^(1/3)-1/6*d^3*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(10/3)+1/2*d^3
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(10/3)-1
/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(2/3)/(-a*d+b*c)^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{bx(6bc-13ad)}{28a^2(a+bx^3)^{4/3}(bc-ad)^2} + \frac{bx(67a^2d^2-57abcd+18b^2c^2)}{28a^3\sqrt[3]{a+bx^3}(bc-ad)^3} - \frac{d^3\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{10/3}} - \frac{d^3\log(c+dx^3)}{6c^{2/3}(bc-ad)^{10/3}} + \frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}} + \frac{bx}{7a(a+bx^3)^{7/3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^3)^(7/3)) + (b*(6*b*c - 13*a*d)*x)/(28*a^2*
(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2
)*x)/(28*a^3*(b*c - a*d)^3*(a + b*x^3)^(1/3)) - (d^3*ArcTan[(1 + (2*(b*c -
a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c
- a*d)^(10/3)) - (d^3*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(10/3)) + (d^3
*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c -
a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}} - \frac{\int \frac{-6bc+7ad-6bdx^3}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx}{7a(bc-ad)}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}} + \frac{\int \frac{18b^2c^2-39abcd+28a^2d^2+3bd(6bc-13ad)x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx}{28a^2(bc-ad)^2}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}} + \frac{b\left(18b^2c^2-57abcd+67a^2d^2\right)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{\int \frac{28a^3d^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{28a^3(bc-ad)^3}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}} + \frac{b\left(18b^2c^2-57abcd+67a^2d^2\right)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{d^3\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{(bc-ad)^3}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2\left(a+bx^3\right)^{4/3}} + \frac{b\left(18b^2c^2-57abcd+67a^2d^2\right)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{10/3}} - \frac{d^3\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{10/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.43 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{1}{84}\left(-\frac{3bx(84a^4d^2+18b^4c^2x^6+3ab^3cx^3(14c-19dx^3)+21a^3bd(-4c+7dx^3}{a^3(-bc+ad)^3(a+bx^3)^{7/3}}\right.$$

$$+\frac{14\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{10/3}}$$

$$-\frac{14i(-i+\sqrt{3})d^3\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{10/3}}$$

$$\left.+\frac{7(1+i\sqrt{3})d^3\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{10/3}}\right)$$

```
[In] Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*b*x*(84*a^4*d^2 + 18*b^4*c^2*x^6 + 3*a*b^3*c*x^3*(14*c - 19*d*x^3) + 2
1*a^3*b*d*(-4*c + 7*d*x^3) + a^2*b^2*(28*c^2 - 133*c*d*x^3 + 67*d^2*x^6)))/
(a^3*(-(b*c) + a*d)^3*(a + b*x^3)^(7/3)) + (14*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3
*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(10/3)) - ((14*I)*(-I
 + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3)]])/(c^(2/3)*(b*c - a*d)^(10/3)) + (7*(1 + I*Sqrt[3])*d^3*Log[2*(b
*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x
^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(b*c - a*d
)^(10/3)))/84
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{3}d^{3}\left(b\,x^{3}+a\right)^{\frac{7}{3}}-9xbc\left(a^{4}d^{2}-\left(-\frac{7d\,x^{3}}{4}+c\right)bd\,a^{3}+\frac{b^{2}\left(\frac{67}{28}d^{2}x^{6}-\frac{19}{4}cd\,x^{3}+c^{2}\right)a^{2}}{3}+\frac{x^{3}b^{3}\left(-\frac{19d\,x^{3}}{14}+\right.}{2}\right.}{3\left(\frac{ad}{}\right.}$

[In] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(7/3)*(ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^3*d^3*(b*x^3+a)^(7/3)-9*x*b*c*(a^4*d^2-(-7/4*d*x^3+c)*b*d*a^3+1/3*b^2*(67/28*d^2*x^6-19/4*c*d*x^3+c^2)*a^2+1/2*x^3*b^3*(-19/14*d*x^3+c)*c*a+3/14*b^4*c^2*x^6)*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b*x^3+a)^(7/3)*d^3*a^3)/(a*d-b*c)^3/c/a^3

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{10/3}\,(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{10}{3}}\,(c+dx^3)}\,dx$$

[In] integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)

[Out] Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{10/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{10/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{10/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

3.93 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.

Time = 10.46 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int\frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)\text{AppellF1}\right.}{\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\left(\right.\right.}\right.}{8d\,(}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a \right)^{\frac{4}{3}}}{d\,x^{3}+c}d x$$

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.94 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(59) = 118$.

Time = 10.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{4acx\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\text{App}\right.\right.}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}d x$$

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.95 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx =$$

$$-\frac{4acx\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}dx$$

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.96 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)} dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a)
^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)} dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)}\,dx}{a\,(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 332 vs. $2(62) = 124$.

Time = 10.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)}\,dx = \frac{x\left(-\frac{bdx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac(2ad-b(2c+dx^3))\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\right.\right.}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\right.\right.}\right.}{8a(-bc+}$$

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)}dx$$

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)}\,dx = \int \frac{1}{\left(a+bx^3\right)^{\frac{5}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\,(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)), x)
```

3.97 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3+
a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 429 vs. $2(62) = 124$.

Time = 10.78 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.92

$$\int\frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx = -\frac{x\left(\frac{bd(-4bc+9ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac\left(10a^3d^2+4b^3cx^3\left(2c+dx^3\right)-a^2bd\left(20c+dx^3\right)+ab^2\left(10c^2-12cdx^3-\right.\right.\right.}{(a+bx^3)(c+dx^3)\left(-4ac\right.}\right.}{40a}$$

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*c*
x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3 - 9
*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(
c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*c*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3
, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}} \left(d\,x^{3}+c\right)}dx$$

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \int \frac{1}{\left(a+bx^{3}\right)^{\frac{8}{3}} \left(c+dx^{3}\right)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \int \frac{1}{\left(bx^{3}+a\right)^{\frac{8}{3}}\left(dx^{3}+c\right)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{8/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)), x)
```

3.98 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 351

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx &= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} \\ &\quad - \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)} - \frac{2b^{5/3}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^3} \\ &\quad + \frac{2(bc-ad)^{5/3}(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^3} \\ &\quad + \frac{(bc-ad)^{5/3}(3bc+ad)\log(c+dx^3)}{9c^{5/3}d^3} \\ &\quad - \frac{(bc-ad)^{5/3}(3bc+ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3} \\ &\quad + \frac{b^{5/3}(3bc-4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3d^3}\end{aligned}$$

```
[Out] 1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3)
/c/d/(d*x^3+c)+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1/3
*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3)
)/c^(5/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
```

```
/2))/d^3*3^(1/2)+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c
)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 542, 544, 245, 384}

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = -\frac{2b^{5/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{3\sqrt{3}d^3} + \frac{2(bc-ad)^{5/3}(ad+3bc)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^3} + \frac{b^{5/3}(3bc-4ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3d^3} + \frac{(bc-ad)^{5/3}(ad+3bc)\log(c+dx^3)}{9c^{5/3}d^3} - \frac{(bc-ad)^{5/3}(ad+3bc)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3} + \frac{bx(a+bx^3)^{2/3}(2bc-ad)}{3cd^2} - \frac{x(a+bx^3)^{5/3}(bc-ad)}{3cd(c+dx^3)}$$

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*c*d^2) - ((b*c - a*d)*x*(a + b*x^3
)^(5/3))/(3*c*d*(c + d*x^3)) - (2*b^(5/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^3) + (2*(b*c - a*d)^(5/3
)*(3*b*c + a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1
/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^3) + ((b*c - a*d)^(5/3)*(3*b*c + a*d)*
Log[c + d*x^3])/(9*c^(5/3)*d^3) - ((b*c - a*d)^(5/3)*(3*b*c + a*d)*Log[((b*
c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(3*c^(5/3)*d^3) + (b^(5/3)*
(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)} + \frac{\int \frac{(a+bx^3)^{2/3}(a(bc+2ad)+3b(2bc-ad)x^3)}{c+dx^3}\,dx}{3cd}$$

$$= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} - \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)} + \frac{\int \frac{-6a(b^2c^2-abcd-a^2d^2)-6b^2c(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9cd^2}$$

$$= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} - \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)} - \frac{(2b^2(3bc-4ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3d^3} + \frac{(2(bc-ad)^2(3bc+ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3cd^3}$$

$$= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} - \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)}$$
$$- \frac{2b^{5/3}(3bc-4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^3}$$
$$+ \frac{2(bc-ad)^{5/3}(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^3}$$
$$+ \frac{(bc-ad)^{5/3}(3bc+ad)\log\left(c+dx^3\right)}{9c^{5/3}d^3}$$
$$- \frac{(bc-ad)^{5/3}(3bc+ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3}$$
$$+ \frac{b^{5/3}(3bc-4ad)\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{3d^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.94 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{1}{18}\left(\frac{6x(a+bx^3)^{2/3}\left(b^2+\frac{(bc-ad)^2}{c(c+dx^3)}\right)}{d^2}\right.$$

$$-\frac{9b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}$$

$$+\frac{12ab^2x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd\sqrt[3]{a+bx^3}}$$

$$+\frac{2a^3\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ac}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{5/3}\sqrt[3]{bc-ad}}$$

$$-\frac{2ab^2\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc}}{\sqrt[3]{b+}}\right.\right.}{d^2\sqrt[3]{bc-ad}}$$

$$+\frac{2a^2b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{2/3}d\sqrt[3]{bc-ad}}$$

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^3
*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/3))
+ (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
```

```
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3)) - (2*a*b
^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)))/18
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$\frac{\frac{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(a\,b^{\frac{5}{3}}d-\frac{3b^{\frac{8}{3}}c}{4}\right)c^{2}\left(d\,x^{3}+c\right)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{9}+\frac{2\left(d\,x^{3}+c\right)(ad+3bc)(ad-bc)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+(}{x}\right.}{9}}{}$

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(2*((a*d-b*c)/c)^(1/3)*(a*b^(5/3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*ln((b^
(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(d*x^3+c)*(a*d+3*
b*c)*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-4*((a*d-b*c)
/c)^(1/3)*3^(1/2)*(a*b^(5/3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*arctan(1/3*3^(1
/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-4*((a*d-b*c)/c)^(1/3)*(a*b^(5/
3)*d-3/4*b^(8/3)*c)*c^2*(d*x^3+c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*x*
(2*b^2*c^2-2*b*d*(-1/2*b*x^3+a)*c+a^2*d^2)*d*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c
)^(1/3)+(a*d+3*b*c)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^
(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c)*(a*d-b*c)
^2)/((a*d-b*c)/c)^(1/3)/d^3/c^2/(d*x^3+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(291) = 582.

Time = 3.35 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{2\sqrt{3}(3\,b^2c^3-2\,abc^2d-a^2cd^2+(3\,b^2c^2d-2\,abcd^2-a^2d^3)x^3)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\right.}{}$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*
c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1
/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4*a*b
*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)
*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^3 - 2*
a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 - 4*a*b*c^
2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c
*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)
^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) +
 (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^2}\,dx$$

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

3.99 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 301

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)} + \frac{b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{(bc-ad)^{2/3}(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^2} - \frac{(bc-ad)^{2/3}(3bc+2ad)\log(c+dx^3)}{18c^{5/3}d^2} + \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2} - \frac{b^{5/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^2}$$

```
[Out] -1/3*(-a*d+b*c)*x*(b*x^3+a)^(2/3)/c/d/(d*x^3+c)-1/18*(-a*d+b*c)^(2/3)*(2*a*
d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^2+1/6*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln((-a*
d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/d^2-1/2*b^(5/3)*ln(-b^(1/3)
*x+(b*x^3+a)^(1/3))/d^2+1/3*b^(5/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))/d^2*3^(1/2)-1/9*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 544, 245, 384}

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \frac{b^{5/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{(bc-ad)^{2/3}(2ad+3bc)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^2} - \frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d^2} - \frac{(bc-ad)^{2/3}(2ad+3bc)\log(c+dx^3)}{18c^{5/3}d^2} + \frac{(bc-ad)^{2/3}(2ad+3bc)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2} - \frac{x(a+bx^3)^{2/3}(bc-ad)}{3cd(c+dx^3)}$$

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (b^(5/3)*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c - a*d
)^(2/3)*(3*b*c + 2*a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^2) - ((b*c - a*d)^(2/3)*(3*b*c
 + 2*a*d)*Log[c + d*x^3])/(18*c^(5/3)*d^2) + ((b*c - a*d)^(2/3)*(3*b*c + 2*
a*d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*d^2
) - (b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)} + \frac{\int \frac{a(bc+2ad)+3b^2cx^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)} + \frac{b^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d^2} - \frac{((bc-ad)(3bc+2ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3cd^2}$$

$$= -\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd\,(c+dx^3)} + \frac{b^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{(bc-ad)^{2/3}(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^2} - \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(c+dx^3\right)}{18c^{5/3}d^2} + \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2} - \frac{b^{5/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.12 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \frac{-\frac{12d(bc-ad)x(a+bx^3)^{2/3}}{c(c+dx^3)} + 12\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \frac{2i\left(3i+\sqrt{3}\right)\left(3b^2c^2-abcd-2a^2d^{2}\right.}{c}}{}$$

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*d*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*(c + d*x^3)) + 12*Sqrt[3]*b^(5/
3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + ((2*I)*(
3*I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(c^(5/3)*(b*
c - a*d)^(1/3)) - 12*b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*I)
*(-I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x
 + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(5/3)*(b*c - a*d)^(1/3))
+ 6*b^(5/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/
3)] + ((1 + I*Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^
(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(5/3)*(b*c - a*d)^(1/3)))/
(36*d^2)
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$-\frac{-\frac{3b^{\frac{5}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)c^2\left(dx^3+c\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}-2\left(ad+\frac{3bc}{2}\right)\left(dx^3+c\right)(ad-bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+\right.}{x}\right.}{}$

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-3/2*b^(5/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)*c^2*(d*x^3+c)*((a*d-b*c)/c)^(1/3)-2*(a*d+3/2*b*c)*(d*x^3+c)*(a*d-b*
c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3*3^(1/2)*b^(5/3)*arctan(1
/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c^2*(d*x^3+c)*((a*d-b*c
```

```
)/c)^(1/3)+3*b^(5/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*c^2*(d*x^3+c)*((a*d
-b*c)/c)^(1/3)+(-3*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)*c*d*x+(a*d+3/2*b*c)*
(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)
/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c))*(a*d-b*c))/((a*d-b*c)/c)^(1/3
)/d^2/c^2/(d*x^3+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(248) = 496$.

Time = 0.52 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.10

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx =$$

$$-\frac{2\sqrt{3}((3\,bcd+2\,ad^2)x^3+3\,bc^2+2\,acd)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right.}{}$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c
^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*
x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^
2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2
 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x
^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^
2))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{(c+dx^3)^2}\,dx$$

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^2}\,dx$$

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

3.100 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx = \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{2a\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)+1/9*a*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3
)-1/3*a*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(
1/3)+2/9*a*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1
/2))/c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

= {386, 384}

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{2a\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c+dx^3\right)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} + \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)}$$

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*
d)^(1/3)) + (a*Log[c + d*x^3])/(9*c^(5/3)*(b*c - a*d)^(1/3)) - (a*Log[((b*c
 - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(1/3)
)
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^3)^{2/3}}{3c\,(c+dx^3)} + \frac{(2a)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3c}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}}{c+dx^3} - \frac{2\sqrt{-6+6i\sqrt{3}}a\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2(a+i\sqrt{3}a)\log\left(2\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*c^(2/3)*x*(a + b*x^3)^(2/3))/(c + d*x^3) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*
a*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) + (2*(a + I*Sqrt[3]*a)*
Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*
c - a*d)^(1/3) - (I*(-I + Sqrt[3])*a*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a*d)^(1/3))/(18*c^(5/3))
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$\frac{-\frac{a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)\left(d\,x^3+c\right)}{9}+\frac{2a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)\left(d\,x^3+c\right)}{9}+\frac{x\left(b\,x^3+a\right)^{\frac{2}{3}}c(\cdots}{3}\cdots}{c^2\left(d\,x^3+c\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(-1/2*a*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)*(d*x^3+c)+a*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1
```

```
/3))/x)*(d*x^3+c)+3/2*x*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+arctan(1/3*3^
(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1
/2)*a*(d*x^3+c))/((a*d-b*c)/c)^(1/3)/c^2/(d*x^3+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^2}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^2}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2} dx = -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)\,(c+dx^3)} + \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}$$

```
[Out] -1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/18*(-2*a*d+3*b*c)*ln(d*x^
3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1
/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/9*(-2*a*d+3*b*c)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*
c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {390, 384}

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \frac{(3bc-2ad)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{dx(a+bx^3)^{2/3}}{3c\,(c+dx^3)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + ((3*b*c - 2*a*d)*Log[c + d*x^3])/(
18*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[((b*c - a*d)^(1/3)*x)/
c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx$$

$$= \frac{-12c^{2/3}d\sqrt[3]{bc-ad}x(a+bx^3)^{2/3} + 2(3-i\sqrt{3})(3bc-2ad)(c+dx^3)\,\text{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right) + \cdots}{\cdots}$$

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] (-12*c^(2/3)*d*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) + 2*(3 - I*Sqrt[3])*(3
*b*c - 2*a*d)*(c + d*x^3)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^
(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]] + 2*(1 + I*Sqrt[3])*(3*b*c - 2*a*d)*
(c + d*x^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)
^(1/3)] - I*(-I + Sqrt[3])*(3*b*c - 2*a*d)*(c + d*x^3)*Log[2*(b*c - a*d)^(2
/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^(4/3)*(
c + d*x^3))
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-\frac{\left(ad-\frac{3bc}{2}\right)\left(d\,x^3+c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9}+\frac{2\left(ad-\frac{3bc}{2}\right)\left(d\,x^3+c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)}{9}}{c^2(ad-bc)(d\,x^3+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-3/2*b*c)*(d*x^3+c)*ln((((a*d-b*c)/c)^(2/
3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-3/2
*b*c)*(d*x^3+c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*(b*x^3+a)
^(2/3)*((a*d-b*c)/c)^(1/3)*c*d*x+arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-
2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)*(a*d-3/2*b*c)*(d*x^3+c))/
c^2/(a*d-b*c)/(d*x^3+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

3.102 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2} dx = \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{2d(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}$$

```
[Out] 1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/(
b*x^3+a)^(1/3)/(d*x^3+c)-1/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^
(7/3)+1/3*d*(-a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
5/3)/(-a*d+b*c)^(7/3)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = -\frac{2d(3bc-ad)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}} + \frac{bx(ad+3bc)}{3ac\sqrt[3]{a+bx^3}(bc-ad)^2} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (b*(3*b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)) - (d*x)/(3*c*(b
*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) - (2*d*(3*b*c - a*d)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^
(5/3)*(b*c - a*d)^(7/3)) - (d*(3*b*c - a*d)*Log[c + d*x^3])/(9*c^(5/3)*(b*c
 - a*d)^(7/3)) + (d*(3*b*c - a*d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a +
b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} + \frac{\int \frac{3bc-2ad-3bdx^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx}{3c(bc-ad)}$$

$$= \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{\int \frac{2ad(3bc-ad)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3ac(bc-ad)^2}$$

$$= \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{(2d(3bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3c(bc-ad)^2}$$

$$= \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{2d(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a^2d^2+abd^2x^3+3b^2c(c+dx^3))}{a(bc-ad)^2\sqrt[3]{a+bx^3}(c+dx^3)} + \frac{2i(3i+\sqrt{3})d(3bc-ad)\operatorname{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+b}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}}}{}$$

[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]

[Out] ((6*c^(2/3)*x*(a^2*d^2 + a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3)))/(a*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((2*I)*(3*I + Sqrt[3])*d*(3*b*c - a*d)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*d*(-3*b*c + a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(7/3) + ((1 + I*Sqrt[3])*d*(3*b*c - a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(18*c^(5/3))

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{\frac{2\left(b\,x^3+a\right)^{\frac{1}{3}} a d \left(d\,x^3+c\right)(ad-3bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)}{9}+\frac{xc\left(a\left(b\,x^3+a\right)d^2+3x^3b^2cd+3b^2c^2\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{3}+\frac{2(ad-3bc)da\left(d\,x^3\right.}{}}{c^2(d\,x^3+c)(ad-bc)^2\left(\frac{ad}{}\right.}$

[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)

[Out] 2/9/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*((b*x^3+a)^(1/3)*a*d*(d*x^3+c)*(a*d-3*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*x*c*(a*(b*x^3+a)*d^2+3*x^3*b^2*c*d+3*b^2*c^2)*((a*d-b*c)/c)^(1/3)+(a*d-3*b*c)*d*a*(d*x^3+c)*(b*x^3+a)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/c^2/(d*x^3+c)/(a*d-b*c)^2/a

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)^2}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

3.103 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 324

$$\begin{aligned}\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx &= \frac{b(3bc+4ad)x}{12ac(bc-ad)^2(a+bx^3)^{4/3}} \\ &+ \frac{b(9b^2c^2-33abcd-4a^2d^2)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)} \\ &+ \frac{d^2(9bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{10/3}} \\ &- \frac{d^2(9bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}\end{aligned}$$

```
[Out] 1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)+1/12*b*(-4*a^2*d^2-
33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+
b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)+1/18*d^2*(-2*a*d+9*b*c)*ln(d*x^3+c)/c^(5/3)/
(-a*d+b*c)^(10/3)-1/6*d^2*(-2*a*d+9*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x
^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(10/3)+1/9*d^2*(-2*a*d+9*b*c)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^
(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \frac{bx(-4a^2d^2-33abcd+9b^2c^2)}{12a^2c\sqrt[3]{a+bx^3}(bc-ad)^3} + \frac{d^2(9bc-2ad)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{10/3}} - \frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}} - \frac{dx}{3c(a+bx^3)^{4/3}(c+dx^3)(bc-ad)} + \frac{bx(4ad+3bc)}{12ac(a+bx^3)^{4/3}(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (b*(3*b*c + 4*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(9*b^2*
c^2 - 33*a*b*c*d - 4*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3))
 - (d*x)/(3*c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + (d^2*(9*b*c - 2*
a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(10/3)) + (d^2*(9*b*c - 2*a*d)*Log[c +
d*x^3])/(18*c^(5/3)*(b*c - a*d)^(10/3)) - (d^2*(9*b*c - 2*a*d)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}+\frac{\int \frac{3bc-2ad-6bdx^3}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx}{3c(bc-ad)}\\
&= \frac{b(3bc+4ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}}-\frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\\
&\quad -\frac{\int \frac{-9b^2c^2+24abcd-8a^2d^2-3bd(3bc+4ad)x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx}{12ac(bc-ad)^2}\\
&= \frac{b(3bc+4ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}}+\frac{b\left(9b^2c^2-33abcd-4a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}}\\
&\quad -\frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}+\frac{\int \frac{4a^2d^2(9bc-2ad)}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{12a^2c(bc-ad)^3}\\
&= \frac{b(3bc+4ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}}+\frac{b\left(9b^2c^2-33abcd-4a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}}\\
&\quad -\frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}+\frac{\left(d^2(9bc-2ad)\right)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{3c(bc-ad)^3}
\end{aligned}$$

$$= \frac{b(3bc+4ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}} + \frac{b\left(9b^2c^2-33abcd-4a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} + \frac{d^2(9bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{10/3}} - \frac{d^2(9bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.11 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx = \frac{\frac{3c^{2/3}x\left(4a^4d^3+8a^3bd^3x^3-9b^4c^2x^3\left(c+dx^3\right)+4a^2b^2d\left(9c^2+9cdx^3+d^2x^6\right)+3ab^3c\left(-4c^2+7cdx^3+11d^2x^6\right)\right)}{a^2(-bc+ad)^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}}{}$$

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] ((3*c^(2/3)*x*(4*a^4*d^3 + 8*a^3*b*d^3*x^3 - 9*b^4*c^2*x^3*(c + d*x^3) + 4*
a^2*b^2*d*(9*c^2 + 9*c*d*x^3 + d^2*x^6) + 3*a*b^3*c*(-4*c^2 + 7*c*d*x^3 + 1
1*d^2*x^6)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((2*I)*
(3*I + Sqrt[3])*d^2*(-9*b*c + 2*a*d)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(10/3) + (2*
(1 + I*Sqrt[3])*d^2*(9*b*c - 2*a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(10/3) + ((1 + I*Sqrt[3])*d^2*
(-9*b*c + 2*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*
c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2
/3)])/(b*c - a*d)^(10/3))/(36*c^(5/3))
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{-\frac{3x\left(4a^{2}b^{2}d^{3}x^{6}+33a\,b^{3}c\,d^{2}x^{6}-9b^{4}c^{2}d\,x^{6}+8a^{3}b\,d^{3}x^{3}+36a^{2}b^{2}c\,d^{2}x^{3}+21a\,b^{3}c^{2}d\,x^{3}-9b^{4}c^{3}x^{3}+4a^{4}d^{3}+36a^{2}b^{2}c^{2}d-12a\,b^{3}c^{3}\right)c\left(\frac{ad-bc}{c}\right.}{2}}{}$

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(4/3)*(-3/2*x*(4*a^2*b^2*d^3*x^6+33*a*b
^3*c*d^2*x^6-9*b^4*c^2*d*x^6+8*a^3*b*d^3*x^3+36*a^2*b^2*c*d^2*x^3+21*a*b^3*
c^2*d*x^3-9*b^4*c^3*x^3+4*a^4*d^3+36*a^2*b^2*c^2*d-12*a*b^3*c^3)*c*((a*d-b*
c)/c)^(1/3)+(b*x^3+a)^(4/3)*a^2*d^2*(d*x^3+c)*(2*a*d-9*b*c)*(-2*arctan(1/3*
3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^
(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*
x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/c^2/(d
*x^3+c)/(a*d-b*c)^3/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)^2}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

3.104 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3
/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(60) = 120.

Time = 10.34 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.68

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{x\left(b(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(3a^2d-b^2cx^3+abd\right.}{(c+dx^3)\left(\right.}}{}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*x^3
*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x
^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a + b*x^3)^
(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)^{2}}d x$$

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{\left(c+dx^3\right)^2}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a + bx^3)^{4/3}}{(c + dx^3)^2} \, dx = \int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^2} \, dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^3)^{4/3}}{(c + dx^3)^2} \, dx = \int \frac{(b\,x^3 + a)^{4/3}}{(d\,x^3 + c)^2} \, dx$$

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^2, x)
```

3.105 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/a
)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.22 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \frac{x\left(\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{5}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}\right)}{c+dx^3}\right)}{12\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{(d\,x^{3}+c)^{2}}dx$$

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^2}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

3.106 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a)
^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)^2}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 393 vs. $2(59) = 118$.

Time = 10.30 (sec) , antiderivative size = 393, normalized size of antiderivative = 6.66

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \frac{4acx\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(4c(-3bc+3ad+bdx^3)+bdx^3\left(1+\right.\right.}{12c^2(bc-}$$

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b*
c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4
/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) + b*
x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)^2}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)
```

3.107 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3+
a)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)^2}\,dx}{a\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 386 vs. $2(62) = 124$.

Time = 10.53 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.23

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx = \frac{x\left(bd(3bc+2ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac(6c\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d*x
^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3
, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

[In] `int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)`

[Out] `int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)`

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

[In] `integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")`

[Out] `Timed out`

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{3}}\,(c+dx^3)^2}\,dx$$

[In] `integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)`

[Out] `Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)`

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^2}\,dx$$

[In] `integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")`

[Out] `integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)`

Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)
```

3.108 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x^
3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)^2}\,dx}{a^2\,(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 550 vs. 2(62) = 124.

Time = 10.98 (sec) , antiderivative size = 550, normalized size of antiderivative = 8.87

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^2}\,dx = \frac{\frac{bd(-6b^2c^2+21abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3} + \frac{4c\left(-4acx(15a^4d^3-6b\right.}{}}{}$$

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] ((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*c*
(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c + 4
*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c^2 +
 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3) + a^2
*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d*x^3 +
7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*
d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(a+bx^{3}\right)^{\frac{8}{3}}\left(c+dx^{3}\right)^{2}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(bx^{3}+a\right)^{\frac{8}{3}}\left(dx^{3}+c\right)^{2}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^2}\,dx$$

[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)^2}\,dx$$

[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x)

[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2), x)

3.109 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 541

$$
\begin{aligned}
\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx &= -\frac{b(2bc-ad)\,(18b^2c^2-18abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^4} \\
&+ \frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3} \\
&- \frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} \\
&+ \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^5} \\
&- \frac{(bc-ad)^{8/3}\,(54b^2c^2+18abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5} \\
&- \frac{(bc-ad)^{8/3}\,(54b^2c^2+18abcd+5a^2d^2)\log\left(c+dx^3\right)}{54c^{8/3}d^5} \\
&+ \frac{(bc-ad)^{8/3}\,(54b^2c^2+18abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5} \\
&- \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^5}
\end{aligned}
$$

```
[Out] -1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c
^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^3-
1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+12
*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(8/3)*(5*a^2*d^2+
18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(5*a^2
*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/
c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d^5+1/27*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^5*3^(1/2)-1/27*(-a*d+b*c)^
(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^5*3^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {424, 540, 542, 544, 245, 384}

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx =$$

$$-\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5}+\frac{b^{8/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(77a^2d^2-126abcd+54b^2c^2\right)}{9\sqrt{3}d^5}-\frac{bx(a+bx^3)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{18c^2d^4}+\frac{bx(a+bx^3)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{18c^2d^3}-\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^5}+\frac{(bc-ad)^{8/3}\left(5a^2d^2+18abcd+54b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5}-\frac{b^{8/3}\left(77a^2d^2-126abcd+54b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18d^5}-\frac{x(a+bx^3)^{8/3}(bc-ad)(5ad+12bc)}{18c^2d^2\left(c+dx^3\right)}-\frac{x(a+bx^3)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/18*(b*(2*b*c - a*d)*(18*b^2*c^2 - 18*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^
(2/3))/(c^2*d^4) + (b*(18*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(
5/3))/(18*c^2*d^3) - ((b*c - a*d)*x*(a + b*x^3)^(11/3))/(6*c*d*(c + d*x^3)^
2) - ((b*c - a*d)*(12*b*c + 5*a*d)*x*(a + b*x^3)^(8/3))/(18*c^2*d^2*(c + d*
x^3)) + (b^(8/3)*(54*b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^5) - ((b*c - a*d)^(8/3)*(
54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c
^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^5) - ((b*c - a*d)
^(8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*d^
5) + ((b*c - a*d)^(8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^5) - (b^(8/3)*(54*
b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(18*d^5)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} + \frac{\int \frac{(a+bx^3)^{8/3}(a(bc+5ad)+6b(2bc-ad)x^3)}{(c+dx^3)^2}\,dx}{6cd} \\
&= -\frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} \\
&\quad - \frac{\int \frac{(a+bx^3)^{5/3}(-2a(6b^2c^2-2abcd+5a^2d^2)-6b(18b^2c^2-10abcd-5a^2d^2)x^3)}{c+dx^3}\,dx}{18c^2d^2} \\
&= \frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3} \\
&\quad - \frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} \\
&\quad - \frac{\int \frac{(a+bx^3)^{2/3}(6a(18b^3c^3-22ab^2c^2d-a^2bcd^2-10a^3d^3)+18b(2bc-ad)(18b^2c^2-18abcd-5a^2d^2)x^3)}{c+dx^3}\,dx}{108c^2d^3} \\
&= -\frac{b(2bc-ad)\,(18b^2c^2-18abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^4} \\
&\quad + \frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3} \\
&\quad - \frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} \\
&\quad - \frac{\int \frac{-36a(18b^4c^4-36ab^3c^3d+15a^2b^2c^2d^2+3a^3bcd^3+5a^4d^4)-36b^3c^2(54b^2c^2-126abcd+77a^2d^2)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{324c^2d^4}
\end{aligned}
$$

$$= -\frac{b(2bc-ad)\left(18b^2c^2-18abcd-5a^2d^2\right)x(a+bx^3)^{2/3}}{18c^2d^4}$$
$$+\frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3}$$
$$-\frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)}$$
$$-\frac{\left((bc-ad)^3\left(54b^2c^2+18abcd+5a^2d^2\right)\right)\int\frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2d^5}$$
$$+\frac{\left(b^3(54b^2c^2-126abcd+77a^2d^2)\right)\int\frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9d^5}$$

$$= -\frac{b(2bc-ad)\left(18b^2c^2-18abcd-5a^2d^2\right)x(a+bx^3)^{2/3}}{18c^2d^4}$$
$$+\frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3}$$
$$-\frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)}$$
$$+\frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^5}$$
$$-\frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5}$$
$$-\frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^5}$$
$$+\frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5}$$
$$-\frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^5}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 12.28 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \frac{1}{108}\left(\frac{6x(a+bx^3)^{2/3}\left(-2b^3(9bc-13ad)+3b^4dx^3+\frac{3(bc-ad)^4}{c(c+dx^3)^2}-\frac{(bc-ad)^3(21bc+5ad)}{c^2(c+dx^3)}\right)}{d^4}\right.$$

$$+\frac{162b^5cx^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^4\sqrt[3]{a+bx^3}}$$

$$-\frac{378ab^4x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^3\sqrt[3]{a+bx^3}}$$

$$+\frac{231a^2b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd^2\sqrt[3]{a+bx^3}}$$

$$+\frac{10a^5\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{8/3}\sqrt[3]{bc-ad}}$$

$$+\frac{36ab^4c^{4/3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{b}}{\sqrt[3]{b}}\right.\right.}{d^4\sqrt[3]{bc-ad}}$$

$$-\frac{72a^2b^3\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{}}{\sqrt[3]{b}}\right.\right.}{d^3\sqrt[3]{bc-ad}}$$

$$+\frac{30a^3b^2\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{2/3}d^2\sqrt[3]{bc-ad}}$$

$$+\frac{6a^4b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{5/3}d\sqrt[3]{bc-ad}}$$

```
[In] Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*x^
3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*(1 +
 (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(
d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/
3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (1
0*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(
1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]
+ Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) + (36*a*b^4
*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3)) - (72*a^2*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/
3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (30*a^3*b
^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (6*a^4*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{8}{3}}\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^3\left(d\,x^3+c\right)^2\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}+\left(5a^2d^2+18abcd+54b^2c^2\right)\left(d\,x^3+\right.}{}$

```
[In] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27/((a*d-b*c)/c)^(1/3)*(1/2*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*(
(a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*
```

```
x+(b*x^3+a)^(2/3))/x^2)+(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*(d*x^3+c)^2*(a*d-
b*c)^3*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-3^(1/2)*b^(8/3)*(77*a^
2*d^2-126*a*b*c*d+54*b^2*c^2)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*arctan(1/
3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-b^(8/3)*(77*a^2*d^2-126*
a*b*c*d+54*b^2*c^2)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((-b^(1/3)*x+(b*x
^3+a)^(1/3))/x)+3/2*x*(3*b^4*c^2*d^3*x^9+26*a*b^3*c^2*d^3*x^6-12*b^4*c^3*d^
2*x^6+5*a^4*d^5*x^3+6*a^3*b*c*d^4*x^3-48*a^2*b^2*c^2*d^3*x^3+110*a*b^3*c^3*
d^2*x^3-54*b^4*c^4*d*x^3+8*a^4*c*d^4-6*a^3*b*c^2*d^3-30*a^2*b^2*c^3*d^2+72*
a*b^3*c^4*d-36*b^4*c^5)*d*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)-1/2*(5*a^2*
d^2+18*a*b*c*d+54*b^2*c^2)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*
(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2
-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c)^2*(
a*d-b*c)^3)/d^5/c^3/(d*x^3+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. $2(473) = 946$.

Time = 94.18 (sec) , antiderivative size = 1555, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*(54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3*
b*c^3*d^3 + 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b^2
*c^2*d^4 + 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c^4*
d^2 + 23*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 2*sqrt(3)*(54*b^4*c^6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^4*d^2 +
 (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^2*d^4)*x^6 + 2*(54*b^4*
c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3)*(-b^2)^(1/3)*arctan(-1
/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(54*
b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d^3 + 5*a^4*c^2
*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b^2*c^2*d^4 + 8*a^3*b*c*
d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c^4*d^2 + 23*a^2*b^2*c^3*
d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 +
a)^(1/3)*(b*c - a*d))/x) - 2*(54*b^4*c^6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^4
*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^2*d^4)*x^6 + 2*(5
4*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3)*(-b^2)^(1/3)*log
(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (54*b^4*c^6 - 126*a*b^3*c^5*d
 + 77*a^2*b^2*c^4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^
```

```
2*d^4)*x^6 + 2*(54*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3)
*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x -
 (b*x^3 + a)^(2/3)*b)/x^2) + (54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*
d^2 + 8*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3
+ 23*a^2*b^2*c^2*d^4 + 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 9
0*a*b^3*c^4*d^2 + 23*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2
 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^
4*c^2*d^4*x^10 - 2*(6*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4)*x^7 - (54*b^4*c^4*d^2
 - 110*a*b^3*c^3*d^3 + 48*a^2*b^2*c^2*d^4 - 6*a^3*b*c*d^5 - 5*a^4*d^6)*x^4
- 2*(18*b^4*c^5*d - 36*a*b^3*c^4*d^2 + 15*a^2*b^2*c^3*d^3 + 3*a^3*b*c^2*d^4
 - 4*a^4*c*d^5)*x)*(b*x^3 + a)^(2/3))/(c^2*d^7*x^6 + 2*c^3*d^6*x^3 + c^4*d^
5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{14/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

3.110 $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 458

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx = \frac{b(18b^2c^2-7abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2\,(c+dx^3)} - \frac{b^{8/3}(9bc-11ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^4} + \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^4} + \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log\left(c+dx^3\right)}{54c^{8/3}d^4} - \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4} + \frac{b^{8/3}(9bc-11ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^4}$$

```
[Out] 1/18*b*(-5*a^2*d^2-7*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c^2/d^3-1/6*(-a*
d+b*c)*x*(b*x^3+a)^(8/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+9*b*c)*x*(b
```

```
*x^3+a)^(5/3)/c^2/d^2/(d*x^3+c)+1/54*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d
+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^4-1/18*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*
b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/d^
4+1/6*b^(8/3)*(-11*a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^4-1/9*b^(8/3
)*(-11*a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^4*3
^(1/2)+1/27*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^4*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {424, 540, 542, 544, 245, 384}

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^4} + \frac{bx(a+bx^3)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{18c^2d^3} + \frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^4} - \frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4} - \frac{b^{8/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(9bc-11ad)}{3\sqrt{3}d^4} + \frac{b^{8/3}(9bc-11ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6d^4} - \frac{x(a+bx^3)^{5/3}(bc-ad)(5ad+9bc)}{18c^2d^2\left(c+dx^3\right)} - \frac{x(a+bx^3)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

```
[In] Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] (b*(18*b^2*c^2 - 7*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*c^2*d^3) -
 ((b*c - a*d)*x*(a + b*x^3)^(8/3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(9*
b*c + 5*a*d)*x*(a + b*x^3)^(5/3))/(18*c^2*d^2*(c + d*x^3)) - (b^(8/3)*(9*b*
c - 11*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[
3]*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*ArcTan[(
1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[
3]*c^(8/3)*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*
```

```
Log[c + d*x^3])/(54*c^(8/3)*d^4) - ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*
c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(1
8*c^(8/3)*d^4) + (b^(8/3)*(9*b*c - 11*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(
1/3)])/(6*d^4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd(c+dx^3)^2} + \frac{\int \frac{(a+bx^3)^{5/3}(a(bc+5ad)+3b(3bc-ad)x^3)}{(c+dx^3)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2(c+dx^3)} - \frac{\int \frac{(a+bx^3)^{2/3}(-a(9b^2c^2-abcd+10a^2d^2)-3b(18b^2c^2-7abcd-5a^2d^2)x^3)}{c+dx^3}\,dx}{18c^2d^2}$$

$$= \frac{b(18b^2c^2-7abcd-5a^2d^2)x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2(c+dx^3)} - \frac{\int \frac{6a(9b^3c^3-8ab^2c^2d-2a^2bcd^2-5a^3d^3)+18b^3c^2(9bc-11ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{54c^2d^3}$$

$$= \frac{b(18b^2c^2-7abcd-5a^2d^2)x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2(c+dx^3)} - \frac{(b^3(9bc-11ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3d^4} + \frac{((bc-ad)^2(27b^2c^2+12abcd+5a^2d^2))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2d^4}$$

$$= \frac{b(18b^2c^2 - 7abcd - 5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd\,(c+dx^3)^2}$$
$$- \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2\,(c+dx^3)} - \frac{b^{8/3}(9bc-11ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^4}$$
$$+ \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^4}$$
$$+ \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}d^4}$$
$$- \frac{(bc-ad)^{5/3}\,(27b^2c^2+12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4}$$
$$+ \frac{b^{8/3}(9bc-11ad)\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{6d^4}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.74 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \frac{1}{108}\left(\frac{6x(a+bx^3)^{2/3}\left(6b^3-\frac{3(bc-ad)^3}{c(c+dx^3)^2}+\frac{5(bc-ad)^2(3bc+ad)}{c^2(c+dx^3)}\right)}{d^3}\right.$$

$$-\frac{81b^4x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^3\sqrt[3]{a+bx^3}}$$

$$+\frac{99ab^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd^2\sqrt[3]{a+bx^3}}$$

$$+\frac{10a^4\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{8/3}\sqrt[3]{bc-ad}}$$

$$-\frac{18ab^3\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{b}}{\sqrt[3]{b-}}\right.\right.}{d^3\sqrt[3]{bc-ad}}$$

$$+\frac{16a^2b^2\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{2/3}d^2\sqrt[3]{bc-ad}}$$

$$+\frac{4a^3b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{b+ax^3}}\right.\right.}{c^{5/3}d\sqrt[3]{bc-ad}}$$

```
[In] Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*(
b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b*x
```

```
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^3*(
a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a^4*(2
*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/S
qrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^
(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1
/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^3*c^(1/3)
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a^2*b^2*(2*Sqr
t[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[
3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3
) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (4*a^3*b*(2*Sqrt[
3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]
] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3)
+ ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)
/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{-3b^{\frac{8}{3}}(11ad-9bc)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^{3}(d\,x^{3}+c)^{2}\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\left(5a^{2}d^{2}+12abcd+27b^{2}c^{2}\right)(d\,x^{3}+c)^{2}(\ldots}{}$

```
[In] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-3*b^(8/3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((b^
(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*(5*a^2*d^2+12*a
*b*c*d+27*b^2*c^2)*(d*x^3+c)^2*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3
+a)^(1/3))/x)+6*b^(8/3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*3^(1/2)*c^3*(d*x
^3+c)^2*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+6*b^(8/
3)*(11*a*d-9*b*c)*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((-b^(1/3)*x+(b*x^3
+a)^(1/3))/x)-3*x*d*c*(6*b^3*c^2*d^2*x^6+5*a^3*d^4*x^3+5*a^2*b*c*d^3*x^3-25
*a*b^2*c^2*d^2*x^3+27*b^3*c^3*d*x^3+8*a^3*c*d^3-4*a^2*b*c^2*d^2-16*a*b^2*c^
3*d+18*b^3*c^4)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+(5*a^2*d^2+12*a*b*c*d+2
7*b^2*c^2)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))
/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(
1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(d*x^3+c)^2*(a*d-b*c)^2)/((a*
d-b*c)/c)^(1/3)/(d*x^3+c)^2/d^4/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. 2(394) = 788.

Time = 18.18 (sec) , antiderivative size = 1246, normalized size of antiderivative = 2.72

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 +
 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)
*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*
d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2
)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
)/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3
 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*
(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a
*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a
*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^
6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)
*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27
*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d
^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d -
15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{11/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

3.111 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 391

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3} dx = -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd(c+dx^3)^2} - \frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2(c+dx^3)} + \frac{b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^3} - \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}d^3} + \frac{(bc-ad)^{2/3}(9b^2c^2+6abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3} - \frac{b^{8/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^3}$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)^(5/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6*
b*c)*x*(b*x^3+a)^(2/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(2/3)*(5*a^2*d^2+6
*a*b*c*d+9*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^3+1/18*(-a*d+b*c)^(2/3)*(5*a^2*d^
2+6*a*b*c*d+9*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/
3)/d^3-1/2*b^(8/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/3*b^(8/3)*arctan(1/
```

```
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)-1/27*(-a*d+b*c)^(2/3
)*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {424, 540, 544, 245, 384}

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx =$$
$$-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^3}-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^3}+\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3}+\frac{b^{8/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^3}-\frac{b^{8/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d^3}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)(5ad+6bc)}{18c^2d^2\left(c+dx^3\right)}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)^2) - ((b*c - a*d)*(
6*b*c + 5*a*d)*x*(a + b*x^3)^(2/3))/(18*c^2*d^2*(c + d*x^3)) + (b^(8/3)*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^3) - ((b*c -
 a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^3) - (
(b*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c
^(8/3)*d^3) + ((b*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^3) - (b^(8/
3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2}+\frac{\int \frac{(a+bx^3)^{2/3}(a(bc+5ad)+6b^2cx^3)}{(c+dx^3)^2}\,dx}{6cd}\\
&= -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2}-\frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2\,(c+dx^3)}\\
&\quad -\frac{\int \frac{-2a(3b^2c^2+ad(bc+5ad))-18b^3c^2x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{18c^2d^2}
\end{aligned}$$

$$= -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2\,(c+dx^3)} + \frac{b^3\int\frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d^3} - \frac{((bc-ad)\,(9b^2c^2+6abcd+5a^2d^2))\int\frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2d^3}$$

$$= -\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2\,(c+dx^3)} + \frac{b^{8/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^3} - \frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\log\left(c+dx^3\right)}{54c^{8/3}d^3} + \frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3} - \frac{b^{8/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.00 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.66

$$\int\frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(-bc+ad)x(a+bx^3)^{2/3}(3bc(2c+3dx^3)+ad(8c+5dx^3))}{d^2(c+dx^3)^2} + \frac{27b^3c^{5/3}x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\right.}{d^2\sqrt[3]{a+bx^3}}}{}$$

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d
*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a + b
*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
```

```
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
 + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6
*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108*c^(8/3))
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{\frac{9 b^{\frac{8}{3}} c^{3} \left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} \left(d \,x^{3}+c \right)^{2} \ln \left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2}}\right)}{2}+\left(5 a^{2} d^{2}+6 a b c d +9 b^{2} c^{2}\right) \left(d \,x^{3}+c \right)^{2} \left(a d -b c \right) \ln \left(\frac{\left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} x \cdots}{x}\right.}{}$

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(9/2*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)^2*ln((b^(2/3)*x^2+b^(1/
3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*
(d*x^3+c)^2*(a*d-b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-9*3^(1/
2)*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)^2*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)-9*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)
^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/2*(3*(b*x^3+a)^(2/3)*x*d*(5*a*d^2*x
^3+9*b*c*d*x^3+8*a*c*d+6*b*c^2)*c*((a*d-b*c)/c)^(1/3)-(-2*arctan(1/3*3^(1/2
)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+
ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*(d*x^3+c)^2)*(a*d-b*c))/((a*d
-b*c)/c)^(1/3)/(d*x^3+c)^2/d^3/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(334) = 668$.

Time = 1.95 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.44

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx =$$

$$-\frac{2\sqrt{3}((9\,b^2c^2d^2+6\,abcd^3+5\,a^2d^4)x^6+9\,b^2c^4+6\,abc^3d+5\,a^2c^2d^2+2\,(9\,b^2c^3d+6\,abc^2d^2+5\,a^2cd^3)x^3)\Big(\frac{b^2}{}}{}$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4
 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d
^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(
b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d
*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4
)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^
2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((
c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c -
a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*lo
g(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x^6 + 2*b^2*c
^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d^2 + 6*a*b*c*d
^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^
3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)
+ (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^
3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a*b*c*d^3 - 5*a^2*d^
4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)*x)*(b*x^3 + a)^(2/3))/
(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

3.112 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{x(a+bx^3)^{5/3}}{6c\,(c+dx^3)^2} + \frac{5ax(a+bx^3)^{2/3}}{18c^2\,(c+dx^3)} + \frac{5a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2\log(c+dx^3)}{54c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*x*(b*x^3+a)^(5/3)/c/(d*x^3+c)^2+5/18*a*x*(b*x^3+a)^(2/3)/c^2/(d*x^3+c)+
5/54*a^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-5/18*a^2*ln((-a*d+b*c)^(1/3)*
x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+5/27*a^2*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{386, 384\}$

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{5a^2\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2\log(c+dx^3)}{54c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} + \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (5*a^2*Log
[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a^2*Log[((b*c - a*d)^(1/3)
*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2} + \frac{(5a)\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx}{6c}$$

$$= \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} + \frac{(5a^2)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2}$$

$$= \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} + \frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2\log(c+dx^3)}{54c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(8acx+3bcx^4+5adx^4)}{(c+dx^3)^2} - \frac{10\sqrt{-6+6i\sqrt{3}}a^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}}{}$$

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(8*a*c*x + 3*b*c*x^4 + 5*a*d*x^4))/(c + d*x^3
)^2 - (10*Sqrt[-6 + (6*I)*Sqrt[3]]*a^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt
[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c
 - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*a^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[
3])*a^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*
c - a*d)^(1/3))/(108*c^(8/3))
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{-\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)a^2\left(dx^3+c\right)^2}{54}+\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)a^2\left(dx^3+c\right)^2}{27}+\frac{4x\left(\frac{(5}{}\right.}{}}{c^3(dx^3+c)^2\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/27*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)*a^2*(d*x^3+c)^2+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)
^(1/3))/x)*a^2*(d*x^3+c)^2+12/5*x*(1/8*(5*a*d+3*b*c)*x^3+a*c)*c*(b*x^3+a)^(
2/3)*((a*d-b*c)/c)^(1/3)+3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-
2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a^2*(d*x^3+c)^2)/((a*d-b*c)/c)^(1
/3)/c^3/(d*x^3+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

3.113 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = & -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)\,(c+dx^3)^2}+\frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)\,(c+dx^3)} \\ & +\frac{a(6bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{4/3}} \\ & -\frac{a(6bc-5ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}\end{aligned}$$

```
[Out] -1/6*d*x*(b*x^3+a)^(5/3)/c/(-a*d+b*c)/(d*x^3+c)^2+1/18*(-5*a*d+6*b*c)*x*(b*
x^3+a)^(2/3)/c^2/(-a*d+b*c)/(d*x^3+c)+1/54*a*(-5*a*d+6*b*c)*ln(d*x^3+c)/c^(
8/3)/(-a*d+b*c)^(4/3)-1/18*a*(-5*a*d+6*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(
b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)+1/27*a*(-5*a*d+6*b*c)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)
^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 386, 384}

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \frac{a(6bc-5ad)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc-5ad)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}} + \frac{x(a+bx^3)^{2/3}(6bc-5ad)}{18c^2(c+dx^3)(bc-ad)} - \frac{dx(a+bx^3)^{5/3}}{6c(c+dx^3)^2(bc-ad)}$$

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] -1/6*(d*x*(a + b*x^3)^(5/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*
d)*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*
d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]
])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) + (a*(6*b*c - 5*a*d)*Log[c + d*x^3
])/(54*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2(bc-ad)}$$

$$= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc-5ad)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc-5ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}(3bc(2c+dx^3)-ad(8c+5dx^3))}{(bc-ad)(c+dx^3)^2} + \frac{2i(3i+\sqrt{3})a(-6bc+5ad)\text{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right.}{(bc-ad)^{4/3}}}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + d*x^3) - a*d*(8*c + 5*d*x^3))
)/((b*c - a*d)*(c + d*x^3)^2) + ((2*I)*(3*I + Sqrt[3])*a*(-6*b*c + 5*a*d)*A
rcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x
))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*a*(6*b*c - 5*a*d)*Log[2
```

```
*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a
*d)^(4/3) + ((1 + I*Sqrt[3])*a*(-6*b*c + 5*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2
 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I +
Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(108*c^(8/3))
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{-\frac{5\left(ad-\frac{6bc}{5}\right)a\left(d\,x^{3}+c\right)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{54}+\frac{5\left(ad-\frac{6bc}{5}\right)a\left(d\,x^{3}+c\right)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}\right.}{x}\right.}{27}}{(ad-bc)c^{3}}$

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/27/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-6/5*b*c)*a*(d*x^3+c)^2*ln((((a*d-b*c)/c
)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*
d-6/5*b*c)*a*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5
*(1/8*(5*a*d^2-3*b*c*d)*x^3+c*(a*d-3/4*b*c))*x*c*(b*x^3+a)^(2/3)*((a*d-b*c)
/c)^(1/3)+(a*d-6/5*b*c)*3^(1/2)*a*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x
-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*(d*x^3+c)^2)/(a*d-b*c)/c^3/(d*x^
3+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^3}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 307

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3} dx = -\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\,(c+dx^3)^2} - \frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\,(c+dx^3)} + \frac{(9b^2c^2-12abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{7/3}} + \frac{(9b^2c^2-12abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{7/3}} - \frac{(9b^2c^2-12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}$$

```
[Out] -1/6*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*(
b*x^3+a)^(2/3)/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(7/3)-1/18*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(7/3
)+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \frac{(5a^2d^2-12abcd+9b^2c^2)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{7/3}} + \frac{(5a^2d^2-12abcd+9b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{7/3}} - \frac{(5a^2d^2-12abcd+9b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}} - \frac{dx(a+bx^3)^{2/3}(9bc-5ad)}{18c^2(c+dx^3)(bc-ad)^2} - \frac{dx(a+bx^3)^{2/3}}{6c(c+dx^3)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)^2) - (d*(9*b*c - 5*
a*d)*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)^2*(c + d*x^3)) + ((9*b^2*c^2
- 12*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
 b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(7/3)) + ((9*b^2*c
^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(7/3))
 - ((9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{\int\frac{6bc-5ad-3bdx^3}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx}{6c(bc-ad)}$$

$$=-\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\left(c+dx^3\right)^2}-\frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\left(c+dx^3\right)}+\frac{\int\frac{2(9b^2c^2-12abcd+5a^2d^2)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{18c^2(bc-ad)^2}$$

$$=-\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\left(c+dx^3\right)^2}-\frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\left(c+dx^3\right)}+\frac{(9b^2c^2-12abcd+5a^2d^2)\int\frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2(bc-ad)^2}$$

$$=-\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\left(c+dx^3\right)^2}-\frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\left(c+dx^3\right)}+\frac{(9b^2c^2-12abcd+5a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{7/3}}+\frac{(9b^2c^2-12abcd+5a^2d^2)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{7/3}}-\frac{(9b^2c^2-12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$

$$= \frac{\frac{6c^{2/3}dx(a+bx^3)^{2/3}(-3bc(4c+3dx^3)+ad(8c+5dx^3))}{(bc-ad)^2(c+dx^3)^2} + \frac{2\left(3-i\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}} + \frac{2(\cdots}{\cdots}}{\cdots}$$

[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]

[Out] ((6*c^(2/3)*d*x*(a + b*x^3)^(2/3)*(-3*b*c*(4*c + 3*d*x^3) + a*d*(8*c + 5*d*x^3)))/((b*c - a*d)^2*(c + d*x^3)^2) + (2*(3 - I*Sqrt[3])*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(7/3) - (I*(-I + Sqrt[3])*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(108*c^(8/3))

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\frac{5\left(a^2d^2-\frac{12}{5}abcd+\frac{9}{5}b^2c^2\right)\left(d\,x^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)}{27}+\frac{4x\left(-\frac{3b\,c^2}{2}+d\left(-\frac{9b\,x^3}{8}+a\right)c+\frac{5a\,d^2x^3}{8}\right)dc\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\cdots}{9}}{\left(\frac{ad-bc}{c}\right)\cdots}$

[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)

[Out] 5/27/((a*d-b*c)/c)^(1/3)*((a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5*x*(-3/2*b*c^2+d*(-9/8*b*x^3+a)*c+5/8*a*d^2*x^3)*d*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+

```
(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*(d*x^3+c)^2)/(a*d
-b*c)^2/c^3/(d*x^3+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

3.115 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 377

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2} + \frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)} + \frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18ac^2(bc-ad)^3\,(c+dx^3)} - \frac{d(27b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{10/3}} - \frac{d(27b^2c^2-18abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}} + \frac{d(27b^2c^2-18abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}}$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(1/3)/(d*x^3+c)^2+1/6*b*(a*d+6*b*c)*x/a/c/(
-a*d+b*c)^2/(b*x^3+a)^(1/3)/(d*x^3+c)+1/18*d*(-5*a^2*d^2+15*a*b*c*d+18*b^2*
c^2)*x*(b*x^3+a)^(2/3)/a/c^2/(-a*d+b*c)^3/(d*x^3+c)-1/54*d*(5*a^2*d^2-18*a*
b*c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(10/3)+1/18*d*(5*a^2*d^2-1
8*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3
)/(-a*d+b*c)^(10/3)-1/27*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = -\frac{d(5a^2d^2-18abcd+27b^2c^2)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{10/3}} + \frac{dx(a+bx^3)^{2/3}(-5a^2d^2+15abcd+18b^2c^2)}{18ac^2(c+dx^3)(bc-ad)^3} - \frac{d(5a^2d^2-18abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}} + \frac{d(5a^2d^2-18abcd+27b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}} + \frac{bx(ad+6bc)}{6ac\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)^2} - \frac{dx}{6c\sqrt[3]{a+bx^3}(c+dx^3)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + (b*(6*b*c + a*
d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + (d*(18*b^2*c^2
+ 15*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*a*c^2*(b*c - a*d)^3*(c +
 d*x^3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c
 - a*d)^(10/3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/
(54*c^(8/3)*(b*c - a*d)^(10/3)) + (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*
Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c -
a*d)^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2}+\frac{\int \frac{6bc-5ad-6bdx^3}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx}{6c(bc-ad)} \\
&= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2} \\
&\quad +\frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)}-\frac{\int \frac{ad(12bc-5ad)-3bd(6bc+ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx}{6ac(bc-ad)^2} \\
&= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2}+\frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)} \\
&\quad +\frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18ac^2(bc-ad)^3\,(c+dx^3)}-\frac{\int \frac{2ad(27b^2c^2-18abcd+5a^2d^2)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{18ac^2(bc-ad)^3} \\
&= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2}+\frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)} \\
&\quad +\frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18ac^2(bc-ad)^3\,(c+dx^3)} \\
&\quad -\frac{(d(27b^2c^2-18abcd+5a^2d^2))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{9c^2(bc-ad)^3}
\end{aligned}$$

$$= -\frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2} + \frac{b(6bc+ad)x}{6ac(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)}$$
$$+ \frac{d(18b^2c^2+15abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18ac^2(bc-ad)^3\,(c+dx^3)}$$
$$- \frac{d(27b^2c^2-18abcd+5a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{10/3}}$$
$$- \frac{d(27b^2c^2-18abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}}$$
$$+ \frac{d(27b^2c^2-18abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 13.68 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+bx^3)^{4/3}\,(c+dx^3)^3}\,dx =$$
$$-\frac{65c^2(a+bx^3)^2\left(-14000a^2c^5-21896abc^5x^3-48104a^2c^4dx^3-8391b^2c^5x^6-70802abc^4dx^6-60807a^2c^3d^2x\right.}{}$$

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/16380*(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104*
a^2*c^4*d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^2*
x^6 - 24417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^9 -
 23409*b^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12 - 74
25*b^2*c^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c + d*x^
3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 +
33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 486*(b*
c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)^3*x^8*(a + b*x^3
)^(7/3)*(c + d*x^3)^2)
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{-2\left(5a^{2}d^{2}-18abcd+27b^{2}c^{2}\right)da\left(d\,x^{3}+c\right)^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-3xc\left(5a^{2}b\,d^{4}x^{6}-15a\,b^{2}c\,d^{3}x^{6}-18b^{3}c}{}$

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-2*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*d*a*(d*x^3+c)^2*(b*x^3+a)^(1/3)
*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-3*x*c*(5*a^2*b*d^4*x^6-15*a*
b^2*c*d^3*x^6-18*b^3*c^2*d^2*x^6+5*a^3*d^4*x^3-7*a^2*b*c*d^3*x^3-18*a*b^2*c
^2*d^2*x^3-36*b^3*c^3*d*x^3+8*a^3*c*d^3-18*a^2*b*c^2*d^2-18*b^3*c^4)*((a*d-
b*c)/c)^(1/3)+(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*d*a*(-2*arctan(1/3*3^(1/2)*
(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln
((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(
2/3))/x^2))*(d*x^3+c)^2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3
)/(d*x^3+c)^2/c^3/(a*d-b*c)^3/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)^{3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^{3}\right)^{4/3}\left(c+dx^{3}\right)^{3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

3.116 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 463

$$
\begin{aligned}
\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = &-\frac{dx}{6c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)^2} \\
&+\frac{b(3bc+2ad)x}{12ac(bc-ad)^2(a+bx^3)^{4/3}(c+dx^3)}+\frac{b(9b^2c^2-42abcd-2a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\,(c+dx^3)} \\
&+\frac{d(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3)\,x(a+bx^3)^{2/3}}{36a^2c^2(bc-ad)^4(c+dx^3)} \\
&+\frac{d^2(54b^2c^2-24abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{13/3}} \\
&+\frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{13/3}} \\
&-\frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}}
\end{aligned}
$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)^2+1/12*b*(2*a*d+3*b*c)*x/a/
c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)/(d*x^3+c)+1/12*b*(-2*a^2*d^2-42*a*b*c*d+9*b^
2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)/(d*x^3+c)+1/36*d*(10*a^3*d^3-42
*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x*(b*x^3+a)^(2/3)/a^2/c^2/(-a*d+b*
c)^4/(d*x^3+c)+1/54*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/
3)/(-a*d+b*c)^(13/3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(13/3)+1/27*d^2*(5*a
^2*d^2-24*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b
*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {425, 541, 12, 384}

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{d^2(5a^2d^2-24abcd+54b^2c^2)\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{13/3}} + \frac{bx(-2a^2d^2-42abcd+9b^2c^2)}{12a^2c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)^3} + \frac{d^2(5a^2d^2-24abcd+54b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{13/3}} - \frac{d^2(5a^2d^2-24abcd+54b^2c^2)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}} + \frac{dx(a+bx^3)^{2/3}(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3)}{36a^2c^2(c+dx^3)(bc-ad)^4} - \frac{dx}{6c(a+bx^3)^{4/3}(c+dx^3)^2(bc-ad)} + \frac{bx(2ad+3bc)}{12ac(a+bx^3)^{4/3}(c+dx^3)(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)^2) + (b*(3*b*c + 2*
a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)*(c + d*x^3)) + (b*(9*b^2*c^
2 - 42*a*b*c*d - 2*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3)*(c
 + d*x^3)) + (d*(27*b^3*c^3 - 135*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 10*a^3*d^3
)*x*(a + b*x^3)^(2/3))/(36*a^2*c^2*(b*c - a*d)^4*(c + d*x^3)) + (d^2*(54*b^
2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(13/3)) + (d
^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c -
 a*d)^(13/3)) - (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(13/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}+\frac{\int \frac{6bc-5ad-9bdx^3}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx}{6c(bc-ad)}\\
&= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}+\frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\\
&\quad-\frac{\int \frac{-2\left(9b^2c^2-24abcd+10a^2d^2\right)-12bd(3bc+2ad)x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx}{24ac(bc-ad)^2}\\
&= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}+\frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\\
&\quad+\frac{b\left(9b^2c^2-42abcd-2a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}+\frac{\int \frac{2ad\left(9b^2c^2+36abcd-10a^2d^2\right)+6bd\left(9b^2c^2-42abcd-2a^2d^2\right)x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx}{24a^2c(bc-ad)^3}
\end{aligned}$$

$$= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2} + \frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} + \frac{b\left(9b^2c^2-42abcd-2a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)} + \frac{d\left(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3\right)x\left(a+bx^3\right)^{2/3}}{36a^2c^2(bc-ad)^4\left(c+dx^3\right)} + \frac{\int \frac{8a^2d^2\left(54b^2c^2-24abcd+5a^2d^2\right)}{\sqrt[3]{a+bx^3}(c+dx^3)}dx}{72a^2c^2(bc-ad)^4}$$

$$= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2} + \frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} + \frac{b\left(9b^2c^2-42abcd-2a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)} + \frac{d\left(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3\right)x\left(a+bx^3\right)^{2/3}}{36a^2c^2(bc-ad)^4\left(c+dx^3\right)} + \frac{\left(d^2\left(54b^2c^2-24abcd+5a^2d^2\right)\right)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}dx}{9c^2(bc-ad)^4}$$

$$= -\frac{dx}{6c(bc-ad)\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2} + \frac{b(3bc+2ad)x}{12ac(bc-ad)^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} + \frac{b\left(9b^2c^2-42abcd-2a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)} + \frac{d\left(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3\right)x\left(a+bx^3\right)^{2/3}}{36a^2c^2(bc-ad)^4\left(c+dx^3\right)} + \frac{d^2\left(54b^2c^2-24abcd+5a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{13/3}} + \frac{d^2\left(54b^2c^2-24abcd+5a^2d^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{13/3}} - \frac{d^2\left(54b^2c^2-24abcd+5a^2d^2\right)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}}$$

Mathematica [A] (warning: unable to verify)

Time = 15.75 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{1}{36}x(a+bx^3)^{2/3}\left(-\frac{9b^3}{a(-bc+ad)^3(a+bx^3)^2}+\frac{27b^3(bc-5ad)}{a^2(bc-ad)^4(a+bx^3)}-\frac{6d^3}{c(bc-ad)^3(c+dx^3)^2}+\frac{2d^3(-21bc+5ad}{c^2(bc-ad)^4(c+d\ldots}\right.$$

$$+\frac{d^2(54b^2c^2-24abcd+5a^2d^2)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\ldots\right.\right.}{54c^{8/3}(bc-ad)^{13/3}}$$

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^3
*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3)))
)/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3)
 - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{5\left(b\,x^3+a\right)^{\frac{4}{3}}\left(a^2d^2-\frac{24}{5}abcd+\frac{54}{5}b^2c^2\right)d^2a^2\left(d\,x^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)}{27}+\frac{4\left(\frac{5a^3x^3\left(b\,x^3+a\right)^2d^5}{8}+\left(b\,x^3+a\right)^2\left(-\frac{21b\,x^3}{8}\ldots\right.\right.}{\ldots}$

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/27/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(4/3)*((b*x^3+a)^(4/3)*(a^2*d^2-24/5*a*b
*c*d+54/5*b^2*c^2)*d^2*a^2*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)+12/5*(5/8*a^3*x^3*(b*x^3+a)^2*d^5+(b*x^3+a)^2*(-21/8*b*x^3+a)*c*a
^2*d^4-3*b*(45/16*b^3*x^9+4*a*b^2*x^6+2*a^2*b*x^3+a^3)*c^2*a*d^3-18*x^3*b^3
*(-3/32*b^2*x^6+13/16*a*b*x^3+a^2)*c^3*d^2-9*b^3*(-3/8*b^2*x^6+7/16*a*b*x^3
```

```
+a^2)*c^4*d+9/4*b^4*(3/4*b*x^3+a)*c^5)*x*c*((a*d-b*c)/c)^(1/3)+(arctan(1/3*
3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^
(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-24/5*a*b*c*d+54/5*b^2*c^2)*d^2*a^2*(b*x^3+
a)^(4/3)*(d*x^3+c)^2)/(d*x^3+c)^2/c^3/(a*d-b*c)^4/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

3.117 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3
/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} \, dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 285 vs. $2(60) = 120$.

Time = 10.47 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.75

$$\int\frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \frac{x\left(b(2bc+5ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left((a+bx^3)(-bc(c-2dx\right.}{}\right.}{72}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + (4*c*((a + b*x^3)*(-(b*c*(c - 2*d*x^3)) + a*d
*(8*c + 5*d*x^3)) + (4*a^2*c*(b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b
*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a
), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)
/c)]))))/(c + d*x^3)^2))/(72*c^3*d*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)^{3}}dx$$

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^3, x)
```

3.118 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx = \frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/a
)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} \, dx = \frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 407 vs. $2(59) = 118$.

Time = 10.62 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$$

$$= \frac{-b(-4bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16acx(-b^2cx^3(7c+4dx^3)+3a^2d(6c+5dx^3)+ab(\right.}{}}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3)) +
 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(a + b*x^3)*(-(b*c*(7
*c + 4*d*x^3)) + a*d*(8*c + 5*d*x^3))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -(
(d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])
)))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)^{3}}dx$$

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^3, x)
```

3.119 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\,(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a)
^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\,(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)^3}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 418 vs. $2(59) = 118$.

Time = 10.68 (sec) , antiderivative size = 418, normalized size of antiderivative = 7.08

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx = \frac{x\left(5bd(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c(-4c}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3) +
b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 + 5*d
^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3*(a
+ b*x^3)*(a*d*(8*c + 5*d*x^3) - b*c*(13*c + 10*d*x^3))*(3*a*d*AppellF1[4/3,
 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])))))/(72*c^3*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{3}}dx$$

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)
```

3.120 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3+
a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}(c+dx^3)^3}\,dx}{a\,(a+bx^3)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 531 vs. $2(62) = 124$.

Time = 10.92 (sec) , antiderivative size = 531, normalized size of antiderivative = 8.56

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^3}\,dx = \frac{\frac{bd(-9b^2c^2-16abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3} - \frac{4c\left(4acx(3a^3d^3(6c+5\ldots\right.}{\ldots}}{\ldots}$$

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] ((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*c*
(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 16*d
^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2 - 43
*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c*d^2*x
^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d*x^3)^2*(
4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{3}}d x$$

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3} (c+dx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3} (c+dx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3} (c+dx^3)^3} \, dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}} (dx^3+c)^3} \, dx$$

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx=\int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^3}\,dx$$

[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx=\int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)^3}\,dx$$

[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x)

[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3), x)

3.121 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x^
3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx=\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}(c+dx^3)^3}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(62) = 124$.

Time = 11.48 (sec) , antiderivative size = 515, normalized size of antiderivative = 8.31

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx = \frac{x\left(bd(36b^3c^3-171ab^2c^2d-110a^2bcd^2+25a^3d^3)\,x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\right.\right.}{}$$

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*(
1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c +
d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*c*d*
x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^2*b^3
*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*x^3) +
 (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^3*b*c*d
^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(3
60*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}} \left(d\,x^{3}+c\right)^{3}}dx$$

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3), x)
```

3.122 $\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} dx = \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}$$

```
[Out] 4/25*x*(b*x^3+a)^(7/4)/c/(d*x^3+c)^(25/12)+84/325*a*x*(b*x^3+a)^(3/4)/c^2/(
d*x^3+c)^(13/12)+189/325*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1
/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(1/4)/(d*x^3+c)^(
1/12)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} dx = \frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}}$$

```
[In] Int[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3
/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*
x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}}+\frac{(21a)\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx}{25c}\\
&= \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}}+\frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}}+\frac{(189a^2)\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx}{325c^2}\\
&= \frac{4x(a+bx^3)^{7/4}}{25c\,(c+dx^3)^{25/12}}+\frac{84ax(a+bx^3)^{3/4}}{325c^2\,(c+dx^3)^{13/12}}+\frac{189a^2x\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx=\frac{ax(a+bx^3)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

```
[In] Integrate[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{7}{4}}}{(d\,x^{3}+c)^{\frac{37}{12}}}dx$$

[In] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)

[Out] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)

Fricas [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="fricas")

[Out] integral((b*x^3 + a)^(7/4)*(d*x^3 + c)^(11/12)/(d^4*x^12 + 4*c*d^3*x^9 + 6*c^2*d^2*x^6 + 4*c^3*d*x^3 + c^4), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \text{Timed out}$$

[In] integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)

Giac [F]

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \int \frac{(b\,x^3+a)^{7/4}}{(d\,x^3+c)^{37/12}}\,dx$$

```
[In] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x)
[Out] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12), x)
```

3.123 $\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} \, dx = \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2(c+dx^3)^{7/12}} + \frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{133c^3(a+bx^3)^{3/4}}$$

```
[Out] 4/19*x*(b*x^3+a)^(5/4)/c/(d*x^3+c)^(19/12)+60/133*a*x*(b*x^3+a)^(1/4)/c^2/(
d*x^3+c)^(7/12)+45/133*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/1
2)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(3
/4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} \, dx = \frac{45a^2x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{133c^3(a+bx^3)^{3/4}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2(c+dx^3)^{7/12}} + \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

```
[In] Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (60*a*x*(a + b*x^3)^(1
/4))/(133*c^2*(c + d*x^3)^(7/12)) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(133*c^3*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}}+\frac{(15a)\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx}{19c}\\ &= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}}+\frac{(45a^2)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{133c^2}\\ &= \frac{4x(a+bx^3)^{5/4}}{19c\,(c+dx^3)^{19/12}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\,(c+dx^3)^{7/12}}\\ &\quad+\frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{133c^3\,(a+bx^3)^{3/4}}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx=\frac{ax\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{7/12}}$$

```
[In] Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c +
d*x^3)^(7/12))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{5}{4}}}{(d\,x^{3}+c)^{\frac{31}{12}}}dx$$

```
[In] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
[Out] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^
2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(b\,x^3+a)^{5/4}}{(d\,x^3+c)^{31/12}}\,dx$$

```
[In] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)
[Out] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)
```

3.124 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} \, dx = \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}} + \frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

```
[Out] 4/13*x*(b*x^3+a)^(3/4)/c/(d*x^3+c)^(13/12)+9/13*a*x*(c*(b*x^3+a)/a/(d*x^3+c
))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x^3
+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} \, dx = \frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}} + \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

```
[In] Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^
3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}} + \frac{(9a)\int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}}\,dx}{13c} \\
&= \frac{4x(a+bx^3)^{3/4}}{13c\,(c+dx^3)^{13/12}} + \frac{9ax\sqrt[4]{\frac{c\,(a+bx^3)}{a\,(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \frac{x(a+bx^3)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

```
[In] Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*x^
3)/c)^(1/4))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{3}{4}}}{(d\,x^{3}+c)^{\frac{25}{12}}}dx$$

```
[In] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
[Out] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(b\,x^3+a)^{3/4}}{(d\,x^3+c)^{25/12}}\,dx$$

```
[In] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x)
[Out] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12), x)
```

3.125 $\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}} + \frac{3ax\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{7c^2(a+bx^3)^{3/4}}$$

```
[Out] 4/7*x*(b*x^3+a)^(1/4)/c/(d*x^3+c)^(7/12)+3/7*a*x*(c*(b*x^3+a)/a/(d*x^3+c))^
(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+
c))/c^2/(b*x^3+a)^(3/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \frac{3ax(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{7c^2(a+bx^3)^{3/4}} + \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}}$$

```
[In] Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/
(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}+\frac{(3a)\int\frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{7c}\\ &= \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}+\frac{3ax\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 3.78 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.73

$$\int\frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx=\frac{x\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{1+\frac{bx^3}{a}}\left(c+dx^3\right)^{7/12}}$$

```
[In] Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*x^
3)^(7/12))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{4}}}{(d\,x^{3}+c)^{\frac{19}{12}}}dx$$

```
[In] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
[Out] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{\frac{19}{12}}}\,dx$$

```
[In] integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
[Out] Integral((a + b*x**3)**(1/4)/(c + d*x**3)**(19/12), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(b\,x^3+a)^{1/4}}{(d\,x^3+c)^{19/12}}\,dx$$

```
[In] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x)
[Out] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12), x)
```

3.126 $\int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^
3/a/(d*x^3+c))/c/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {388}

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

```
[In] Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3,
 -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^(1
/12))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
```

```
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Mathematica [A] (warning: unable to verify)

Time = 3.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \frac{x\sqrt[4]{1+\frac{bx^3}{a}}\left(1+\frac{dx^3}{c}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}$$

```
[In] Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3)^
(13/12))
```

Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{1}{4}}\,(d\,x^3+c)^{\frac{13}{12}}}dx$$

```
[In] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
[Out] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d^
2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{\frac{13}{12}}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
[Out] Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/4}\,(d\,x^3+c)^{13/12}}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x)
[Out] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)), x)
```

3.127 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{3/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/
3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(3/4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {388}

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \frac{x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\,(a+bx^3)^{3/4}}$$

```
[In] Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometri
c2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(
3/4))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{3/4}}$$

Mathematica [A] (warning: unable to verify)

Time = 5.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}$$

```
[In] Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3)^
(7/12))
```

Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{3}{4}}(d\,x^3+c)^{\frac{7}{12}}}dx$$

```
[In] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

```
[Out] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{4}}(c+dx^3)^{\frac{7}{12}}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
[Out] Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(b\,x^3+a)^{3/4}\,(d\,x^3+c)^{7/12}}\,dx$$

```
[In] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x)
[Out] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)), x)
```

3.128 $\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{5/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4
/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(5/4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {388}

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\,(a+bx^3)^{5/4}}$$

```
[In] Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometr
ic2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^
(5/4))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\,(a+bx^3)^{5/4}}$$

Mathematica [A] (warning: unable to verify)

Time = 3.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \frac{x\sqrt[4]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

```
[In] Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*x
^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3)/
c)^(1/4))
```

Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{5}{4}}\,(d\,x^3+c)^{\frac{1}{12}}}dx$$

```
[In] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

```
[Out] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b*
d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{4}}\sqrt[12]{c+dx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
[Out] Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/4}\,(d\,x^3+c)^{1/12}}\,dx$$

```
[In] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x)
[Out] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)), x)
```

3.129 $\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx = \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}} + \frac{5x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a(a+bx^3)^{3/4}}$$

```
[Out] 4/9*x*(d*x^3+c)^(5/12)/a/(b*x^3+a)^(3/4)+5/9*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3
/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c)
)/a/(b*x^3+a)^(3/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx = \frac{5x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

```
[In] Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a
*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(
((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4x(c+dx^3)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{(5c)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{9a}\\ &= \frac{4x(c+dx^3)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{5x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a\left(a+bx^3\right)^{3/4}}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\left(c+dx^3\right)^{5/12}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\left(a+bx^3\right)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

```
[In] Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3)/
c)^(3/4))
```

Maple [F]

$$\int \frac{\left(d\,x^{3}+c\right)^{\frac{5}{12}}}{\left(b\,x^{3}+a\right)^{\frac{7}{4}}}d x$$

```
[In] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
[Out] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx=\int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx=\int \frac{(c+dx^3)^{\frac{5}{12}}}{(a+bx^3)^{\frac{7}{4}}}\,dx$$

```
[In] integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
[Out] Integral((c + d*x**3)**(5/12)/(a + b*x**3)**(7/4), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx=\int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(d\,x^3+c)^{5/12}}{(b\,x^3+a)^{7/4}}\,dx$$

```
[In] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x)
[Out] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4), x)
```

3.130 $\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{4x(c+dx^3)^{11/12}}{15a\,(a+bx^3)^{5/4}} + \frac{11x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a\,(a+bx^3)^{5/4}}$$

```
[Out] 4/15*x*(d*x^3+c)^(11/12)/a/(b*x^3+a)^(5/4)+11/15*x*(c*(b*x^3+a)/a/(d*x^3+c)
)^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x
^3+c))/a/(b*x^3+a)^(5/4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{11x(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{15a\,(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{11/12}}{15a\,(a+bx^3)^{5/4}}$$

```
[In] Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}} + \frac{(11c)\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{15a}\\ &= \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}} + \frac{11x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a(a+bx^3)^{5/4}}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{x\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{9}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\sqrt[4]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

```
[In] Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*x^
3)/c)^(5/4))
```

Maple [F]

$$\int \frac{(d\,x^3+c)^{\frac{11}{12}}}{(b\,x^3+a)^{\frac{9}{4}}}dx$$

```
[In] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
[Out] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(d\,x^3+c)^{11/12}}{(b\,x^3+a)^{9/4}}\,dx$$

```
[In] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x)
[Out] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4), x)
```

3.131 $\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}} + \frac{85cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{189a^2(a+bx^3)^{3/4}}$$

```
[Out] 68/189*c*x*(d*x^3+c)^(5/12)/a^2/(b*x^3+a)^(3/4)+4/21*x*(d*x^3+c)^(17/12)/a/
(b*x^3+a)^(7/4)+85/189*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)
*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(3/4
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{85cx(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{189a^2(a+bx^3)^{3/4}} + \frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}}$$

```
[In] Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (68*c*x*(c + d*x^3)^(5/12))/(189*a^2*(a + b*x^3)^(3/4)) + (4*x*(c + d*x^3)^
(17/12))/(21*a*(a + b*x^3)^(7/4)) + (85*c*x*((c*(a + b*x^3))/(a*(c + d*x^3)
))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(189*a^2*(a + b*x^3)^(3/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} + \frac{(17c)\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx}{21a} \\
&= \frac{68cx(c+dx^3)^{5/12}}{189a^2\,(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} + \frac{(85c^2)\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx}{189a^2} \\
&= \frac{68cx(c+dx^3)^{5/12}}{189a^2\,(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a\,(a+bx^3)^{7/4}} \\
&\quad + \frac{85cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{189a^2\,(a+bx^3)^{3/4}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \frac{cx\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\,(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

```
[In] Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

[Out] (c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4, 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (d*x^3)/c)^(3/4))

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{17}{12}}}{(b\,x^{3}+a)^{\frac{11}{4}}}dx$$

[In] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)

[Out] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)

Fricas [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")

[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \text{Timed out}$$

[In] integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)

Giac [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(d\,x^3+c)^{17/12}}{(b\,x^3+a)^{11/4}}\,dx$$

```
[In] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)
[Out] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)
```

3.132 $\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}} dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}} dx = \frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}} + \frac{253cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{405a^2(a+bx^3)^{5/4}}$$

```
[Out] 92/405*c*x*(d*x^3+c)^(11/12)/a^2/(b*x^3+a)^(5/4)+4/27*x*(d*x^3+c)^(23/12)/a
/(b*x^3+a)^(9/4)+253/405*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/
12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(
5/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {386, 388}

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}} dx = \frac{253cx(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2(a+bx^3)^{5/4}} + \frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}}$$

```
[In] Int[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (92*c*x*(c + d*x^3)^(11/12))/(405*a^2*(a + b*x^3)^(5/4)) + (4*x*(c + d*x^3)
^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (253*c*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a
*d)*x^3)/(a*(c + d*x^3)))])/(405*a^2*(a + b*x^3)^(5/4))
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} + \frac{(23c)\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx}{27a} \\
&= \frac{92cx(c+dx^3)^{11/12}}{405a^2\,(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} + \frac{(253c^2)\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{405a^2} \\
&= \frac{92cx(c+dx^3)^{11/12}}{405a^2\,(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a\,(a+bx^3)^{9/4}} \\
&\quad + \frac{253cx\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{405a^2\,(a+bx^3)^{5/4}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \frac{cx\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^3\sqrt[4]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

```
[In] Integrate[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (c*x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 13/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^3*(a + b*x^3)^(1/4)*(1 + (d
*x^3)/c)^(5/4))
```

Maple [F]

$$\int \frac{(d\,x^3+c)^{\frac{23}{12}}}{(b\,x^3+a)^{\frac{13}{4}}}dx$$

```
[In] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

```
[Out] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(23/12)/(b^4*x^12 + 4*a*b^3*x^9 + 6*
a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(23/12)/(b*x**3+a)**(13/4),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(dx^3+c)^{\frac{23}{12}}}{(bx^3+a)^{\frac{13}{4}}}\,dx$$

[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="giac")

[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx = \int \frac{(d\,x^3+c)^{23/12}}{(b\,x^3+a)^{13/4}}\,dx$$

[In] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x)

[Out] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4), x)

3.133 $\int (a+bx^3)^m (c+dx^3)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

```
[Out] x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3,-m,-p,4/3,-b*x^3/a,-d*x^3/c)/((1+b*x
^3/a)^m)/((1+d*x^3/c)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} (c+dx^3)^p \left(\frac{dx^3}{c}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((
d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \left(1+\frac{bx^3}{a}\right)^m \left(c+dx^3\right)^p\,dx$$

$$= \left(\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(c+dx^3\right)^p \left(1+\frac{dx^3}{c}\right)^{-p}\right) \int \left(1+\frac{bx^3}{a}\right)^m \left(1+\frac{dx^3}{c}\right)^p dx$$

$$= x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(c+dx^3\right)^p \left(1+\frac{dx^3}{c}\right)^{-p} F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p\,dx$$

$$= \frac{4acx(a+bx^3)^m (c+dx^3)^p \operatorname{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac \operatorname{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(bcm \operatorname{AppellF1}\left(\frac{4}{3},1-m,-p,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adp \operatorname{Appell}\right.}$$

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)
/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{p}dx$$

```
[In] int((b*x^3+a)^m*(d*x^3+c)^p,x)
[Out] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \,dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \,dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \,dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \,dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Giac [F]

$$\int \left(a + bx^3\right)^m \left(c + dx^3\right)^p \, dx = \int \left(bx^3 + a\right)^m \left(dx^3 + c\right)^p \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + bx^3\right)^m \left(c + dx^3\right)^p \, dx = \int \left(b\,x^3 + a\right)^m \left(d\,x^3 + c\right)^p \, dx$$

```
[In] int((a + b*x^3)^m*(c + d*x^3)^p,x)
[Out] int((a + b*x^3)^m*(c + d*x^3)^p, x)
```

3.134 $\int (a+bx^3)^2 (c+dx^3)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int (a+bx^3)^2 (c+dx^3)^q \, dx = -\frac{b(4bc-ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2-2abcd(7+3q)+a^2d^2(28+33q+9q^2))\,x(c+dx^3)^{1+q}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+q,\frac{4}{3},-\frac{dx^3}{c}\right)}{cd^2(4+3q)(7+3q)}$$

```
[Out] -b*(4*b*c-a*d*(10+3*q))*x*(d*x^3+c)^(1+q)/d^2/(9*q^2+33*q+28)+b*x*(b*x^3+a)
*(d*x^3+c)^(1+q)/d/(7+3*q)+(4*b^2*c^2-2*a*b*c*d*(7+3*q)+a^2*d^2*(9*q^2+33*q
+28))*x*(d*x^3+c)^(1+q)*hypergeom([1, 4/3+q],[4/3],-d*x^3/c)/c/d^2/(9*q^2+3
3*q+28)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 396, 252, 251}

$$\int (a+bx^3)^2 (c+dx^3)^q \, dx$$

$$= \frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} (a^2d^2(9q^2+33q+28)-2abcd(3q+7)+4b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\right.}{d^2(3q+4)(3q+7)} - \frac{bx(c+dx^3)^{q+1}(4bc-ad(3q+10))}{d^2(3q+4)(3q+7)} + \frac{bx(a+bx^3)(c+dx^3)^{q+1}}{d(3q+7)}$$

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*
q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 -
2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*Hypergeo
metric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3
)/c)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{bx(a+bx^3)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\int \left(c+dx^3\right)^q \left(-a(bc-ad(7+3q))-b(4bc-ad(10+3q))x^3\right)\,dx}{d(7+3q)}$$

$$= -\frac{b(4bc - ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2 - 2abcd(7+3q) + a^2d^2(28+33q+9q^2))\int (c+dx^3)^q\,dx}{d^2(4+3q)(7+3q)}$$

$$= -\frac{b(4bc - ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)} + \frac{\left((4b^2c^2 - 2abcd(7+3q) + a^2d^2(28+33q+9q^2))(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \left(1+\frac{dx^3}{c}\right)^q dx}{d^2(4+3q)(7+3q)}$$

$$= -\frac{b(4bc - ad(10+3q))x(c+dx^3)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx(a+bx^3)(c+dx^3)^{1+q}}{d(7+3q)} + \frac{(4b^2c^2 - 2abcd(7+3q) + a^2d^2(28+33q+9q^2))\,x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)}{d^2(4+3q)(7+3q)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int (a+bx^3)^2 (c+dx^3)^q\,dx = \frac{1}{14}x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\left(14a^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right) + bx^3\left(7a\,\text{Hypergeometric2F1}\left(\frac{4}{3},-q,\frac{7}{3},-\frac{dx^3}{c}\right) + 2bx^3\,\text{Hypergeometric2F1}\left(\frac{7}{3},-q,\frac{10}{3},-\frac{dx^3}{c}\right)\right)\right)$$

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)] + b*
x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)] + 2*b*x^3*Hypergeome
tric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(1 + (d*x^3)/c)^q)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{2} \left(d\,x^{3}+c\right)^{q}\,dx$$

[In] int((b*x^3+a)^2*(d*x^3+c)^q,x)

[Out] int((b*x^3+a)^2*(d*x^3+c)^q,x)

Fricas [F]

$$\int \left(a+bx^{3}\right)^{2}\left(c+dx^{3}\right)^{q}\,dx=\int \left(bx^{3}+a\right)^{2}\left(dx^{3}+c\right)^{q}\,dx$$

[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="fricas")

[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 90.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \left(a+bx^{3}\right)^{2}\left(c+dx^{3}\right)^{q}\,dx=\frac{a^{2}c^{q}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},-q\\ \frac{4}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2abc^{q}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},-q\\ \frac{7}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^{2}c^{q}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{3},-q\\ \frac{10}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

[In] integrate((b*x**3+a)**2*(d*x**3+c)**q,x)

[Out] a**2*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(4/3)) + 2*a*b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(7/3)) + b**2*c**q*x**7*gamma(7/3)*hyper((7/3, -q), (10/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(10/3))

Maxima [F]

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^2 \left(dx^3+c\right)^q \, dx$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(bx^3+a\right)^2 \left(dx^3+c\right)^q \, dx$$

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx = \int \left(b\,x^3+a\right)^2 \left(d\,x^3+c\right)^q \, dx$$

```
[In] int((a + b*x^3)^2*(c + d*x^3)^q,x)
[Out] int((a + b*x^3)^2*(c + d*x^3)^q, x)
```

3.135 $\int (a+bx^3)(c+dx^3)^q\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int (a+bx^3)(c+dx^3)^q\,dx = \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \frac{(bc-ad(4+3q))x(c+dx^3)^{1+q}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+q,\frac{4}{3},-\frac{dx^3}{c}\right)}{cd(4+3q)}$$

```
[Out] b*x*(d*x^3+c)^(1+q)/d/(4+3*q)-(b*c-a*d*(4+3*q))*x*(d*x^3+c)^(1+q)*hypergeom
([1, 4/3+q],[4/3],-d*x^3/c)/c/d/(4+3*q)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {396, 252, 251}

$$\int (a+bx^3)(c+dx^3)^q\,dx = x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right) + \frac{bx(c+dx^3)^{q+1}}{d(3q+4)}$$

```
[In] Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*q))*x*(c +
 d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(1 + (d*x^3)/c)^q
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(-a+\frac{bc}{4d+3dq}\right)\int \left(c+dx^3\right)^q\,dx\\
&= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(\left(-a+\frac{bc}{4d+3dq}\right)\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \left(1+\frac{dx^3}{c}\right)^q\,dx\\
&= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} + \left(a-\frac{bc}{4d+3dq}\right)x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx$$
$$=\frac{x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\left(b(c+dx^3)\left(1+\frac{dx^3}{c}\right)^q+(-bc+ad(4+3q))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-q,\frac{4}{3},-\frac{dx^3}{c}\right)\right)}{d(4+3q)}$$

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(b*(c + d*x^3)*(1 + (d*x^3)/c)^q + (-(b*c) + a*d*(4 + 3*q)
)*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]))/(d*(4 + 3*q)*(1 + (d*x^3)
/c)^q)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)\left(d\,x^{3}+c\right)^{q}dx$$

```
[In] int((b*x^3+a)*(d*x^3+c)^q,x)
[Out] int((b*x^3+a)*(d*x^3+c)^q,x)
```

Fricas [F]

$$\int \left(a+bx^{3}\right)\left(c+dx^{3}\right)^{q}\,dx=\int \left(bx^{3}+a\right)\left(dx^{3}+c\right)^{q}dx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="fricas")
[Out] integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \left(a+bx^{3}\right)\left(c+dx^{3}\right)^{q}\,dx=\frac{ac^{q}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},-q\\\frac{4}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{bc^{q}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},-q\\\frac{7}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x**3+a)*(d*x**3+c)**q,x)
[Out] a*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*g
amma(4/3)) + b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_pol
ar(I*pi)/c)/(3*gamma(7/3))
```

Maxima [F]

$$\int (a+bx^3)(c+dx^3)^q\,dx = \int (bx^3+a)(dx^3+c)^q\,dx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Giac [F]

$$\int (a+bx^3)(c+dx^3)^q\,dx = \int (bx^3+a)(dx^3+c)^q\,dx$$

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="giac")
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)(c+dx^3)^q\,dx = \int (b\,x^3+a)\,(d\,x^3+c)^q\,dx$$

```
[In] int((a + b*x^3)*(c + d*x^3)^q,x)
[Out] int((a + b*x^3)*(c + d*x^3)^q, x)
```

3.136 $\int \frac{(c+dx^3)^q}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx=\frac{x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{3},1,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,1,-q,4/3,-b*x^3/a,-d*x^3/c)/a/((1+d*x^3/c)^q)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx=\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{3},1,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[In] Int[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*
(1 + (d*x^3)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^q}{a+bx^3}\,dx\\ &= \frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx$$
$$= \frac{4acx(c+dx^3)^q \operatorname{AppellF1}\left(\frac{1}{3},-q,1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(4ac\operatorname{AppellF1}\left(\frac{1}{3},-q,1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(adq\operatorname{AppellF1}\left(\frac{4}{3},1-q,1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bc\,\mathrm{Ap}\right.\right.}$$

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)] -
 b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [F]

$$\int \frac{(d\,x^3+c)^q}{b\,x^3+a}dx$$

```
[In] int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**q/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \int \frac{(dx^3+c)^q}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{a+bx^3}\,dx = \int \frac{(d\,x^3+c)^q}{b\,x^3+a}\,dx$$

```
[In] int((c + d*x^3)^q/(a + b*x^3),x)
[Out] int((c + d*x^3)^q/(a + b*x^3), x)
```

3.137 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx = \frac{x(c+dx^3)^q \left(1+\frac{dx^3}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,2,-q,4/3,-b*x^3/a,-d*x^3/c)/a^2/((1+d*x^3/c)^q)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} \, dx = \frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[In] Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^
2*(1 + (d*x^3)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int\frac{\left(1+\frac{dx^3}{c}\right)^q}{\left(a+bx^3\right)^2}\,dx$$

$$=\frac{x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{\left(c+dx^3\right)^q}{\left(a+bx^3\right)^2}\,dx$$

$$=\frac{4acx\left(c+dx^3\right)^q\,\text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^2\left(4ac\,\text{AppellF1}\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(adq\,\text{AppellF1}\left(\frac{4}{3},2,1-q,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bc\ldots\right.\right.}$$

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int\frac{\left(d\,x^3+c\right)^q}{\left(b\,x^3+a\right)^2}dx$$

```
[In] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2} dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="fricas")
[Out] integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2} dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx = \int \frac{(dx^3+c)^q}{(bx^3+a)^2} dx$$

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^q}{(b\,x^3+a)^2}\,dx$$

```
[In] int((c + d*x^3)^q/(a + b*x^3)^2,x)
```

```
[Out] int((c + d*x^3)^q/(a + b*x^3)^2, x)
```

3.138 $\int (a+bx^3)^m (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 298

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx = \frac{d(28a^2d^2-abcd(92+15m)+b^2c^2(118+60m+9m^2))\,x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)} - \frac{(28a^3d^3-12a^2bcd^2(10+3m)+3ab^2c^2d(70+51m+9m^2)-b^3c^3(280+414m+189m^2+27m^3))\,x(a+}{b^3(4+3m)(7+3m)(10+3m)}$$

```
[Out] d*(28*a^2*d^2-a*b*c*d*(92+15*m)+b^2*c^2*(9*m^2+60*m+118))*x*(b*x^3+a)^(1+m)
/b^3/(10+3*m)/(9*m^2+33*m+28)-d*(7*a*d-b*c*(16+3*m))*x*(b*x^3+a)^(1+m)*(d*x
^3+c)/b^2/(9*m^2+51*m+70)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)^2/b/(10+3*m)-(28*a^
3*d^3-12*a^2*b*c*d^2*(10+3*m)+3*a*b^2*c^2*d*(9*m^2+51*m+70)-b^3*c^3*(27*m^3
+189*m^2+414*m+280))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^3/
(10+3*m)/(9*m^2+33*m+28)/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{427, 542, 396, 252, 251\}$

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx$$

$$= \frac{dx(a+bx^3)^{m+1} \left(28a^2d^2 - abcd(15m+92) + b^2c^2(9m^2+60m+118)\right)}{b^3(3m+4)(3m+7)(3m+10)}$$

$$- \frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(28a^3d^3 - 12a^2bcd^2(3m+10) + 3ab^2c^2d(9m^2+51m+70) - b^3c^3(27m^3+18\right.}{b^3(3m+4)(3m+7)(3m+10)}$$

$$- \frac{dx(c+dx^3)\left(a+bx^3\right)^{m+1} (7ad - bc(3m+16))}{b^2(3m+7)(3m+10)} + \frac{dx(c+dx^3)^2 \left(a+bx^3\right)^{m+1}}{b(3m+10)}$$

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a +
 b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16
 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d
*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^
2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 4
14*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -
((b*x^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)} + \frac{\int (a+bx^3)^m (c+dx^3)\left(-c(ad-bc(10+3m))-d(7ad-bc(16+3m))x^3\right)\,dx}{b(10+3m)}$$

$$= -\frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)} + \frac{\int (a+bx^3)^m \left(c(7a^2d^2-abcd(23+6m)+b^2c^2(70+51m+9m^2))+d(28a^2d^2-abcd(92+15m)\right.}{b^2(7+3m)(10+3m)}$$

$$= \frac{d(28a^2d^2-abcd(92+15m)+b^2c^2(118+60m+9m^2))x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)} - \frac{(28a^3d^3-12a^2bcd^2(10+3m)+3ab^2c^2d(70+51m+9m^2)-b^3c^3(280+414m+189m^2+27m^3}{b^3(4+3m)(7+3m)(10+3m)}$$

$$= \frac{d(28a^2d^2-abcd(92+15m)+b^2c^2(118+60m+9m^2))x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)} - \frac{\left((28a^3d^3-12a^2bcd^2(10+3m)+3ab^2c^2d(70+51m+9m^2)-b^3c^3(280+414m+189m^2+27m\right.}{b^3(4+3m)(7+3m)(10+3m)}$$

$$= \frac{d(28a^2d^2 - abcd(92+15m) + b^2c^2(118+60m+9m^2))\,x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)}$$
$$- \frac{d(7ad - bc(16+3m))x(a+bx^3)^{1+m}(c+dx^3)}{b^2(7+3m)(10+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)^2}{b(10+3m)}$$
$$- \frac{(28a^3d^3 - 12a^2bcd^2(10+3m) + 3ab^2c^2d(70+51m+9m^2) - b^3c^3(280+414m+189m^2+27m}{b^3(4+3m)(7+3m)(10+3m)}$$

Mathematica [A] (verified)

Time = 8.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.46

$$\int (a+bx^3)^m (c+dx^3)^3\,dx$$
$$= \frac{1}{140}x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(140c^3\,\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)\right.$$
$$+ dx^3\left(105c^2\,\text{Hypergeometric2F1}\left(\frac{4}{3},-m,\frac{7}{3},-\frac{bx^3}{a}\right)\right.$$
$$+2dx^3\left(30c\,\text{Hypergeometric2F1}\left(\frac{7}{3},-m,\frac{10}{3},-\frac{bx^3}{a}\right)+7dx^3\,\text{Hypergeometric2F1}\left(\frac{10}{3},-m,\frac{13}{3},-\frac{bx^3}{a}\right)\right)$$

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30*c
*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric2F1
[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

Maple [F]

$$\int (b\,x^3+a)^m\,(d\,x^3+c)^3\,dx$$

```
[In] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

Fricas [F]

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx = \int (dx^3+c)^3 (bx^3+a)^m \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx = \int (dx^3+c)^3 (bx^3+a)^m \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int (a+bx^3)^m (c+dx^3)^3 \, dx = \int (dx^3+c)^3 (bx^3+a)^m \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^3 dx$$

```
[In] int((a + b*x^3)^m*(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^m*(c + d*x^3)^3, x)
```

3.139 $\int (a+bx^3)^m (c+dx^3)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^3)^m (c+dx^3)^2\,dx$$
$$= -\frac{d(4ad-bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)}$$
$$+ \frac{(4a^2d^2-2abcd(7+3m)+b^2c^2(28+33m+9m^2))\,x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{3},-m\right.}{b^2(4+3m)(7+3m)}$$

```
[Out] -d*(4*a*d-b*c*(10+3*m))*x*(b*x^3+a)^(1+m)/b^2/(9*m^2+33*m+28)+d*x*(b*x^3+a)
^(1+m)*(d*x^3+c)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33*m
+28))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^2/(9*m^2+33*m+28)
/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 396, 252, 251}

$$\int (a+bx^3)^m (c+dx^3)^2\,dx$$
$$= \frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}(4a^2d^2-2abcd(3m+7)+b^2c^2(9m^2+33m+28))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\right.}{b^2(3m+4)(3m+7)}$$
$$-\frac{dx(a+bx^3)^{m+1}(4ad-bc(3m+10))}{b^2(3m+4)(3m+7)} + \frac{dx(c+dx^3)(a+bx^3)^{m+1}}{b(3m+7)}$$

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*
m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 -
2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeo
metric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3
)/a)^m)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)} + \frac{\int (a+bx^3)^m(-c(ad-bc(7+3m))-d(4ad-bc(10+3m))x^3)\,dx}{b(7+3m)}$$

$$= -\frac{d(4ad - bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)} + \frac{(4a^2d^2 - 2abcd(7+3m) + b^2c^2(28+33m+9m^2)) \int (a+bx^3)^m \, dx}{b^2(4+3m)(7+3m)}$$

$$= -\frac{d(4ad - bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)} + \frac{\left((4a^2d^2 - 2abcd(7+3m) + b^2c^2(28+33m+9m^2))(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \left(1+\frac{bx^3}{a}\right)^m dx}{b^2(4+3m)(7+3m)}$$

$$= -\frac{d(4ad - bc(10+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx(a+bx^3)^{1+m}(c+dx^3)}{b(7+3m)} + \frac{(4a^2d^2 - 2abcd(7+3m) + b^2c^2(28+33m+9m^2))\, x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right.}{b^2(4+3m)(7+3m)}$$

Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \frac{1}{14}x(a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \left(14c^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, -m, \frac{4}{3}, -\frac{bx^3}{a}\right) + dx^3\left(7c \operatorname{Hypergeometric2F1}\left(\frac{4}{3}, -m, \frac{7}{3}, -\frac{bx^3}{a}\right) + 2dx^3 \operatorname{Hypergeometric2F1}\left(\frac{7}{3}, -m, \frac{10}{3}, -\frac{bx^3}{a}\right)\right)\right)$$

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d*
x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeome
tric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{2}dx$$

[In] int((b*x^3+a)^m*(d*x^3+c)^2,x)

[Out] int((b*x^3+a)^m*(d*x^3+c)^2,x)

Fricas [F]

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \int (dx^3+c)^2(bx^3+a)^m \, dx$$

[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")

[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 89.96 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \frac{a^m c^2 x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^m cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^m d^2 x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{7}{3}, -m \\ \frac{10}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

[In] integrate((b*x**3+a)**m*(d*x**3+c)**2,x)

[Out] a**m*c**2*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a**m*c*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**m*d**2*x**7*gamma(7/3)*hyper((7/3, -m), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(dx^3+c\right)^2 \left(bx^3+a\right)^m \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(dx^3+c\right)^2 \left(bx^3+a\right)^m \, dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^2 \, dx$$

```
[In] int((a + b*x^3)^m*(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^m*(c + d*x^3)^2, x)
```

3.140 $\int (a+bx^3)^m (c+dx^3)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^3)^m (c+dx^3)\,dx = \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \frac{(ad-bc(4+3m))x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)}{b(4+3m)}$$

```
[Out] d*x*(b*x^3+a)^(1+m)/b/(4+3*m)-(a*d-b*c*(4+3*m))*x*(b*x^3+a)^m*hypergeom([1/
3, -m],[4/3],-b*x^3/a)/b/(4+3*m)/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$\int (a+bx^3)^m (c+dx^3)\,dx = x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c - \frac{ad}{3bm+4b}\right) \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right) + \frac{dx(a+bx^3)^{m+1}}{b(3m+4)}$$

```
[In] Int[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a +
 b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^m
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \left(-c+\frac{ad}{4b+3bm}\right)\int \left(a+bx^3\right)^m\,dx \\
&= \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} - \left(\left(-c+\frac{ad}{4b+3bm}\right)\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m\,dx \\
&= \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} + \left(c-\frac{ad}{4b+3bm}\right)x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$
\int \left(a+bx^3\right)^m\left(c+dx^3\right)\,dx
= \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^m+(-ad+bc(4+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\right.\right.}{b(4+3m)}
$$

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m)
)*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^3)
/a)^m)
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)dx$$

```
[In] int((b*x^3+a)^m*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx = \frac{a^m cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^m dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x**3+a)**m*(d*x**3+c),x)
```

```
[Out] a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^m\,\left(d\,x^3+c\right)\,dx$$

```
[In] int((a + b*x^3)^m*(c + d*x^3),x)
[Out] int((a + b*x^3)^m*(c + d*x^3), x)
```

3.141 $\int (a+bx^3)^m \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^3)^m \, dx = x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)$$

```
[Out] x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$\int (a+bx^3)^m \, dx = x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)$$

```
[In] Int[(a + b*x^3)^m,x]
```

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3
)/a)^m
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int\left(1+\frac{bx^3}{a}\right)^m dx$$

$$= x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.61

$$\int\left(a+bx^3\right)^m dx$$

$$= \frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-m}\left(a+bx^3\right)^m\text{AppellF1}\left(1+m,-m,-m,\right.}{\sqrt[3]{b}(1+m)}$$

```
[In] Integrate[(a + b*x^3)^m,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2 +
 m, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/
3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^m*b^(1
/3)*(1 + m)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^m
*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^m)
```

Maple [F]

$$\int\left(b\,x^3+a\right)^m dx$$

```
[In] int((b*x^3+a)^m,x)
```

```
[Out] int((b*x^3+a)^m,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m,x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^3\right)^m\,dx=\frac{a^m x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},-m\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b*x**3+a)**m,x)
[Out] a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^m\,dx=\int \left(bx^3+a\right)^m\,dx$$

```
[In] integrate((b*x^3+a)^m,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^3)^m \, dx = \frac{x\,(b\,x^3+a)^m\,{}_2\mathrm{F}_1\left(\frac{1}{3},-m;\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^m}$$

```
[In] int((a + b*x^3)^m,x)
[Out] (x*(a + b*x^3)^m*hypergeom([1/3, -m], 4/3, -(b*x^3)/a))/((b*x^3)/a + 1)^m
```

3.142 $\int \frac{(a+bx^3)^m}{c+dx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{c+dx^3} dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^m}{c+dx^3} dx = \frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[In] Int[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*
(1 + (b*x^3)/a)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^m}{c+dx^3}\,dx\\ &= \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(a+bx^3)^m}{c+dx^3}\,dx =$$

$$-\frac{4acx(a+bx^3)^m\,\text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(-bcm\,\text{AppellF1}\left(\frac{4}{3},1-m,1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.\right.}$$

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int\frac{(b\,x^3+a)^m}{d\,x^3+c}dx$$

```
[In] int((b*x^3+a)^m/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx=\int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**3+a)**m/(d*x**3+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx=\int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx=\int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^m}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^m/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3), x)
```

3.143 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx = \frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
2*(1 + (b*x^3)/a)^m)
```

<u>Rule 440</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^m}{(c+dx^3)^2}\,dx\\ &= \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = -\frac{4acx(a+bx^3)^m\,\text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^2\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcm\,\text{AppellF1}\left(\frac{4}{3},1-m,2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-\right.\right.}$$

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int\frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}dx$$

```
[In] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2} \, dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2} \, dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2} \, dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}\,dx$$

```
[In] int((a + b*x^3)^m/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

3.144 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/((1+b*x^3/a)^m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
3*(1 + (b*x^3)/a)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^m}{(c+dx^3)^3}\,dx$$

$$=\frac{x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.55 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx =$$

$$-\frac{4acx(a+bx^3)^m\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^3\left(-4ac\operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(bcm\operatorname{AppellF1}\left(\frac{4}{3},1-m,3,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-\right.\right.}$$

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int\frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}dx$$

```
[In] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
[Out] integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}\,dx$$

```
[In] int((a + b*x^3)^m/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

3.145 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx$

Optimal result

Integrand size = 50, antiderivative size = 53

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[Out] x*(d*x^3+c)^(a*d/(-3*a*d+3*b*c))/a/c/((b*x^3+a)^(b*c/(-3*a*d+3*b*c)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {389}

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[In] Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c
 - 3*a*d)),x]
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\text{integral} = \frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{x(a+bx^3)^{\frac{bc}{-3bc+3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

[In] Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c - 3*a*d)),x]

[Out] (x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c)

Maple [A] (verified)

Time = 7.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
gosper	$\frac{x\left(b\,x^3+a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}}\left(d\,x^3+c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$	71

[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x,method=_RETURNVERBOSE)

[Out] x/a/c*(b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*b*c)/(a*d-b*c))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{bdx^7+(bc+ad)x^4+acx}{(bx^3+a)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}(dx^3+c)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x, algorithm="fricas")

[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c - a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3
*b*c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx = \int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx = \int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx = \frac{x\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}+\frac{x^4\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\,\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^7\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{\left(d\,x^3+c\right)^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

```
[In] int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*d
 - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d - 3*
b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

3.146 $\int (a+bx^4)(c+dx^4)^4\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^4)(c+dx^4)^4\,dx = ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}+\frac{1}{21}bd^4x^{21}$$

```
[Out] a*c^4*x+1/5*c^3*(4*a*d+b*c)*x^5+2/9*c^2*d*(3*a*d+2*b*c)*x^9+2/13*c*d^2*(2*a
*d+3*b*c)*x^13+1/17*d^3*(a*d+4*b*c)*x^17+1/21*b*d^4*x^21
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^4)(c+dx^4)^4\,dx = \frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

```
[In] Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^4 + c^3(bc+4ad)x^4 + 2c^2d(2bc+3ad)x^8 + 2cd^2(3bc+2ad)x^{12} + d^3(4bc+ad)x^{16} + bd^4x^{20}\right) dx \\ &= ac^4x + \frac{1}{5}c^3(bc+4ad)x^5 + \frac{2}{9}c^2d(2bc+3ad)x^9 + \frac{2}{13}cd^2(3bc+2ad)x^{13} + \frac{1}{17}d^3(4bc+ad)x^{17} + \frac{1}{21}bd^4x^{21}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^4\, dx = ac^4x + \frac{1}{5}c^3(bc+4ad)x^5 + \frac{2}{9}c^2d(2bc+3ad)x^9 + \frac{2}{13}cd^2(3bc+2ad)x^{13} + \frac{1}{17}d^3(4bc+ad)x^{17} + \frac{1}{21}bd^4x^{21}$$

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$a\,c^4x + \left(\frac{4}{5}a\,c^3d + \frac{1}{5}b\,c^4\right)x^5 + \left(\frac{2}{3}a\,c^2d^2 + \frac{4}{9}b\,c^3d\right)x^9 + \left(\frac{4}{13}ac\,d^3 + \frac{6}{13}b\,c^2d^2\right)x^{13} + \left(\frac{1}{17}a\,d^4 + \frac{4}{17}b\right.$
default	$\frac{b\,d^4x^{21}}{21} + \frac{(a\,d^4+4bc\,d^3)x^{17}}{17} + \frac{(4ac\,d^3+6b\,c^2d^2)x^{13}}{13} + \frac{(6a\,c^2d^2+4b\,c^3d)x^9}{9} + \frac{(4a\,c^3d+b\,c^4)x^5}{5} + a\,c^4x$
gosper	$a\,c^4x + \frac{4}{5}x^5a\,c^3d + \frac{1}{5}x^5b\,c^4 + \frac{2}{3}x^9a\,c^2d^2 + \frac{4}{9}x^9b\,c^3d + \frac{4}{13}x^{13}ac\,d^3 + \frac{6}{13}x^{13}b\,c^2d^2 + \frac{1}{17}x^{17}a\,d^4 +$
risch	$a\,c^4x + \frac{4}{5}x^5a\,c^3d + \frac{1}{5}x^5b\,c^4 + \frac{2}{3}x^9a\,c^2d^2 + \frac{4}{9}x^9b\,c^3d + \frac{4}{13}x^{13}ac\,d^3 + \frac{6}{13}x^{13}b\,c^2d^2 + \frac{1}{17}x^{17}a\,d^4 +$
parallelrisch	$a\,c^4x + \frac{4}{5}x^5a\,c^3d + \frac{1}{5}x^5b\,c^4 + \frac{2}{3}x^9a\,c^2d^2 + \frac{4}{9}x^9b\,c^3d + \frac{4}{13}x^{13}ac\,d^3 + \frac{6}{13}x^{13}b\,c^2d^2 + \frac{1}{17}x^{17}a\,d^4 +$

```
[In] int((b*x^4+a)*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
[Out] a*c^4*x+(4/5*a*c^3*d+1/5*b*c^4)*x^5+(2/3*a*c^2*d^2+4/9*b*c^3*d)*x^9+(4/13*a
*c*d^3+6/13*b*c^2*d^2)*x^13+(1/17*a*d^4+4/17*b*c*d^3)*x^17+1/21*b*d^4*x^21
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)\,(c+dx^4)^4\,dx = \frac{1}{21}\,bd^4x^{21} + \frac{1}{17}\,(4\,bcd^3 + ad^4)x^{17} + \frac{2}{13}\,(3\,bc^2d^2 + 2\,acd^3)x^{13} + \frac{2}{9}\,(2\,bc^3d + 3\,ac^2d^2)x^9 + ac^4x + \frac{1}{5}\,(bc^4 + 4\,ac^3d)x^5$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int (a+bx^4)\,(c+dx^4)^4\,dx = ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\cdot\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\cdot\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\cdot\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

```
[In] integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a
*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x*
*5*(4*a*c**3*d/5 + b*c**4/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)\,(c+dx^4)^4\,dx = \frac{1}{21}\,bd^4x^{21} + \frac{1}{17}\,(4\,bcd^3 + ad^4)x^{17} + \frac{2}{13}\,(3\,bc^2d^2 + 2\,acd^3)x^{13} + \frac{2}{9}\,(2\,bc^3d + 3\,ac^2d^2)x^9 + ac^4x + \frac{1}{5}\,(bc^4 + 4\,ac^3d)x^5$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx=\frac{1}{21}\,bd^4x^{21}+\frac{4}{17}\,bcd^3x^{17}+\frac{1}{17}\,ad^4x^{17}+\frac{6}{13}\,bc^2d^2x^{13}+\frac{4}{13}\,acd^3x^{13}+\frac{4}{9}\,bc^3dx^9+\frac{2}{3}\,ac^2d^2x^9+\frac{1}{5}\,bc^4x^5+\frac{4}{5}\,ac^3dx^5+ac^4x$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13
 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5
+ 4/5*a*c^3*d*x^5 + a*c^4*x
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx=x^5\left(\frac{b\,c^4}{5}+\frac{4\,a\,d\,c^3}{5}\right)+x^{17}\left(\frac{a\,d^4}{17}+\frac{4\,b\,c\,d^3}{17}\right)+\frac{b\,d^4\,x^{21}}{21}+a\,c^4\,x+\frac{2\,c^2\,d\,x^9\,\left(3\,a\,d+2\,b\,c\right)}{9}+\frac{2\,c\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

```
[In] int((a + b*x^4)*(c + d*x^4)^4,x)
```

```
[Out] x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*d
^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(2*
a*d + 3*b*c))/13
```

3.147 $\int (a+bx^4)(c+dx^4)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^4)\left(c+dx^4\right)^3\,dx = ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}$$

```
[Out] a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)*
x^13+1/17*b*d^3*x^17
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^4)\left(c+dx^4\right)^3\,dx = \frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

```
[In] Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^3+c^2(bc+3ad)x^4+3cd(bc+ad)x^8+d^2(3bc+ad)x^{12}+bd^3x^{16}\right)\,dx\\ &= ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx = ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}$$

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
norman	$a\,c^3x+\left(\frac{3}{5}a\,c^2d+\frac{1}{5}c^3b\right)x^5+\left(\frac{1}{3}ac\,d^2+\frac{1}{3}b\,c^2d\right)x^9+\left(\frac{1}{13}a\,d^3+\frac{3}{13}bc\,d^2\right)x^{13}+\frac{b\,d^3x^{17}}{17}$	72
default	$\frac{b\,d^3x^{17}}{17}+\frac{(a\,d^3+3bc\,d^2)x^{13}}{13}+\frac{(3ac\,d^2+3b\,c^2d)x^9}{9}+\frac{(3a\,c^2d+c^3b)x^5}{5}+a\,c^3x$	73
gosper	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75
risch	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75
parallelrisch	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75

```
[In] int((b*x^4+a)*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^3*x+(3/5*a*c^2*d+1/5*c^3*b)*x^5+(1/3*a*c*d^2+1/3*b*c^2*d)*x^9+(1/13*a*d
^3+3/13*b*c*d^2)*x^13+1/17*b*d^3*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3\,dx=\frac{1}{17}bd^3x^{17}+\frac{1}{13}(3\,bcd^2+ad^3)x^{13}+\frac{1}{3}(bc^2d+acd^2)x^9+\frac{1}{5}(bc^3+3\,ac^2d)x^5+ac^3x$$

[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")

[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int (a+bx^4)(c+dx^4)^3\,dx=ac^3x+\frac{bd^3x^{17}}{17}+x^{13}\left(\frac{ad^3}{13}+\frac{3bcd^2}{13}\right)+x^9\left(\frac{acd^2}{3}+\frac{bc^2d}{3}\right)+x^5\cdot\left(\frac{3ac^2d}{5}+\frac{bc^3}{5}\right)$$

[In] integrate((b*x**4+a)*(d*x**4+c)**3,x)

[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c*d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3\,dx=\frac{1}{17}bd^3x^{17}+\frac{1}{13}(3\,bcd^2+ad^3)x^{13}+\frac{1}{3}(bc^2d+acd^2)x^9+\frac{1}{5}(bc^3+3\,ac^2d)x^5+ac^3x$$

[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")

[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^4) (c+dx^4)^3 \, dx = \frac{1}{17} bd^3x^{17} + \frac{3}{13} bcd^2x^{13} + \frac{1}{13} ad^3x^{13} + \frac{1}{3} bc^2dx^9 + \frac{1}{3} acd^2x^9 + \frac{1}{5} bc^3x^5 + \frac{3}{5} ac^2dx^5 + ac^3x$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 + 1
/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^4) (c+dx^4)^3 \, dx = x^5 \left(\frac{b\,c^3}{5} + \frac{3\,a\,d\,c^2}{5}\right) + x^{13} \left(\frac{a\,d^3}{13} + \frac{3\,b\,c\,d^2}{13}\right) + \frac{b\,d^3\,x^{17}}{17} + a\,c^3\,x + \frac{c\,d\,x^9\,(a\,d+b\,c)}{3}$$

```
[In] int((a + b*x^4)*(c + d*x^4)^3,x)
```

```
[Out] x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*d
^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

3.148 $\int (a+bx^4)(c+dx^4)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)(c+dx^4)^2\,dx = ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}$$

```
[Out] a*c^2*x+1/5*c*(2*a*d+b*c)*x^5+1/9*d*(a*d+2*b*c)*x^9+1/13*b*d^2*x^13
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^4)(c+dx^4)^2\,dx = \frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

```
[In] Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^2+c(bc+2ad)x^4+d(2bc+ad)x^8+bd^2x^{12}\right)\,dx\\ &= ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)\,(c+dx^4)^2\,dx = ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}$$

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^2x^{13}}{13}+\frac{(a\,d^2+2bcd)x^9}{9}+\frac{(2acd+b\,c^2)x^5}{5}+a\,c^2x$	49
norman	$\frac{b\,d^2x^{13}}{13}+\left(\frac{1}{9}a\,d^2+\frac{2}{9}bcd\right)x^9+\left(\frac{2}{5}acd+\frac{1}{5}b\,c^2\right)x^5+a\,c^2x$	49
gosper	$\frac{1}{13}b\,d^2x^{13}+\frac{1}{9}x^9a\,d^2+\frac{2}{9}x^9bcd+\frac{2}{5}x^5acd+\frac{1}{5}x^5b\,c^2+a\,c^2x$	51
risch	$\frac{1}{13}b\,d^2x^{13}+\frac{1}{9}x^9a\,d^2+\frac{2}{9}x^9bcd+\frac{2}{5}x^5acd+\frac{1}{5}x^5b\,c^2+a\,c^2x$	51
parallelrisch	$\frac{1}{13}b\,d^2x^{13}+\frac{1}{9}x^9a\,d^2+\frac{2}{9}x^9bcd+\frac{2}{5}x^5acd+\frac{1}{5}x^5b\,c^2+a\,c^2x$	51

```
[In] int((b*x^4+a)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)\,(c+dx^4)^2\,dx = \frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\,(2\,bcd+ad^2)x^9+\frac{1}{5}\,(bc^2+2\,acd)x^5+ac^2x$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right) \left(c+dx^4\right)^2 \, dx = ac^2x + \frac{bd^2x^{13}}{13} + x^9\left(\frac{ad^2}{9} + \frac{2bcd}{9}\right) + x^5 \cdot \left(\frac{2acd}{5} + \frac{bc^2}{5}\right)$$

```
[In] integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
+ b*c**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right) \left(c+dx^4\right)^2 \, dx = \frac{1}{13}\,bd^2x^{13} + \frac{1}{9}\left(2\,bcd + ad^2\right)x^9 + \frac{1}{5}\left(bc^2 + 2\,acd\right)x^5 + ac^2x$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right) \left(c+dx^4\right)^2 \, dx = \frac{1}{13}\,bd^2x^{13} + \frac{2}{9}\,bcdx^9 + \frac{1}{9}\,ad^2x^9 + \frac{1}{5}\,bc^2x^5 + \frac{2}{5}\,acdx^5 + ac^2x$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d
*x^5 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4) (c+dx^4)^2 \, dx = x^5 \left(\frac{b\,c^2}{5}+\frac{2\,a\,d\,c}{5}\right)+x^9 \left(\frac{a\,d^2}{9}+\frac{2\,b\,c\,d}{9}\right)+\frac{b\,d^2\,x^{13}}{13}+a\,c^2\,x$$

```
[In] int((a + b*x^4)*(c + d*x^4)^2,x)
```

```
[Out] x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^13
)/13 + a*c^2*x
```

3.149 $\int (a+bx^4)(c+dx^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^4)(c+dx^4)\,dx = acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

```
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

```
[In] Int[(a + b*x^4)*(c + d*x^4),x]
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac+(bc+ad)x^4+bdx^8\right)dx\\ &= acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

```
[In] Integrate[(a + b*x^4)*(c + d*x^4),x]
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^5}{5}+\frac{bd\,x^9}{9}$	25
norman	$\frac{bd\,x^9}{9}+\left(\frac{ad}{5}+\frac{bc}{5}\right)x^5+acx$	26
gosper	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
risch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
parallelrisch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27

```
[In] int((b*x^4+a)*(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=acx+\frac{bdx^9}{9}+x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

```
[In] integrate((b*x**4+a)*(d*x**4+c),x)
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=\frac{1}{9}\,bdx^9+\frac{1}{5}\,bcx^5+\frac{1}{5}\,adx^5+acx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=\frac{b\,d\,x^9}{9}+\left(\frac{a\,d}{5}+\frac{b\,c}{5}\right)\,x^5+a\,c\,x$$

```
[In] int((a + b*x^4)*(c + d*x^4),x)
[Out] x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

3.150 $\int \frac{a+bx^4}{c+dx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 223

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d}+\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}$$

```
[Out] b*x/d-1/4*(-a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)-1/4*(-a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)+1/8*(-a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^
(3/4)/d^(5/4)*2^(1/2)-1/8*(-a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x
^2*d^(1/2))/c^(3/4)/d^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {396, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} + \frac{bx}{d}$$

```
[In] Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^4}\,dx}{d}\\
&= \frac{bx}{d} - \frac{(bc-ad)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d} - \frac{(bc-ad)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d}\\
&= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{3/2}} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{3/2}}\\
&\quad + \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{3/4}d^{5/4}}
\end{aligned}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{8bc^{3/4}\sqrt[4]{d}x + 2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) - 2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + \sqrt{2}(bc-ad)\,\mathrm{lo}}{8c^{3/4}d^{5/4}}$$

```
[In] Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
 + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(8*c^(3/4)*d^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^3}}{4d^2}$	42
default	$\frac{bx}{d}+\frac{(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8dc}$	120

```
[In] int((b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] b*x/d+1/4/d^2*sum((a*d-b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.51

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}\log\left(cd\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)+i\,d}{}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] 1/4*(d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + I*d*(-(b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d
^5))^(1/4)*log(I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - I*d*(-(b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)
*log(-I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - d*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(-c*d*(
-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c
^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} + \mathrm{RootSum}\left(256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(\frac{4tcd}{ad-bc} + x\right)\right)\right)$$

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a
*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8d}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2
)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^
(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*
c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^2} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8cd^2}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)
^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*
d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{b\,x}{d} - \frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}\right)\cdots}{2(-c)^{3/4}d^{5/4}} - \frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-b\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-b\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}{4(-c)^{3/4}d^{5/4}}\cdots}\right)\cdots}{2(-c)^{3/4}d^{5/4}}$$

```
[In] int((a + b*x^4)/(c + d*x^4),x)

[Out] (b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d
^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-
c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^
2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (atan((((x*
(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d
 - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)) + (
(x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a*c*d^3)*(
a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))
/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3
)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(
5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a
*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/
4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

3.151 $\int \frac{a+bx^4}{(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 245

$$\begin{aligned}\int \frac{a+bx^4}{(c+dx^4)^2} \, dx = &-\frac{(bc-ad)x}{4cd(c+dx^4)} - \frac{(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} \\ &+ \frac{(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} \\ &- \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} \\ &+ \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}\end{aligned}$$

```
[Out] -1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2
)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/16*(3*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1
/2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)-1/32*(3*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x
*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)+1/32*(3*a*d+b*c)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx=-\frac{(3ad+bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{5/4}}-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

```
[In] Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x)/(c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*
d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7
/4)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{4cd\,(c+dx^4)}+\frac{(bc+3ad)\int\frac{1}{c+dx^4}\,dx}{4cd}\\
&= -\frac{(bc-ad)x}{4cd\,(c+dx^4)}+\frac{(bc+3ad)\int\frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d}+\frac{(bc+3ad)\int\frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d}\\
&= -\frac{(bc-ad)x}{4cd\,(c+dx^4)}+\frac{(bc+3ad)\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}}+\frac{(bc+3ad)\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}}\\
&\quad-\frac{(bc+3ad)\int\frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{(bc+3ad)\int\frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}d^{5/4}}
\end{aligned}$$

$$= -\frac{(bc-ad)x}{4cd(c+dx^4)} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}$$

$$= -\frac{(bc-ad)x}{4cd(c+dx^4)} - \frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int\frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{-\frac{8c^{3/4}\sqrt[4]{d}(bc-ad)x}{c+dx^4} - 2\sqrt{2}(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt{2}(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) - \sqrt{2}(bc\cdots}{32c^{7/4}d^{5/4}}$$

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^4+c)}\frac{(3ad+bc)\ln(x-\_R)}{\_R^3}}{16c\,d^2}$	65
default	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{(3ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2d}$	140

```
[In] int((b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*d-b*c)/c*x/(d*x^4+c)+1/16/c/d^2*sum((3*a*d+b*c)/_R^3*ln(x-_R),_R=R
ootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.64

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$= \frac{(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right.\right.}{}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
+ 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 12*
a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))
^(1/4) + (b*c + 3*a*d)*x) - (-I*c*d^2*x^4 - I*c^2*d)*(-(b^4*c^4 + 12*a*b^3*
c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)
*log(I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c
*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - (I*c*d^2*x^4 + I*c
^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 +
81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*
a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3
*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*d*(-(b^4*c^4
 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7
*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4 + c^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{x(ad-bc)}{4c^2d+4cd^2x^4} + \text{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+\right.\right.\right.$$

```
[In] integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*
d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x}{4\,(cd^2x^4+c^2d)} + \frac{\frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc+3\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\,cd}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d)*log(
sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{bcx-adx}{4\,(dx^4+c)cd}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)^(
1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt
(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/
4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.02

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}-\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}\right)(3\,a\,d+b\,c)}{16\,(-c)^{7/4}\,d^{5/4}}+\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}+\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}\right)}{16\,(-c)^{7/4}\,d^{5/4}}}{\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}-\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}\right)(3\,a\,d+b\,c)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}-\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}+\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}\right)}{16\,(-c)^{7/4}\,d^{5/4}}}\right.}{8\,(-c)^{7/4}\,d^{5/4}}$$

$$+\frac{x\,(a\,d-b\,c)}{4\,c\,d\,\left(d\,x^4+c\right)}$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}-\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)}{16\,(-c)^{7/4}\,d^{5/4}}\right)(3\,a\,d+b\,c)\,1\mathrm{i}}{16\,(-c)^{7/4}\,d^{5/4}}+\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}+\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)}{16\,(-c)^{7/4}\,d^{5/4}}\right)}{16\,(-c)^{7/4}\,d^{5/4}}}{\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}-\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)}{16\,(-c)^{7/4}\,d^{5/4}}\right)(3\,a\,d+b\,c)}{16\,(-c)^{7/4}\,d^{5/4}}-\frac{\left(\frac{x\left(9\,a^2\,d^3+6\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{4\,c^2}+\frac{(3\,a\,d+b\,c)\left(12\,a\,d^3+4\,b\,c\,d^2\right)}{16\,(-c)^{7/4}\,d^{5/4}}\right)}{16\,(-c)^{7/4}\,d^{5/4}}}\right.}{8\,(-c)^{7/4}\,d^{5/4}}$$

```
[In] int((a + b*x^4)/(c + d*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)
*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c
)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((
3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)
*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/
(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(
3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b
*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^
(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)/(8*(-c)^
(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2)
 - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*
d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(
5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*
d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4))
))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c))/(4*c*d*(c + d*x^
4))
```

3.152 $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 273

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x}{8cd(c+dx^4)^2} + \frac{(bc+7ad)x}{32c^2d(c+dx^4)} - \frac{3(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}$$

```
[Out] -1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128*
(7*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)+3
/128*(7*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/
2)-3/256*(7*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(
11/4)/d^(5/4)*2^(1/2)+3/256*(7*a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2
)+x^2*d^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {393, 205, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx=-\frac{3(7ad+bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d\left(c+dx^4\right)}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

```
[In] Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] -1/8*((b*c - a*d)*x)/(c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c +
 d*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sq
rt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*
(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*
Sqrt[2]*c^(11/4)*d^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2}+\frac{(bc+7ad)\int\frac{1}{(c+dx^4)^2}\,dx}{8cd}\\ &= -\frac{(bc-ad)x}{8cd\,(c+dx^4)^2}+\frac{(bc+7ad)x}{32c^2d\,(c+dx^4)}+\frac{(3(bc+7ad))\int\frac{1}{c+dx^4}\,dx}{32c^2d}\end{aligned}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}} - \frac{(3(bc+7ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{128\sqrt{2}c^{11/4}d^{5/4}} - \frac{(3(bc+7ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{128\sqrt{2}c^{11/4}d^{5/4}}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{(3(bc+7ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} - \frac{(3(bc+7ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{-\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)x}{(c+dx^4)^2}+\frac{8c^{3/4}\sqrt[4]{d}(bc+7ad)x}{c+dx^4}-6\sqrt{2}(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+6\sqrt{2}(bc+7ad)\arctan\left(1+\right.}{256c}$$

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b*
c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.93 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.31

method	result	siz
risch	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(dx^4+c)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(d\_Z^4+c)}\frac{(7ad+bc)\ln(x-\_R)}{\_R^3}\right)}{128c^2d^2}$	84
default	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(dx^4+c)^2}+\frac{3(7ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{256c^3d}$	159

```
[In] int((b*x^4+a)/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+3/128/c^2/
d^2*sum((7*a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.71

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{4\,(bcd+7\,ad^2)x^5+3\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\log\left(3\,c^3d\left(\right.\right.}{}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-
(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a
^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2
*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c
+ 7*a*d)*x) - 3*(-I*c^2*d^3*x^8 - 2*I*c^3*d^2*x^4 - I*c^4*d)*(-(b^4*c^4 + 2
8*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^1
1*d^5))^(1/4)*log(3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d
^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x
) - 3*(I*c^2*d^3*x^8 + 2*I*c^3*d^2*x^4 + I*c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3
*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/
4)*log(-3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*
a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 3*(c^2
*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2
*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*
(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a*c*d)*x
)/(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.55

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{x^5\cdot(7ad^2+bcd)+x(11acd-3bc^2)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8} + \mathrm{RootSum}\left(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2268ab^3c^3d+81b^4c^4,\left(\right.\right.$$

```
[In] integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d*
*2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 194481*
```

```
a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b**3*
c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x
)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{(bcd+7\,ad^2)x^5-(3\,bc^2-11\,acd)x}{32\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)} + \frac{3\left(\frac{2\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc+7\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right.}{256\,c^2d}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*
d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqr
t(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt
(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - s
qrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx = \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} - \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} + \frac{bcdx^5+7\,ad^2x^5-3\,bc^2x+11\,acdx}{32\left(dx^4+c\right)^2c^2d}$$

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)
^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*
sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*x +
 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.79

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{\frac{x^5\,(7\,a\,d+b\,c)}{32\,c^2}+\frac{x\,(11\,a\,d-3\,b\,c)}{32\,c\,d}}{c^2+2\,c\,d\,x^4+d^2\,x^8} - \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\cdots\right.}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\cdots\right.}{128\,(-c)^{11/4}\,d^{5/4}}}\cdots\right)}{64\,(-c)^{11/4}\,d^{5/4}} - \frac{3\operatorname{atan}\left(\frac{\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\cdots\right.}{256\,(-c)^{15/4}\,d^{5/4}}\cdots\right.}{128\,(-c)^{11/4}\,d^{5/4}}}{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)9\mathrm{i}}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\cdots\right.}{256\,(-c)^{15/4}\,d^{5/4}}\cdots\right.}{128\,(-c)^{11/4}\,d^{5/4}}}\cdots\right)}{64\,(-c)^{11/4}\,d^{5/4}}$$

```
[In] int((a + b*x^4)/(c + d*x^4)^3,x)
```

```
[Out] ((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*x
^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*
(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*c^2*
d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(
-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3*((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a
*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)
*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (
9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4)*d^(5/4))
 - (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((
7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c
*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*
d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x*(49*a^2*d^3 + b
^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i
)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) -
(((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*
(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(
-c)^(11/4)*d^(5/4))))*(7*a*d + b*c))/(64*(-c)^(11/4)*d^(5/4))
```

3.153 $\int (a+bx^4)^2 (c+dx^4)^4 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2(b^2c^2+8abcd+6a^2d^2)\,x^9 + \frac{4}{13}cd(b^2c^2+3abcd+a^2d^2)\,x^{13} + \frac{1}{17}d^2(6b^2c^2+8abcd+a^2d^2)\,x^{17} + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}$$

```
[Out] a^2*c^4*x+2/5*a*c^3*(2*a*d+b*c)*x^5+1/9*c^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x
^9+4/13*c*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^13+1/17*d^2*(a^2*d^2+8*a*b*c*d+6*
b^2*c^2)*x^17+2/21*b*d^3*(a*d+2*b*c)*x^21+1/25*b^2*d^4*x^25
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = \frac{1}{17}d^2x^{17}(a^2d^2+8abcd+6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2+3abcd+b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2+8abcd+b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5(2ad+bc) + \frac{2}{21}bd^3x^{21}(ad+2bc) + \frac{1}{25}b^2d^4x^{25}$$

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2c^4 + 2ac^3(bc+2ad)x^4 + c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^8 \right. \\ &\quad + 4cd\left(b^2c^2+3abcd+a^2d^2\right)x^{12} + d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{16} \\ &\quad \left. + 2bd^3(2bc+ad)x^{20} + b^2d^4x^{24}\right)\,dx \\ &= a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 + \frac{4}{13}cd\left(b^2c^2+3abcd \right. \\ &\quad \left. + a^2d^2\right)x^{13} + \frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^4\,dx &= a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 \\ &\quad + \frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} \\ &\quad + \frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} \\ &\quad + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}\end{aligned}$$

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
norman	$\frac{b^2d^4x^{25}}{25}+\left(\frac{2}{21}ab\,d^4+\frac{4}{21}b^2c\,d^3\right)x^{21}+\left(\frac{1}{17}a^2d^4+\frac{8}{17}abc\,d^3+\frac{6}{17}b^2c^2d^2\right)x^{17}+\left(\frac{4}{13}a^2c\,d^3+\frac{12}{13}ab\,c^2\right.$
default	$\frac{b^2d^4x^{25}}{25}+\frac{(2ab\,d^4+4b^2c\,d^3)x^{21}}{21}+\frac{(a^2d^4+8abc\,d^3+6b^2c^2d^2)x^{17}}{17}+\frac{(4a^2c\,d^3+12ab\,c^2d^2+4b^2c^3d)x^{13}}{13}+\frac{(6a^2c^2d^2+8ab\,c}{9}$
gosper	$\frac{1}{25}b^2d^4x^{25}+\frac{2}{21}x^{21}ab\,d^4+\frac{4}{21}x^{21}b^2c\,d^3+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d^3+\frac{6}{17}x^{17}b^2c^2d^2+\frac{4}{13}x^{13}a^2c\,d^3$
risch	$\frac{1}{25}b^2d^4x^{25}+\frac{2}{21}x^{21}ab\,d^4+\frac{4}{21}x^{21}b^2c\,d^3+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d^3+\frac{6}{17}x^{17}b^2c^2d^2+\frac{4}{13}x^{13}a^2c\,d^3$
parallelrisch	$\frac{1}{25}b^2d^4x^{25}+\frac{2}{21}x^{21}ab\,d^4+\frac{4}{21}x^{21}b^2c\,d^3+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d^3+\frac{6}{17}x^{17}b^2c^2d^2+\frac{4}{13}x^{13}a^2c\,d^3$

```
[In] int((b*x^4+a)^2*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*b^2*d^4*x^25+(2/21*a*b*d^4+4/21*b^2*c*d^3)*x^21+(1/17*a^2*d^4+8/17*a*b
*c*d^3+6/17*b^2*c^2*d^2)*x^17+(4/13*a^2*c*d^3+12/13*a*b*c^2*d^2+4/13*b^2*c^
3*d)*x^13+(2/3*a^2*c^2*d^2+8/9*a*b*c^3*d+1/9*b^2*c^4)*x^9+a^2*c^4*x+(4/5*a^
2*c^3*d+2/5*a*b*c^4)*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^4\,dx=&\ \frac{1}{25}\,b^2d^4x^{25}+\frac{2}{21}\left(2\,b^2cd^3+abd^4\right)x^{21}\\&+\frac{1}{17}\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^{17}\\&+\frac{4}{13}\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^{13}\\&+\frac{1}{9}\left(b^2c^4+8\,abc^3d+6\,a^2c^2d^2\right)x^9\\&+a^2c^4x+\frac{2}{5}\left(abc^4+2\,a^2c^3d\right)x^5\end{aligned}$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = a^2c^4x + \frac{b^2d^4x^{25}}{25} + x^{21}\cdot\left(\frac{2abd^4}{21}+\frac{4b^2cd^3}{21}\right) + x^{17}\left(\frac{a^2d^4}{17}+\frac{8abcd^3}{17}+\frac{6b^2c^2d^2}{17}\right) + x^{13}\cdot\left(\frac{4a^2cd^3}{13}+\frac{12abc^2d^2}{13}+\frac{4b^2c^3d}{13}\right) + x^9\cdot\left(\frac{2a^2c^2d^2}{3}+\frac{8abc^3d}{9}+\frac{b^2c^4}{9}\right) + x^5\cdot\left(\frac{4a^2c^3d}{5}+\frac{2abc^4}{5}\right)$$

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21)
 + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(4*
a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*c**
2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c*
*4/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \frac{1}{25}\,b^2d^4x^{25} + \frac{2}{21}\left(2\,b^2cd^3+abd^4\right)x^{21} + \frac{1}{17}\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^{17} + \frac{4}{13}\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^{13} + \frac{1}{9}\left(b^2c^4+8\,abc^3d+6\,a^2c^2d^2\right)x^9 + a^2c^4x + \frac{2}{5}\left(abc^4+2\,a^2c^3d\right)x^5$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \frac{1}{25} b^2 d^4 x^{25} + \frac{4}{21} b^2 c d^3 x^{21} + \frac{2}{21} a b d^4 x^{21} + \frac{6}{17} b^2 c^2 d^2 x^{17} + \frac{8}{17} a b c d^3 x^{17} + \frac{1}{17} a^2 d^4 x^{17} + \frac{4}{13} b^2 c^3 d x^{13} + \frac{12}{13} a b c^2 d^2 x^{13} + \frac{4}{13} a^2 c d^3 x^{13} + \frac{1}{9} b^2 c^4 x^9 + \frac{8}{9} a b c^3 d x^9 + \frac{2}{3} a^2 c^2 d^2 x^9 + \frac{2}{5} a b c^4 x^5 + \frac{4}{5} a^2 c^3 d x^5 + a^2 c^4 x$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2*
d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13 +
12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b*c^
3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c
^4*x
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = x^9 \left(\frac{2\,a^2\,c^2\,d^2}{3} + \frac{8\,a\,b\,c^3\,d}{9} + \frac{b^2\,c^4}{9}\right) + x^{17} \left(\frac{a^2\,d^4}{17} + \frac{8\,a\,b\,c\,d^3}{17} + \frac{6\,b^2\,c^2\,d^2}{17}\right) + a^2\,c^4\,x + \frac{b^2\,d^4\,x^{25}}{25} + \frac{2\,a\,c^3\,x^5\,(2\,a\,d+b\,c)}{5} + \frac{2\,b\,d^3\,x^{21}\,(a\,d+2\,b\,c)}{21} + \frac{4\,c\,d\,x^{13}\,(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2)}{13}$$

```
[In] int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

```
[Out] x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/1
7 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/25
+ (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c*d*
x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

3.154 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9 + \frac{1}{13}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17}+\frac{1}{21}b^2d^3x^{21}$$

```
[Out] a^2*c^3*x+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^9+1/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^13+1/17*b*d^2*(2*a*d+3*b*c)*x^17+
1/21*b^2*d^3*x^21
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right) +a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2c^3 + ac^2(2bc+3ad)x^4 + c\left(b^2c^2+6abcd+3a^2d^2\right)x^8 \right. \\ &\quad \left. + d\left(3b^2c^2+6abcd+a^2d^2\right)x^{12} + bd^2(3bc+2ad)x^{16} + b^2d^3x^{20}\right) dx \\ &= a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9 \\ &\quad + \frac{1}{13}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3 dx &= a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9 \\ &\quad + \frac{1}{13}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{13} \\ &\quad + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}\end{aligned}$$

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x + \left(\frac{3}{5}a^2c^2d + \frac{2}{5}ab\,c^3\right)x^5 + \left(\frac{1}{3}c\,a^2d^2 + \frac{2}{3}ab\,c^2d + \frac{1}{9}b^2c^3\right)x^9 + \left(\frac{1}{13}a^2d^3 + \frac{6}{13}abc\,d^2 + \frac{3}{13}b^2c\right.$
default	$\frac{b^2d^3x^{21}}{21} + \frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{17}}{17} + \frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{13}}{13} + \frac{\left(3c\,a^2d^2+6ab\,c^2d+b^2c^3\right)x^9}{9} + \frac{\left(3a^2c^2d+2ab\,c^3\right)x^5}{5} +$
gosper	$a^2c^3x + \frac{3}{5}x^5a^2c^2d + \frac{2}{5}x^5ab\,c^3 + \frac{1}{3}x^9c\,a^2d^2 + \frac{2}{3}x^9ab\,c^2d + \frac{1}{9}x^9b^2c^3 + \frac{1}{13}x^{13}a^2d^3 + \frac{6}{13}x^{13}abc\,d^2$
risch	$a^2c^3x + \frac{3}{5}x^5a^2c^2d + \frac{2}{5}x^5ab\,c^3 + \frac{1}{3}x^9c\,a^2d^2 + \frac{2}{3}x^9ab\,c^2d + \frac{1}{9}x^9b^2c^3 + \frac{1}{13}x^{13}a^2d^3 + \frac{6}{13}x^{13}abc\,d^2$
parallelrisch	$a^2c^3x + \frac{3}{5}x^5a^2c^2d + \frac{2}{5}x^5ab\,c^3 + \frac{1}{3}x^9c\,a^2d^2 + \frac{2}{3}x^9ab\,c^2d + \frac{1}{9}x^9b^2c^3 + \frac{1}{13}x^{13}a^2d^3 + \frac{6}{13}x^{13}abc\,d^2$

```
[In] int((b*x^4+a)^2*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^3*x+(3/5*a^2*c^2*d+2/5*a*b*c^3)*x^5+(1/3*c*a^2*d^2+2/3*a*b*c^2*d+1/9*
b^2*c^3)*x^9+(1/13*a^2*d^3+6/13*a*b*c*d^2+3/13*b^2*c^2*d)*x^13+(2/17*a*b*d^
3+3/17*b^2*c*d^2)*x^17+1/21*b^2*d^3*x^21
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx=\frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+a^2c^3x+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx=a^2c^3x+\frac{b^2d^3x^{21}}{21}+x^{17}\cdot\left(\frac{2abd^3}{17}+\frac{3b^2cd^2}{17}\right)+x^{13}\left(\frac{a^2d^3}{13}+\frac{6abcd^2}{13}+\frac{3b^2c^2d}{13}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^5\cdot\left(\frac{3a^2c^2d}{5}+\frac{2abc^3}{5}\right)$$

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
 + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{21}\,b^2d^3x^{21} + \frac{1}{17}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{17} + \frac{1}{13}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{13} + \frac{1}{9}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^9 + a^2c^3x + \frac{1}{5}\left(2\,abc^3 + 3\,a^2c^2d\right)x^5$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{21}\,b^2d^3x^{21} + \frac{3}{17}\,b^2cd^2x^{17} + \frac{2}{17}\,abd^3x^{17} + \frac{3}{13}\,b^2c^2dx^{13} + \frac{6}{13}\,abcd^2x^{13} + \frac{1}{13}\,a^2d^3x^{13} + \frac{1}{9}\,b^2c^3x^9 + \frac{2}{3}\,abc^2dx^9 + \frac{1}{3}\,a^2cd^2x^9 + \frac{2}{5}\,abc^3x^5 + \frac{3}{5}\,a^2c^2dx^5 + a^2c^3x$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2*
d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + a^2
*c^3*x
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = x^9 \left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right) + x^{13} \left(\frac{a^2\,d^3}{13}+\frac{6\,a\,b\,c\,d^2}{13}+\frac{3\,b^2\,c^2\,d}{13}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^{21}}{21} + \frac{a\,c^2\,x^5\,(3\,a\,d+2\,b\,c)}{5} + \frac{b\,d^2\,x^{17}\,(2\,a\,d+3\,b\,c)}{17}$$

```
[In] int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
(3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a*c
^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

3.155 $\int (a+bx^4)^2 (c+dx^4)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = a^2c^2x + \frac{2}{5}ac(bc+ad)x^5 + \frac{1}{9}(b^2c^2+4abcd+a^2d^2)\,x^9 + \frac{2}{13}bd(bc+ad)x^{13} + \frac{1}{17}b^2d^2x^{17}$$

```
[Out] a^2*c^2*x+2/5*a*c*(a*d+b*c)*x^5+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2/13*b*
d*(a*d+b*c)*x^13+1/17*b^2*d^2*x^17
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = \frac{1}{9}x^9(a^2d^2+4abcd+b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad+bc) + \frac{2}{5}acx^5(ad+bc) + \frac{1}{17}b^2d^2x^{17}$$

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2c^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{12}+b^2d^2x^{16}\right)\,dx\\
&= a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^2d^2x^{17}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$
\int \left(a+bx^4\right)^2\left(c+dx^4\right)^2\,dx = a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9 + \frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^2d^2x^{17}
$$

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$a^{2}c^{2}x+\left(\frac{2}{5}a^{2}cd+\frac{2}{5}b\,c^{2}a\right)x^{5}+\left(\frac{1}{9}a^{2}d^{2}+\frac{4}{9}abcd+\frac{1}{9}b^{2}c^{2}\right)x^{9}+\left(\frac{2}{13}ab\,d^{2}+\frac{2}{13}b^{2}cd\right)x^{13}+\frac{b^{2}d^{2}x^{17}}{17}$
default	$\frac{b^{2}d^{2}x^{17}}{17}+\frac{\left(2ab\,d^{2}+2b^{2}cd\right)x^{13}}{13}+\frac{\left(a^{2}d^{2}+4abcd+b^{2}c^{2}\right)x^{9}}{9}+\frac{\left(2a^{2}cd+2b\,c^{2}a\right)x^{5}}{5}+a^{2}c^{2}x$
gosper	$a^{2}c^{2}x+\frac{2}{5}x^{5}a^{2}cd+\frac{2}{5}x^{5}b\,c^{2}a+\frac{1}{9}x^{9}a^{2}d^{2}+\frac{4}{9}x^{9}abcd+\frac{1}{9}x^{9}b^{2}c^{2}+\frac{2}{13}x^{13}ab\,d^{2}+\frac{2}{13}x^{13}b^{2}cd+\frac{1}{17}b^{2}$
risch	$a^{2}c^{2}x+\frac{2}{5}x^{5}a^{2}cd+\frac{2}{5}x^{5}b\,c^{2}a+\frac{1}{9}x^{9}a^{2}d^{2}+\frac{4}{9}x^{9}abcd+\frac{1}{9}x^{9}b^{2}c^{2}+\frac{2}{13}x^{13}ab\,d^{2}+\frac{2}{13}x^{13}b^{2}cd+\frac{1}{17}b^{2}$
parallelrisch	$a^{2}c^{2}x+\frac{2}{5}x^{5}a^{2}cd+\frac{2}{5}x^{5}b\,c^{2}a+\frac{1}{9}x^{9}a^{2}d^{2}+\frac{4}{9}x^{9}abcd+\frac{1}{9}x^{9}b^{2}c^{2}+\frac{2}{13}x^{13}ab\,d^{2}+\frac{2}{13}x^{13}b^{2}cd+\frac{1}{17}b^{2}$

```
[In] int((b*x^4+a)^2*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c^2*x+(2/5*a^2*c*d+2/5*b*c^2*a)*x^5+(1/9*a^2*d^2+4/9*a*b*c*d+1/9*b^2*c^
2)*x^9+(2/13*a*b*d^2+2/13*b^2*c*d)*x^13+1/17*b^2*d^2*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \,dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,\left(b^2cd+abd^2\right)x^{13} + \frac{1}{9}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^9 + \frac{2}{5}\,\left(abc^2+a^2cd\right)x^5 + a^2c^2x$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \,dx = a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13}\cdot\left(\frac{2abd^2}{13}+\frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right) + x^5\cdot\left(\frac{2a^2cd}{5}+\frac{2abc^2}{5}\right)$$

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a*b
*c**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \,dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,\left(b^2cd+abd^2\right)x^{13} + \frac{1}{9}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^9 + \frac{2}{5}\,\left(abc^2+a^2cd\right)x^5 + a^2c^2x$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,b^2cdx^{13} + \frac{2}{13}\,abd^2x^{13} + \frac{1}{9}\,b^2c^2x^9 + \frac{4}{9}\,abcdx^9 + \frac{1}{9}\,a^2d^2x^9 + \frac{2}{5}\,abc^2x^5 + \frac{2}{5}\,a^2cdx^5 + a^2c^2x$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 +
a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = x^9 \left(\frac{a^2\,d^2}{9} + \frac{4\,a\,b\,c\,d}{9} + \frac{b^2\,c^2}{9}\right) + a^2\,c^2\,x + \frac{b^2\,d^2\,x^{17}}{17} + \frac{2\,a\,c\,x^5\,(a\,d+b\,c)}{5} + \frac{2\,b\,d\,x^{13}\,(a\,d+b\,c)}{13}$$

```
[In] int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^17
)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

3.156 $\int (a+bx^4)^2 (c+dx^4) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)^2 (c+dx^4) \, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

```
[Out] a^2*c*x+1/5*a*(a*d+2*b*c)*x^5+1/9*b*(2*a*d+b*c)*x^9+1/13*b^2*d*x^13
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^4)^2 (c+dx^4) \, dx = a^2cx + \frac{1}{9}bx^9(2ad+bc) + \frac{1}{5}ax^5(ad+2bc) + \frac{1}{13}b^2dx^{13}$$

```
[In] Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2c + a(2bc+ad)x^4 + b(bc+2ad)x^8 + b^2dx^{12}\right) dx \\ &= a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4) \, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4),x]
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^{13}}{13}+\frac{(2abd+b^2c)x^9}{9}+\frac{(a^2d+2abc)x^5}{5}+a^2cx$	49
norman	$\frac{b^2 d\,x^{13}}{13}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{5}a^2d+\frac{2}{5}abc\right)x^5+a^2cx$	49
gosper	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51
risch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51
parallelrisch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+a^2cx$	51

```
[In] int((b*x^4+a)^2*(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (c+dx^4) \, dx = \frac{1}{13}\,b^2dx^{13} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}\left(2\,abc + a^2d\right)x^5 + a^2cx$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a
^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\,dx = a^2cx+\frac{b^2dx^{13}}{13}+x^9\cdot\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

```
[In] integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5 +
 2*a*b*c/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\,dx = \frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\left(2\,abc+a^2d\right)x^5+a^2cx$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a
^2*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)\,dx = \frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,b^2cx^9+\frac{2}{9}\,abdx^9+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+a^2cx$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*d
*x^5 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right) \, dx = x^5 \left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^9 \left(\frac{c\,b^2}{9}+\frac{2\,a\,d\,b}{9}\right)+\frac{b^2\,d\,x^{13}}{13}+a^2\,c\,x$$

```
[In] int((a + b*x^4)^2*(c + d*x^4),x)

[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^13
)/13 + a^2*c*x
```

3.157 $\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 253

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*arctan(-1+d^(1/4)*x*2^
(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+d^(1/4)*x*
2^(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*
ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = -\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{9/4}} -\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} +\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

```
[In] Int[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^4\right)}\right)\,dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^4}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d^2}+\frac{(bc-ad)^2\int\frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^5}{5d} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{5/2}} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{5/2}} - \frac{(bc-ad)^2 \int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} + 2x}{-\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2} dx}{4\sqrt{2}c^{3/4}d^{9/4}} - \frac{(bc-ad)^2 \int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} - 2x}{-\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2} dx}{4\sqrt{2}c^{3/4}d^{9/4}}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^5}{5d} - \frac{(bc-ad)^2 \log\left(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{(bc-ad)^2 \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^5}{5d} - \frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{(bc-ad)^2 \log\left(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^2}{c+dx^4} dx$$

$$= \frac{-40bc^{3/4}\sqrt[4]{d}(bc-2ad)x + 8b^2c^{3/4}d^{5/4}x^5 - 10\sqrt{2}(bc-ad)^2 \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 10\sqrt{2}(bc-ad)^2 \arctan}{4}$$

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(b*
c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c -
 a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)
*d^(9/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

method	result
risch	$\frac{b^2x^5}{5d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4d^3}$
default	$\frac{b\left(\frac{1}{5}bd\,x^5+2adx-bcx\right)}{d^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8d^2c}$

```
[In] int((b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*c*x+1/4/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1092, normalized size of antiderivative = 4.32

$$\int\frac{\left(a+bx^4\right)^2}{c+dx^4}\,dx = \frac{4\,b^2dx^5+5\,d^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^3d^9}\right)^{\frac{1}{4}}\log\left(cd^2\left(-\right.\right.}{}$$

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 + 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 -
8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3
*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*d^2*(-(b^8*c^8 - 8*
```

```
a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d
^9))^(1/4)*log(I*c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*x) - 5*I*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a
^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-I*c*d^2*(-(b^8*c^8 - 8*
a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d
^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))
^(1/4)*log(-c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x) - 20*(b^2*c - 2*a*b*d)*x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2x^5}{5d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)$$
$$+\text{RootSum}\left(256t^4c^3d^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6\right.$$

```
[In] integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9 +
 a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d
**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d
**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*d**2
 - 2*a*b*c*d + b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2dx^5-5\,(b^2c-2\,abd)x}{5\,d^2} + \frac{\frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2-2\,abc}{}}{8\,d^2}$$

[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")

[Out] 1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^2

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{(a+bx^4)^2}{c+dx^4}\,dx &= \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3} \\ &+ \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3} \\ &+ \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3} \\ &- \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3} \\ &+ \frac{b^2d^4x^5-5\,b^2cd^3x+10\,abd^4x}{5\,d^5}\end{aligned}$$

[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")

```
[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d
^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
 - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) + 1/5*(b
^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.27

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx=\frac{b^2\,x^5}{5\,d}-x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)$$
$$+\frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{(a\,d-b\,c)^2\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,(-c)^{3/4}\,d^{9/4}}\right)1\mathrm{i}}{(-c)^{3/4}\,d^{9/4}}+\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\cdots\right.}{}\right.}{}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{(a\,d-b\,c)^2\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,(-c)^{3/4}\,d^{9/4}}\right)}{(-c)^{3/4}\,d^{9/4}}-\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\cdots\right.}{}\right.}{}}\right)\cdots}{2\,(-c)^{3/4}\,d^{9/4}}$$
$$+\frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{(a\,d-b\,c)^2\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)1\mathrm{i}}{4\,(-c)^{3/4}\,d^{9/4}}\right)}{(-c)^{3/4}\,d^{9/4}}+\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\cdots\right.}{}\right.}{}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{(a\,d-b\,c)^2\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)1\mathrm{i}}{4\,(-c)^{3/4}\,d^{9/4}}\right)1\mathrm{i}}{(-c)^{3/4}\,d^{9/4}}-\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\cdots\right.}{}\right.}{}}\right)\cdots}{2\,(-c)^{3/4}\,d^{9/4}}$$

```
[In] int((a + b*x^4)^2/(c + d*x^4),x)
```

```
[Out] (b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d -
 ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*
d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4
*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c
)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2
*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^
2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
```

```
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)))/(((a*d - b
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/
(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a
*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d
^(9/4)))*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)/(2*(-c)^(3/4)*d^(9/4))
```

3.158 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 291

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)}+\frac{(bc-ad)(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$
$$-\frac{(bc-ad)(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$
$$+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}$$
$$-\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}$$

```
[Out] b^2*x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*
arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)-1/16*(-a*d+b*c
)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)
+1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^
(1/2))/c^(7/4)/d^(9/4)*2^(1/2)-1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{(bc-ad)(3ad+5bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} + \frac{x(bc-ad)^2}{4cd^2(c+dx^4)} + \frac{b^2x}{d^2}$$

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^4}{d^2\left(c + dx^4\right)^2}\right) dx$$

$$=\frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{(c+dx^4)^2}\,dx}{d^2}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{c+dx^4}\,dx}{4cd^2}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}}+\frac{((bc-ad)(5bc+3ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{((bc-ad)(5bc+3ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}d^{9/4}}$$

$$=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\,(c+dx^4)}+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{((bc-ad)(5bc+3ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{((bc-ad)(5bc+3ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2 x}{4cd^2(c+dx^4)} + \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$
$$- \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$
$$+ \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}$$
$$- \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$

$$= \frac{32b^2\sqrt[4]{d}x + \frac{8\sqrt[4]{d}(bc-ad)^2x}{c(c+dx^4)} + \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} +}{32d^{9/4}}$$

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[2
]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (S
qrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.96 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

method	result
risch	$\frac{b^{2}x}{d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{4c\,d^{2}\left(d\,x^{4}+c\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{4}+c\right)}\frac{\left(3a^{2}d^{2}+2abcd-5b^{2}c^{2}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16d^{3}c}$
default	$\frac{b^{2}x}{d^{2}}+\frac{\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{4c\left(d\,x^{4}+c\right)}+\frac{\left(3a^{2}d^{2}+2abcd-5b^{2}c^{2}\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^{2}}}{d^{2}}$

```
[In] int((b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^4+c)+1/16/d^3/c*sum(
(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1210, normalized size of antiderivative = 4.16

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c*d*x^5 + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^
7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 98
4*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^
7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c
^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 -
 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*
b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (-I*c*d^3*x^4 - I*c^2*d^2)*(-(625*b^8
*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*
a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d
^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(I*c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7
*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/
(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (I*c*d^3*x^4 +
I*c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a
^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^
2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-I*c^2*d^2*(-(62
5*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 +
 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*
b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2
)*x) - (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^
```

```
6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^
5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*lo
g(-c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a
^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^
2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b
*c*d - 3*a^2*d^2)*x) + 4*(5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^4 +
c^2*d^2)
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{4c^2d^2+4cd^3x^4}$$
$$+\text{RootSum}\Big(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+646a^4b^4c^4d^4+1640a^3b^5c^5$$

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**3
*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7 -
 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**
4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d
 + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*
d - 5*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(cd^3x^4+c^2d^2)} + \frac{b^2x}{d^2}$$
$$-\frac{\frac{2\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(5\,b^2c^2-}{}}{32\,cd^2}$$

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 - 1
/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqr
```

```
t(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sq
rt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)
*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(dx^4+c)cd^2}$$

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^
(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((d*x^4 + c)*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 1254, normalized size of antiderivative = 4.31

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4)) +
 (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
+ 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 2
0*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5
*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*
a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9
/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((x*(9*a^4
*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/
(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b
*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/
4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*
c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d
^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a
*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(
-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d
^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*
b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))
*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 2
5*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d
) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*
1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^
(9/4)))/((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
 + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 -
20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a
^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)
*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4))))*(a*d
 - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))
```

3.159 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 349

$$\begin{aligned}\int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx = &-\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} \\ &- \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ &+ \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ &- \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ &+ \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}\end{aligned}$$

```
[Out] -1/8*(-a*d+b*c)*x*(b*x^4+a)/c/d/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+5*b*c)*x
/c^2/d^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(-1+d^(1/4)
*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*
b^2*c^2)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/256
*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2
*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*l
n(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {424, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(21a^2d^2+6abcd+5b^2c^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(21a^2d^2+6abcd+5b^2c^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{9/4}} - \frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} + \frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} - \frac{x(bc-ad)(7ad+5bc)}{32c^2d^2(c+dx^4)} - \frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] -1/8*((b*c - a*d)*x*(a + b*x^4))/(c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c
+ 7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) +
((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4
)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} + \frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{(c+dx^4)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{c+dx^4}\,dx}{32c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d^2} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{5/2}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{5/2}} - \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{128\sqrt{2}c^{11/4}d^{9/4}} - \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{128\sqrt{2}c^{11/4}d^{9/4}}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} - \frac{(5b^2c^2+6abcd+21a^2d^2)\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd\,(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\,(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \frac{\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)^2x}{(c+dx^4)^2} - \frac{8c^{3/4}\sqrt[4]{d}(9b^2c^2-2abcd-7a^2d^2)x}{c+dx^4} - 2\sqrt{2}(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt{}}{}$$

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(9
*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 + 6
```

*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(9/4))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128c^2d^3}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{256c^3d^2}$

[In] int((b*x^4+a)^2/(d*x^4+c)^3,x,method=_RETURNVERBOSE)

[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-5*b^2*c^2)/c/d^2*x)/(d*x^4+c)^2+1/128/c^2/d^3*sum((21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.71

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx = \text{Too large to display}$$

[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")

[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (-I*c^2*d^4*x^8 - 2*

```
I*c^3*d^3*x^4 - I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^
6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3
*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c
^11*d^9))^(1/4)*log(I*c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2
*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*
b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)
/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (I*c^2*d^4*x
^8 + 2*I*c^3*d^3*x^4 + I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900
*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*
a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*
d^8)/(c^11*d^9))^(1/4)*log(-I*c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 1
5900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176
904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*
a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2
*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 159
00*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 17690
4*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^
8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 1
5900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176
904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*
a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5
*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^
4*d^2)
```

Sympy [A] (verification not implemented)

Time = 85.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \frac{x^5\cdot(7a^2d^3+2abcd^2-9b^2c^2d)+x(11a^2cd^2-6abc^2d-5b^2c^3)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8} + \mathrm{RootSum}\left(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7bcd^7+280476a^6b^2c^2d^6+176904a^5b^3c^3d^5+112\right.$$

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6*
a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d**4
*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*a**
7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 + 1128
06*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d
**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**
2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3)x^5+(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2)x}{32\,(c^2d^4x^8+2\,c^3d^3x^4+c^4d^2)} + \frac{\frac{2\,\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(5\,b^2c}{}}{256\,c^2d^2}$$

[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")

[Out] -1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\Big(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}$$
$$+\frac{\sqrt{2}\Big(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}$$
$$+\frac{\sqrt{2}\Big(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}$$
$$-\frac{\sqrt{2}\Big(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}$$
$$-\frac{9\,b^2c^2dx^5-2\,abcd^2x^5-7\,a^2d^3x^5+5\,b^2c^3x+6\,abc^2dx-11\,a^2cd^2x}{32\,(dx^4+c)^2c^2d^2}$$

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*x^5 - 7*a
^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c
^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 1401, normalized size of antiderivative = 4.01

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

```
[Out] - ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((
```

((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d
^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2
*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*
b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2
*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4
)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*
c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
1i)/(128(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(2
1*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441
*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*
d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d
^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d
 + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 +
 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*
a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^
2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(25
6*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2
 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2
+ 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*
b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4
)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(
-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 +
 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(2
56*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4
)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*
a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 2
46*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^
2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2
+ 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4))

3.160 $\int \frac{(c+dx^4)^4}{a+bx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\begin{aligned}\int \frac{(c+dx^4)^4}{a+bx^4} dx = {} & \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} \\ & + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} \\ & - \frac{(bc-ad)^4\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} \\ & - \frac{(bc-ad)^4\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ & + \frac{(bc-ad)^4\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}\end{aligned}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^13/b+1/4*(-a*
d+b*c)^4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)+1/4*
(-a*d+b*c)^4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)-1
/8*(-a*d+b*c)^4*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/
b^(17/4)*2^(1/2)+1/8*(-a*d+b*c)^4*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}} - \frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}} + \frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4} + \frac{d^2x^5(a^2d^2-4abcd+6b^2c^2)}{5b^3} + \frac{d^3x^9(4bc-ad)}{9b^2} + \frac{d^4x^{13}}{13b}$$

```
[In] Int[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{b^3}+\frac{d^3(4bc-ad)x^8}{b^2}+\frac{d^4x^{12}}{b}+\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{b^4\left(a+bx^4\right)}\right)dx$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc-ad)^4\int\frac{1}{a+bx^4}\,dx}{b^4}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc-ad)^4\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^4}+\frac{(bc-ad)^4\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^4}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc-ad)^4\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{9/2}}+\frac{(bc-ad)^4\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{9/2}}-\frac{(bc-ad)^4\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{17/4}}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}-\frac{(bc-ad)^4\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}-\frac{(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$= \frac{4680\sqrt[4]{b}d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+936b^{5/4}d^2(6b^2c^2-4abcd+a^2d^2)\,x^5+520b^{9/4}d^3(4bc-ad}{}$$

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 9
36*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4*b
*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt
[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4))/(4680*b^(17/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.61

method	result
risch	$\frac{d^4x^{13}}{13b}-\frac{d^4x^9a}{9b^2}+\frac{4d^3x^9c}{9b}-\frac{4d^3ac\,x^5}{5b^2}+\frac{6d^2c^2x^5}{5b}+\frac{d^4a^2x^5}{5b^3}-\frac{d^4a^3x}{b^4}+\frac{4d^3a^2cx}{b^3}-\frac{6d^2a\,c^2x}{b^2}+\frac{4d\,c^3x}{b}+\frac{\sum_{\_R=\text{RootOf}(}}{}$
default	$-\frac{d\left(-\frac{b^3d^3x^{13}}{13}+\frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^9}{9}+\frac{\left(2(ad-2bc)b^2cd-bd\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^5}{5}+(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4}+\frac{(a^4}{}$

```
[In] int((d*x^4+c)^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*d^4*x^13/b-1/9*d^4/b^2*x^9*a+4/9*d^3/b*x^9*c-4/5*d^3/b^2*a*c*x^5+6/5*d
^2/b*c^2*x^5+1/5*d^4/b^3*a^2*x^5-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*
a*c^2*x+4*d/b*c^3*x+1/4/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*
a*b^3*c^3*d+b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 2190, normalized size of antiderivative = 6.60

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*c
^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*b^4*(-(b^16*c^16 - 16*a*b^15*
c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12
*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*
d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^1
0 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13
 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*
log(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3
*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6
*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*
b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b
^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d
^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c
^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 585*I*b^4*(-(b^16*c^16 - 16*a*b^15*c
^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*
d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d
^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10
 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13
+ 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*l
og(I*a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^
3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^
6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9
*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*
b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*
d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) - 585*I*b^4*(-(b^16*c^16 - 16*a*b^15*
c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12
*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*
d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^1
0 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13
 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*
log(-I*a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*
a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*
a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a
^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^1
```

```
2*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*
c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*(-(b^16*c^16 - 16*a*b^15*
c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12
*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*
d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^1
0 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13
 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*
log(-a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^
3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^
6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9
*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*
b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*
d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2
 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

Sympy [A] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$
$$= x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right)+x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3b^{17}+a^{16}d^{16}-16a^{15}bcd^{15}+120a^{14}b^2c^2d^{14}-560a^{13}b^3c^3d^{13}+1820a^{12}b^4c^4d^{12}-4368\right.$$
$$+\frac{d^4x^{13}}{13b}$$

```
[In] integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*c
*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b*
*3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a**1
6*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c
**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*
a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d*
*8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a**5*b**
11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d**3 + 12
0*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda(_t, _t*l
og(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \frac{45\,b^3d^4x^{13}+65\,(4\,b^3cd^3-ab^2d^4)x^9+117\,(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4)x^5+585\,(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2b}{585\,b^4} + \frac{\frac{2\sqrt{2}(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\,\mathrm{arcta}}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(
sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(261) = 522$.

Time = 0.29 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.86

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}} b^4c^4 - 4\,(ab^3)^{\frac{1}{4}} ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}} a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}} a^3bcd^3 + (ab^3)^{\frac{1}{4}} a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}} b^4c^4 - 4\,(ab^3)^{\frac{1}{4}} ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}} a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}} a^3bcd^3 + (ab^3)^{\frac{1}{4}} a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\right.}{2\left(\frac{a}{b}\right.}\right.}{4\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}} b^4c^4 - 4\,(ab^3)^{\frac{1}{4}} ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}} a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}} a^3bcd^3 + (ab^3)^{\frac{1}{4}} a^4d^4\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right.\right.}{8\,ab^5}$$

$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}} b^4c^4 - 4\,(ab^3)^{\frac{1}{4}} ab^3c^3d + 6\,(ab^3)^{\frac{1}{4}} a^2b^2c^2d^2 - 4\,(ab^3)^{\frac{1}{4}} a^3bcd^3 + (ab^3)^{\frac{1}{4}} a^4d^4\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right.\right.}{8\,ab^5}$$

$$+ \frac{45\,b^{12}d^4x^{13} + 260\,b^{12}cd^3x^9 - 65\,ab^{11}d^4x^9 + 702\,b^{12}c^2d^2x^5 - 468\,ab^{11}cd^3x^5 + 117\,a^2b^{10}d^4x^5 + 2340\,b^{12}c^3d}{585\,b^{13}}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3
)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d
^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1
/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)
^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/
8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^
(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^
2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12
*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^
5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*x + 2340
*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 1822, normalized size of antiderivative = 5.49

$$\int \frac{(c + dx^4)^4}{a + bx^4} \, dx = \text{Too large to display}$$

[In] int((c + d*x^4)^4/(a + b*x^4),x)

[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b) - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4*1i)/(2*(-a)^(3/4)*b^(17/4)) + (atan(((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^

```
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 +
 a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)
^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4
)/(2*(-a)^(3/4)*b^(17/4))
```

3.161 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\begin{aligned}\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = {} & \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b} \\ & -\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} \\ & -\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} \\ & +\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}\end{aligned}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/5*d^2*(-a*d+3*b*c)*x^5/b^2+1/9*d^3*
x^9/b+1/4*(-a*d+b*c)^3*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(13/4
)*2^(1/2)+1/4*(-a*d+b*c)^3*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1
3/4)*2^(1/2)-1/8*(-a*d+b*c)^3*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))/a^(3/4)/b^(13/4)*2^(1/2)+1/8*(-a*d+b*c)^3*ln(a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}} - \frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3} + \frac{d^2x^5(3bc-ad)}{5b^2} + \frac{d^3x^9}{9b}$$

```
[In] Int[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^8}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\,(a+bx^4)}\right)dx$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^4}\,dx}{b^3}$$

$$=\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^3}+\frac{(bc-ad)^3\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^3}$$

$$= \frac{d(3b^2c^2 - 3abcd + a^2d^2)\,x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc-ad)^3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{4\sqrt{a}b^{7/2}} + \frac{(bc-ad)^3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{4\sqrt{a}b^{7/2}} - \frac{(bc-ad)^3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{4\sqrt{2}a^{3/4}b^{13/4}} - \frac{(bc-ad)^3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{4\sqrt{2}a^{3/4}b^{13/4}}$$

$$= \frac{d(3b^2c^2 - 3abcd + a^2d^2)\,x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc-ad)^3 \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} - \frac{(bc-ad)^3 \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}$$

$$= \frac{d(3b^2c^2 - 3abcd + a^2d^2)\,x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc-ad)^3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} - \frac{(bc-ad)^3 \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$

$$= \frac{360a^{3/4}\sqrt[4]{b}d(3b^2c^2-3abcd+a^2d^2)\,x-72a^{3/4}b^{5/4}d^2(-3bc+ad)x^5+40a^{3/4}b^{9/4}d^3x^9-90\sqrt{2}(bc-ad)^3\,\mathrm{arct}}{}$$

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(
5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/
4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.45

method	result
risch	$\frac{d^3x^9}{9b}-\frac{d^3a\,x^5}{5b^2}+\frac{3d^2c\,x^5}{5b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^4}$
default	$\frac{d\left(\frac{1}{9}b^2d^2x^9-\frac{1}{5}ab\,d^2x^5+\frac{3}{5}b^2cd\,x^5+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{8b^3a}$

```
[In] int((d*x^4+c)^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*d^3*x^9/b-1/5*d^3/b^2*a*x^5+3/5*d^2/b*c*x^5+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x+1/4/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/_
R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1642, normalized size of antiderivative = 5.70

$$\int \frac{(c+dx^4)^3}{a+bx^4} dx = \text{Too large to display}$$

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 45*b^3*(-(b^12*c^1
2 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 12*a
*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 49
5*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*
c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*x) - 45*I*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*
b^13))^(1/4)*log(I*a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10
*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 92
4*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3
*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))
^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*I*b^3*
(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-I*a*b^3*(-(b^
12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)
+ 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-220a^9b^3c^3d^9+495a^8b^4c^4d^8-792a^7b^5c^5d^7\right.$$
$$+\frac{d^3x^9}{9b}$$

```
[In] integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b
*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5
*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a*
*2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-
4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x
))) + d**3*x**9/(9*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \frac{5\,b^2d^3x^9+9\,(3\,b^2cd^2-abd^3)x^5+45\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{45\,b^3}$$
$$+\frac{\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{8\,}$$

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(219) = 438$.

Time = 0.29 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$+ \frac{5\,b^8d^3x^9 + 27\,b^8cd^2x^5 - 9\,ab^7d^3x^5 + 135\,b^8c^2dx - 135\,ab^7cd^2x + 45\,a^2b^6d^3x}{45\,b^9}$$

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3
)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^8*d^3*x^9
+ 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x
+ 45*a^2*b^6*d^3*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1433, normalized size of antiderivative = 4.98

$$\int \frac{(c+dx^4)^3}{a+bx^4} \, dx = \text{Too large to display}$$

```
[In] int((c + d*x^4)^3/(a + b*x^4),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
- (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2
*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^
(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d
 - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4
*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((((x*(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b
^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d
- b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*
a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4))
))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x*(a^6*d^6 + b^6*c
^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5
*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12
*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3
)/((-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3
 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*
d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/(((
(x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3
 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13
/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b
^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(
(-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/(2*(-a)^(3/4)*b^(13/4))
```

3.162 $\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 253

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/5*d^2*x^5/b+1/4*(-a*d+b*c)^2*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+b^(1/4)*x*2
^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {398, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}} - \frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

```
[In] Int[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^4}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^4\right)}\right)dx\\ &= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int\frac{1}{a+bx^4}\,dx}{b^2}\\ &= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^2}+\frac{(bc-ad)^2\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^2}\end{aligned}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{5/2}} + \frac{(bc-ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{5/2}} - \frac{(bc-ad)^2 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{9/4}} - \frac{(bc-ad)^2 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{9/4}}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2 \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} - \frac{(bc-ad)^2 \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} - \frac{(bc-ad)^2 \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^4)^2}{a+bx^4} dx$$

$$= \frac{-40a^{3/4}\sqrt[4]{b}d(-2bc+ad)x + 8a^{3/4}b^{5/4}d^2x^5 - 10\sqrt{2}(bc-ad)^2 \arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 10\sqrt{2}(bc-ad)^2 \text{arcta}}{}$$

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*(b
*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a*d)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4
)*b^(9/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

method	result
risch	$\frac{d^2x^5}{5b} - \frac{d^2ax}{b^2} + \frac{2dcx}{b} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)} \frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^3}}{4b^3}$
default	$-\frac{d(-\frac{1}{5}bd\,x^5+adx-2bcx)}{b^2} + \frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-\cdots\right.\right.}{8b^2a}$

```
[In] int((d*x^4+c)^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*c*x+1/4/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.32

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{4\,bd^2x^5+5\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac14}\log\left(ab^2\left(-\cdots\right.\right.}{\cdots}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 + 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 -
8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3
*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*b^2*(-(b^8*c^8 - 8*
```

```
a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b
^9))^(1/4)*log(I*a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*x) - 5*I*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a
^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-I*a*b^2*(-(b^8*c^8 - 8*
a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b
^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))
^(1/4)*log(-a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x) + 20*(2*b*c*d - a*d^2)*x)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)$$
$$+\text{RootSum}\left(256t^4a^3b^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6\right.$$
$$+\frac{d^2x^5}{5b}$$

```
[In] integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a*
*7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**
4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c*
*7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2) + x))) + d**2*x**5/(5*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{bd^2x^5+5\,(2\,bcd-ad^2)x}{5\,b^2} + \frac{\frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(b^2c^2-2\,abc}{}}{8\,b^2}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{(c+dx^4)^2}{a+bx^4}\,dx &= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c^2-2\,(ab^3)^{\frac{1}{4}}\,abcd+(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\ &+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c^2-2\,(ab^3)^{\frac{1}{4}}\,abcd+(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\ &+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c^2-2\,(ab^3)^{\frac{1}{4}}\,abcd+(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} \\ &- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c^2-2\,(ab^3)^{\frac{1}{4}}\,abcd+(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} \\ &+ \frac{b^4d^2x^5+10\,b^4cdx-5\,ab^3d^2x}{5\,b^5}\end{aligned}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d +
(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/5*(b
^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 1081, normalized size of antiderivative = 4.27

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{d^2\,x^5}{5\,b} - x\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b} - \frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{(-a)^{3/4}\,b^{9/4}} + \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b} - \frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}} - \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c \cdots\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{2\,(-a)^{3/4}\,b^{9/4}} + \frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b} - \frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)1\mathrm{i}}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}} + \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b} - \frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)1\mathrm{i}}{4\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{(-a)^{3/4}\,b^{9/4}} - \frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3 \cdots\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{2\,(-a)^{3/4}\,b^{9/4}}$$

```
[In] int((c + d*x^4)^2/(a + b*x^4),x)
```

```
[Out] (d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b -
 ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*
b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4
*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a
)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3
*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*
d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
```

```
a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)))/(((a*d - b
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/
(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a
*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b
^(9/4)))*1i)/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2)/(2*(-a)^(3/4)*b^(9/4))
```

3.163 $\int \frac{c+dx^4}{a+bx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 223

$$\int \frac{c+dx^4}{a+bx^4}\,dx=\frac{dx}{b}-\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

```
[Out] d*x/b+1/4*(-a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)+1/4*(-a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)-1/8*(-a*d+b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^
(3/4)/b^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used

= {396, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{c+dx^4}{a+bx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{dx}{b}$$

```
[In] Int[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^4}\,dx}{b} \\
&= \frac{dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b} + \frac{(bc-ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b} \\
&= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}} \\
&\quad - \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}
\end{aligned}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{8a^{3/4}\sqrt[4]{b}dx - 2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}(bc-ad)\log}{8a^{3/4}b^{5/4}}$$

```
[In] Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(8*a^(3/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.94 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$	42
default	$\frac{dx}{b}+\frac{(-ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ba}$	120

```
[In] int((d*x^4+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*x/b+1/4/b^2*sum((-a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.51

$$\int\frac{c+dx^4}{a+bx^4}\,dx=$$
$$-\frac{b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)+\cdots}{\cdots}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) + I*b*(-(b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*
b^5))^(1/4)*log(I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - I*b*(-(b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4
)*log(-I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - b*(-(b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(-a*b*
(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \text{RootSum}\left(256t^4a^3b^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(-\frac{4tab}{ad-bc}+x\right)\right)\right) + \frac{dx}{b}$$

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*
c) + x))) + d*x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} + \frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b*
c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^4}{a+bx^4}\,dx=\frac{dx}{b}+\frac{\sqrt{2}\Big((ab^3)^{\frac14}\,bc-(ab^3)^{\frac14}\,ad\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\,ab^2}+\frac{\sqrt{2}\Big((ab^3)^{\frac14}\,bc-(ab^3)^{\frac14}\,ad\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\,ab^2}+\frac{\sqrt{2}\Big((ab^3)^{\frac14}\,bc-(ab^3)^{\frac14}\,ad\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2}-\frac{\sqrt{2}\Big((ab^3)^{\frac14}\,bc-(ab^3)^{\frac14}\,ad\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)
^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*
b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int \frac{c+dx^4}{a+bx^4}\,dx=\frac{d\,x}{b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}+\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}\right)\cdots}{2\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}+\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}{\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)-\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\right)\,\left(a\,d-b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\left(x\,\left(4\,a^2\,b\,d^2-8\,a\,b^2\,c\,d+4\,b^3\,c^2\right)+\frac{\left(16\,a^2\,b^2\,d-16\,a\,b^3\,c\right)\,\left(a\,d-b\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}\cdots}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}}\right)\cdots}{2\,{\left(-a\right)}^{3/4}\,b^{5/4}}$$

```
[In] int((c + d*x^4)/(a + b*x^4),x)

[Out] (d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2
*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-
a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*
b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (atan((((x*
(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d
 - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)) + (
(x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a*b^3*c)*(
a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))
/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c
)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(
5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a
*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/
4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

3.164 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 449

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {400, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}\\ &= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)}\end{aligned}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)}$$
$$- \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)} - \frac{\sqrt{d}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}(bc-ad)} - \frac{\sqrt{d}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}(bc-ad)}$$
$$+ \frac{d^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}$$
$$+ \frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$
$$+ \frac{b^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}$$
$$- \frac{d^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}$$
$$+ \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$
$$- \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}$$
$$+ \frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a^{3/4}\cdots}{\cdots}$$

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a}+\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+\cdots\right)}{8\cdots}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4c^3d^4-4c^4d^3a^3b+6c^5d^2a^2b^2-4c^6da\,b^3+b^4c^7\right)\_Z^4+d^3\right)}\_R\ln\left(\left(\left(\left(-a^7d^7+4a^6bc\,d^6-6a^5b^2c^2d^5+3a^4b^3c^3d^4+3a^3b^4c^4d^3\cdots\right.\right.\right.\right.\right.$

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^
(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2
)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1
/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.61

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^
3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^
3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/
(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^
4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d +
6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^
5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^3*b^4*c^4
 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*lo
g(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b
^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
 + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a
^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2
- a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d
^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (I*b*c^2 - I*a*
c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 +
 a^4*c^3*d^4))^(1/4)) - 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^
5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (-I*b*c^2 + I*a*c*d
)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^
4*c^3*d^4))^(1/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}\right.}{a^{\frac{3}{4}}}}{8(bc-ad)}$$

$$- \frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}} - \frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2\right.}{c}}{8(bc-ad)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
```

```
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 6153, normalized size of antiderivative = 13.70

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c
^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^
7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d
^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11
 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2
*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
```

3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^1
0) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*
a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7
*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*
b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b
^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256
*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 10
24*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3
*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^
4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10

*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^
4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d
^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*
a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^
6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7
x)(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^
2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^
4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((
-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2
*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 102
4*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 +
256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*
d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1
536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (12
8*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^
5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*
a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)
- (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^
3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(2
56*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4
- 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*
b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a
^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256
*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3
) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3
*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*
b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*
d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*
c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^
2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^
3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3
*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^
7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^
3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d

```
^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*
c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (
768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3
+ 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c
^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b
^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3
*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7
+ 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*
c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2
560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*
b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*
a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 +
2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
```

3.165 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 513

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = &-\frac{dx}{4c(bc-ad)(c+dx^4)}-\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}\\&+\frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}\\&+\frac{d^{3/4}(7bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\\&-\frac{d^{3/4}(7bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\\&-\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}\\&+\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}\\&+\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}\\&-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}\end{aligned}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(
1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a
```

```
^(1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctan(-1+
d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*
d+7*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1
/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-a*d
+b*c)^2*2^(1/2)+1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/32*d^(3/4)*(-3*a*d+7*b*c)*ln(-c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/32*d^(3/
4)*(-3*a*d+7*b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)
/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = -\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} - \frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{d^{3/4}(7bc-3ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{dx}{4c(c+dx^4)(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sq
```

```
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*
b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b
*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1
/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b
*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqr
t[2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4c(bc-ad)}\\
&= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{1}{a+bx^4}\,dx}{(bc-ad)^2}-\frac{(d(7bc-3ad))\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^2}\\
&= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)^2}+\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)^2}\\
&\quad -\frac{(d(7bc-3ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}(bc-ad)^2}-\frac{(d(7bc-3ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}(bc-ad)^2}
\end{aligned}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)^2} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)^2}$$

$$- \frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)^2} - \frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)^2}$$

$$- \frac{\left(\sqrt{d}(7bc-3ad)\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}(bc-ad)^2} - \frac{\left(\sqrt{d}(7bc-3ad)\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}(bc-ad)^2}$$

$$+ \frac{\left(d^{3/4}(7bc-3ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}(bc-ad)^2} + \frac{\left(d^{3/4}(7bc-3ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{16\sqrt{2}c^{7/4}(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}$$

$$+ \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}$$

$$+ \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}$$

$$- \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}$$

$$+ \frac{b^{7/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} - \frac{b^{7/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}$$

$$- \frac{\left(d^{3/4}(7bc-3ad)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}$$

$$+ \frac{\left(d^{3/4}(7bc-3ad)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}$$
$$+ \frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}$$
$$- \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}$$
$$+ \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}$$
$$- \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$$
$$= \frac{8a^{3/4}c^{3/4}d(-bc+ad)x - 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1+\right.}{}$$

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
 d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)
*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*S
qrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(3/4)*
d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*
c - a*d)^2*(c + d*x^4))
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.51

method	result
default	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)^2a}+\frac{d\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\frac{(3ad-7bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\ldots\right.\right.}{\ldots}\right.}{\ldots}$
risch	Expression too large to display

[In] int(1/(b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+d/(a*d-b*c)^2*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.02 (sec) , antiderivative size = 2955, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+bx^4)\,(c+dx^4)^2}\,dx = \text{Too large to display}$$

[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] 1/16*(4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^

2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(-I*(b*c^2*d - a*c*d^2)*x^4 - I*b
*c^3 + I*a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5
*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5
 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(I*a*b^2*c^2 - 2*
I*a^2*b*c*d + I*a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5
*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5
 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(I*(b*c^2*d - a*c
*d^2)*x^4 + I*b*c^3 - I*a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7
*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*
a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(-
I*a*b^2*c^2 + 2*I*a^2*b*c*d - I*a^3*d^2)) + ((b*c^2*d - a*c*d^2)*x^4 + b*c^
3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^
5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c
^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 +
 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d -
 3*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b
^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a
^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)) - ((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2
401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d
^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*
b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d
^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x - (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7
*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))
^(1/4)) + (-I*(b*c^2*d - a*c*d^2)*x^4 - I*b*c^3 + I*a*c^2*d)*(-(2401*b^4*c^
4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^
4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d
^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7
*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x - (I*b^2*c^4 -
2*I*a*b*c^3*d + I*a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2
646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^1
4*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*
a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/
4)) + (I*(b*c^2*d - a*c*d^2)*x^4 + I*b*c^3 - I*a*c^2*d)*(-(2401*b^4*c^4*d^3
 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7
)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 +
70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^
8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x - (-I*b^2*c^4 + 2*I*
a*b*c^3*d - I*a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*
a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d
+ 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*

b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = -\frac{dx}{4\left((bc^2d-acd^2)x^4+bc^3-ac^2d\right)} + \frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2\right.}{}}{8\left(b^2c^2-2abcd+a^2d^2\right)} - \frac{\frac{2\sqrt{2}(7bcd-3ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(7bcd-3ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(7bcd-3ad^2)\log\left(\sqrt{d}x^2+\sqrt{\right.}}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^2c^3-2abc^2d+a^2cd^2\right)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] -1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/32*(2*sq
rt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2
)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b
*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(
```

```
3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} + \frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} + \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} + \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{dx}{4\,(dx^4+c)(bc^2-acd)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b
```

```
^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sq
rt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4
)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqr
t(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2
)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1
/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/16*(7*(c
*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/
4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 21975, normalized size of antiderivative = 42.84

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

```
[Out] 2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a
^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458752
0*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 5
24288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^
8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b
^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008
*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28
*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a
^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
```

4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
- 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5
+ 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c
^9*d)))*1i) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8
- 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c
^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6553
6*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^1
3*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*
a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^
8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2
*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c
^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(2867
2*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 9011
2*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 32
9728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^1
6*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 280985
6*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^1
0 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b
^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 46694
4*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*
d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^

2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i)*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^1

2*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b
^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((((81*a^4*b^7*d^10)/16 + 28*b^11*
c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9
*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)
 + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2
*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*
c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4
*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288
*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d
^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^
7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c
^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6
*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*
a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672
*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253
952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 +
3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2
*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^
15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 33628
16*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d
^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*
b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864
*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*
a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))
)*1i)*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 -
756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4) - atan(((-(81*a^4*d^7 + 240
1*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6
)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*
b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 367001
6*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*
((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d
^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^
8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b
^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a
*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10
*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7
 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6
5536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*

c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) +

(x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^
6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^1
2*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 2528
0512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^
14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*
c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d
^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i + (x*(8
1*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d
^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*
d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^
5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a
^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524
288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4
587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^
6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 655
36*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*
a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18
35008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16
+ 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (19
71*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
- 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 +
15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*
d)))) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756
*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*
a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^
4 - 6*a*b^5*c^9*d))) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^

4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7 *d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^ 12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6* b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8* c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^1 0*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7 *d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d ^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2* c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^1 5 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3 670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d ^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13* c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11* c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^ 8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10 *b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c ^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13* c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 2419097 6*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5* c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b* c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6* a*b^5*c^9*d)))) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3 *d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^ 4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4* b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a* b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 655 36*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016* a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18 35008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*2i - atan(((-b^7/(256*a ^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 143 36*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a ^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b ^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 20 48*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b ^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4

*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)

/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4
*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 +
 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9
*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a
^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*
b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*
a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14
336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*
a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*
b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3
 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2
048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4
*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c
^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))
^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^1
2*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*
c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^
6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 -
524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*
d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*
b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17
833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*
d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 +
 a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 +
15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4
*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^
2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) - (x*
(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c
*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d
^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5
*c^9*d))) + (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 71
68*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*
a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(
256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2
 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7
168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*
a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5
*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6
 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048
*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b
^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d
^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12
*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*

b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*2i + 2*atan(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6

*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d
^4 - 6*a*b^5*c^9*d))) - (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^
7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*
d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1
/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*
b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3
*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^1
1*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336
*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9
*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^
8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 +
17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 204
8*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78
848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 -
229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11
+ 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^
14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*
b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 346931
2*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9
- 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b
^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 26091
52*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*
1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20
*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7
*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9
)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*
d^2 - 3*a*b^2*c^6*d)) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^
12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 +
a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 1
5*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^
8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 179
20*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^
10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^
7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4
- 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*
((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*
c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3
*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d
^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^
6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^
2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^1
4*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*
b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a
^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*
a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*

d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i + (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4) + (d*x)/(4*c*(c + d*x^4)*(a*d - b*c))

3.166 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 407

$$\begin{aligned}\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx &= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\ &\quad+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}+\frac{d^4(5bc-2ad)x^9}{9b^3}+\frac{d^5x^{13}}{13b^2} \\ &\quad+\frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)}-\frac{(bc-ad)^4(3bc+17ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ &\quad+\frac{(bc-ad)^4(3bc+17ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ &\quad-\frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\ &\quad+\frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}\end{aligned}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/5*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^5/b^4+1/9*d^4*(-2*a*d+5*b*c)*x^9/b^3+1/13*
d^5*x^13/b^2+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^4+a)+1/16*(-a*d+b*c)^4*(17*a*d+3
*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21/4)*2^(1/2)+1/16*(-
a*d+b*c)^4*(17*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21
/4)*2^(1/2)-1/32*(-a*d+b*c)^4*(17*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1/2)+1/32*(-a*d+b*c)^4*(17*a*d+3*b
*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}} - \frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} + \frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} + \frac{d^3x^5(3a^2d^2-10abcd+10b^2c^2)}{5b^4} + \frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{4ab^5(a+bx^4)} + \frac{d^4x^9(5bc-2ad)}{9b^3} + \frac{d^5x^13}{13b^2}$$

```
[In] Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{b^4}\right.\\
&\qquad \left.+\frac{d^4(5bc-2ad)x^8}{b^3}+\frac{d^5x^{12}}{b^2}+\frac{(bc-ad)^4(bc+4ad)+5bd(bc-ad)^4x^4}{b^5\,(a+bx^4)^2}\right)dx\\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}\\
&\quad +\frac{d^4(5bc-2ad)x^9}{9b^3}+\frac{d^5x^{13}}{13b^2}+\frac{\int\frac{(bc-ad)^4(bc+4ad)+5bd(bc-ad)^4x^4}{(a+bx^4)^2}\,dx}{b^5}\\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}\\
&\quad +\frac{d^4(5bc-2ad)x^9}{9b^3}+\frac{d^5x^{13}}{13b^2}+\frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)}+\frac{((bc-ad)^4(3bc+17ad))\int\frac{1}{a+bx^4}\,dx}{4ab^5}\\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4}\\
&\quad +\frac{d^4(5bc-2ad)x^9}{9b^3}+\frac{d^5x^{13}}{13b^2}+\frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)}\\
&\quad +\frac{((bc-ad)^4(3bc+17ad))\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^5}+\frac{((bc-ad)^4(3bc+17ad))\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\
&\quad + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4} + \frac{d^4(5bc-2ad)x^9}{9b^3} + \frac{d^5x^{13}}{13b^2} \\
&\quad + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)} + \frac{((bc-ad)^4(3bc+17ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{11/2}} \\
&\quad + \frac{((bc-ad)^4(3bc+17ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{11/2}} \\
&\quad - \frac{((bc-ad)^4(3bc+17ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad - \frac{((bc-ad)^4(3bc+17ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{21/4}} \\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4} \\
&\quad + \frac{d^4(5bc-2ad)x^9}{9b^3} + \frac{d^5x^{13}}{13b^2} + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)} \\
&\quad - \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad + \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad + \frac{((bc-ad)^4(3bc+17ad))\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad - \frac{((bc-ad)^4(3bc+17ad))\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\
&\quad + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4} + \frac{d^4(5bc-2ad)x^9}{9b^3} + \frac{d^5x^{13}}{13b^2} \\
&\quad + \frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)} - \frac{(bc-ad)^4(3bc+17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad + \frac{(bc-ad)^4(3bc+17ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad - \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\
&\quad + \frac{(bc-ad)^4(3bc+17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.96

$$
\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx
$$

$$
= \frac{18720\sqrt[4]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+3744b^{5/4}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5+2080b^9}{}
$$

```
[In] Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*b^
(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(b*c
 - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*
d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (5
85*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(18720*b
^(21/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.75

method	result
risch	$\frac{d^5x^{13}}{13b^2}-\frac{2d^5a\,x^9}{9b^3}+\frac{5d^4c\,x^9}{9b^2}+\frac{3d^5a^2x^5}{5b^4}-\frac{2d^4ac\,x^5}{b^3}+\frac{2d^3c^2x^5}{b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(a^5d^5}{}$
default	$-\frac{d^2\left(-\frac{1}{13}b^3d^3x^{13}+\frac{2}{9}a\,b^2d^3x^9-\frac{5}{9}b^3c\,d^2x^9-\frac{3}{5}a^2b\,d^3x^5+2a\,b^2c\,d^2x^5-2b^3c^2d\,x^5+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+\frac{-\frac{(a}{}}{}$

```
[In] int((d*x^4+c)^5/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*d^5*x^13/b^2-2/9*d^5/b^3*a*x^9+5/9*d^4/b^2*c*x^9+3/5*d^5/b^4*a^2*x^5-2
*d^4/b^3*a*c*x^5+2*d^3/b^2*c^2*x^5-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-20*d^
3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/4*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^4+a)+1/16/b^6/a*su
m((17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d^3-50*a^2*b^3*c^3*d^2+5*a*b^4*
c^4*d+3*b^5*c^5)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 2884, normalized size of antiderivative = 7.09

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 20
8*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a*b
^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 + 58
5*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18
*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*
b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500
110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^
10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 54823
1440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c
^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 909483
0*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^
```

```
(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*
d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c
^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^
8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^1
0 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a
^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^1
5 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18
*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)
+ (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65
*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(-I*a*b^6*x^4 - I*a^2*b^5)*(-(81*b^20*c
^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3
+ 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^1
4*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9
*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^1
1 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a
^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^1
6 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b
*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(I*a^2*b^5*(-(81*b^20*c^20
+ 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 13
2205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^
6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^1
1*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 +
517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*
b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 -
39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d
^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 58
5*(I*a*b^6*x^4 + I*a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*
b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*
a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^1
0*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5
48231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b
^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90
94830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^2
1))^(1/4)*log(-I*a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d
^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20
```

```
*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^
3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c
^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a
^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d
^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840
*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d
^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19
*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(-a^2*b^5*(-(81*b^20*c^20
 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 1
32205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d
^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^
11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 +
 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14
*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 -
 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*
d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50
*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) + 2
340*(b^5*c^5 - 5*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a^3*b^2*c^2*d^3 + 65
*a^4*b*c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{4\,(ab^6x^4 + a^2b^5)} + \frac{45\,b^3d^5x^{13} + 65\,(5\,b^3cd^4 - 2\,ab^2d^5)x^9 + 117\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^5 + 585\,(10\,b^3c^3d^2 - 20\,ab^2\cdots}{585\,b^5} + \frac{\frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 + 6
5*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 +
3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4
- 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*
b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*
x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b
^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b
*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/(a*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(334) = 668$.

Time = 0.29 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.96

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$$

$$= \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^5c^5 + 5\,(ab^3)^{\frac{1}{4}}\,ab^4c^4d - 50\,(ab^3)^{\frac{1}{4}}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac{1}{4}}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac{1}{4}}\,a^4bcd^4 + 17\,(ab^3)}{16\,a^2b^6}$$

$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^5c^5 + 5\,(ab^3)^{\frac{1}{4}}\,ab^4c^4d - 50\,(ab^3)^{\frac{1}{4}}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac{1}{4}}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac{1}{4}}\,a^4bcd^4 + 17\,(a}{16\,a^2b^6}$$

$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^5c^5 + 5\,(ab^3)^{\frac{1}{4}}\,ab^4c^4d - 50\,(ab^3)^{\frac{1}{4}}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac{1}{4}}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac{1}{4}}\,a^4bcd^4 + 17\,(a}{32\,a^2b^6}$$

$$- \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^5c^5 + 5\,(ab^3)^{\frac{1}{4}}\,ab^4c^4d - 50\,(ab^3)^{\frac{1}{4}}\,a^2b^3c^3d^2 + 90\,(ab^3)^{\frac{1}{4}}\,a^3b^2c^2d^3 - 65\,(ab^3)^{\frac{1}{4}}\,a^4bcd^4 + 17\,(a}{32\,a^2b^6}$$

$$+ \frac{b^5c^5x - 5\,ab^4c^4dx + 10\,a^2b^3c^3d^2x - 10\,a^3b^2c^2d^3x + 5\,a^4bcd^4x - a^5d^5x}{4\,(bx^4+a)ab^5}$$

$$+ \frac{45\,b^{24}d^5x^{13} + 325\,b^{24}cd^4x^9 - 130\,ab^{23}d^5x^9 + 1170\,b^{24}c^2d^3x^5 - 1170\,ab^{23}cd^4x^5 + 351\,a^2b^{22}d^5x^5 + 5850\,b^{2}}{585\,b^{26}}$$

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a
*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)
^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b
^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 9
0*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)
^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*
c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3
 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5
*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*
(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(
1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) + 1/4*
(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*(45*b^24*d^5*x^1
3 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*
a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^2
3*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 2490, normalized size of antiderivative = 6.12

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b^3
- (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x^13)
/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3
 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((((x*(2
89*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 301
0*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^
7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/
(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*
(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b
^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^
4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 221
0*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 +
```

3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(8*(-a)^(7/4)*b^(21/4)) + (atan(((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(21/4))

3.167 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 357

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)} - \frac{(bc-ad)^3(3bc+13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(3bc+13ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} - \frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9*
d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+3*
b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)+1/16*(-a
*d+b*c)^3*(13*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/
4)*2^(1/2)-1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+1/32*(-a*d+b*c)^3*(13*a*d+3*b*
c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}} - \frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}} + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{4ab^4(a+bx^4)} + \frac{2d^3x^5(2bc-ad)}{5b^3} + \frac{d^4x^9}{9b^2}$$

```
[In] Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4}+\frac{2d^3(2bc-ad)x^4}{b^3}+\frac{d^4x^8}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{b^4(a+bx^4)^2}\right)dx$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{\int\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{(a+bx^4)^2}dx}{b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4(a+bx^4)}+\frac{((bc-ad)^3(3bc+13ad))\int\frac{1}{a+bx^4}dx}{4ab^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4(a+bx^4)}+\frac{((bc-ad)^3(3bc+13ad))\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx}{8a^{3/2}b^4}+\frac{((bc-ad)^3(3bc+13ad))\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}dx}{8a^{3/2}b^4}$$

$$=\frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4(a+bx^4)}+\frac{((bc-ad)^3(3bc+13ad))\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16a^{3/2}b^{9/2}}+\frac{((bc-ad)^3(3bc+13ad))\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16a^{3/2}b^{9/2}}-\frac{((bc-ad)^3(3bc+13ad))\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{16\sqrt{2}a^{7/4}b^{17/4}}-\frac{((bc-ad)^3(3bc+13ad))\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{16\sqrt{2}a^{7/4}b^{17/4}}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)}$$
$$-\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$
$$+\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$
$$+\frac{((bc-ad)^3(3bc+13ad))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}$$
$$-\frac{((bc-ad)^3(3bc+13ad))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}$$
$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}$$
$$+\frac{(bc-ad)^4x}{4ab^4\,(a+bx^4)}-\frac{(bc-ad)^3(3bc+13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}$$
$$+\frac{(bc-ad)^3(3bc+13ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}$$
$$-\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$
$$+\frac{(bc-ad)^3(3bc+13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96

$$\int\frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$= \frac{1440\sqrt[4]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+576b^{5/4}d^3(2bc-ad)x^5+160b^{9/4}d^4x^9+\frac{360\sqrt[4]{b}(bc-ad)^4x}{a(a+bx^4)}+\frac{90\sqrt{2}(-bc+ad)}{}}{}$$

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(
2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(
```

```
a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c)
+ a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.61

method	result
risch	$\frac{d^4x^9}{9b^2} - \frac{2d^4a\,x^5}{5b^3} + \frac{4d^3c\,x^5}{5b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)x}{4a\,b^4(b\,x^4+a)} - \frac{\sum_{\_R=\text{RootO}}}{}$
default	$\frac{d^2\left(\frac{1}{9}b^2d^2x^9-\frac{2}{5}ab\,d^2x^5+\frac{4}{5}b^2cd\,x^5+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{4a\left(b\,x^4+a\right)}+\frac{\left(13a^4d^4-36a^3bc\,d^3\right.}{}}{}$

```
[In] int((d*x^4+c)^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*d^4*x^9/b^2-2/5*d^4/b^3*a*x^5+4/5*d^3/b^2*c*x^5+3*d^4/b^4*a^2*x-8*d^3/b
^3*a*c*x+6*d^2/b^2*c^2*x+1/4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)/a*x/b^4/(b*x^4+a)-1/16/b^5/a*sum((13*a^4*d^4-36*a^3*b*c*d
^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(_Z^4
*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 2315, normalized size of antiderivative = 6.48

$$\int \frac{\left(c+dx^4\right)^4}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*(
30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 45*(a*b^5*x^4 + a
^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304
*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 43
4808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
```

4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5
*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^1
4*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*
log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 -
8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5
- 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^
8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5
*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784
*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1
/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13
*a^4*d^4)*x) - 45*(I*a*b^5*x^4 + I*a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^
15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^1
2*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9
*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*
b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 465
1504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 +
 28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(I*a^2*b^4*(-(81*b^16*c^16 + 432*a*b
^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^
12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a
^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912
*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12
 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*
d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30
*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) - 45*(-I*a*b^5*x^4 - I*a
^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304
*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 43
4808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5
*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^1
4*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*
log(-I*a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2
 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d
^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8
*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*
b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577
784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))
^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 -
 13*a^4*d^4)*x) + 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^1
5*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12
*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*
c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b
^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651
504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 +
28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(-a^2*b^4*(-(81*b^16*c^16 + 432*a*b^1
5*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12

```
*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7
*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a
^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 -
 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^
15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a
^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4*a*b^3*c
^3*d + 30*a^2*b^2*c^2*d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*x^4 + a^
2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{4\,(ab^5x^4 + a^2b^4)} + \frac{5\,b^2d^4x^9 + 18\,(2\,b^2cd^3 - abd^4)x^5 + 45\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{45\,b^4} + \frac{\frac{2\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3}{\sqrt{a}\sqrt{\sqrt{}}}}{}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^4)
*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqrt(2
)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 + 4*a*b
^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
```

```
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*
c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^
(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d -
 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(286) = 572$.

Time = 0.29 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$= \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4+4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{16\,a^2b^5}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4+4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\arctan\left(\frac{\sqrt{2}}{}\right.}{16\,a^2b^5}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4+4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2+\sqrt{}\right.}{32\,a^2b^5}$$
$$-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^4c^4+4\,(ab^3)^{\frac{1}{4}}\,ab^3c^3d-30\,(ab^3)^{\frac{1}{4}}\,a^2b^2c^2d^2+36\,(ab^3)^{\frac{1}{4}}\,a^3bcd^3-13\,(ab^3)^{\frac{1}{4}}\,a^4d^4\right)\log\left(x^2-\sqrt{}\right.}{32\,a^2b^5}$$
$$+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{4\,(bx^4+a)ab^4}$$
$$+\frac{5\,b^{16}d^4x^9+36\,b^{16}cd^3x^5-18\,ab^{15}d^4x^5+270\,b^{16}c^2d^2x-360\,ab^{15}cd^3x+135\,a^2b^{14}d^4x}{45\,b^{18}}$$

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a
*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/
4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*
d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a
*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a
*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^
3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c
^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13
```

```
*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b
^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^
3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16*c*d^3
*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c*d^3*x + 135*a^
2*b^14*d^4*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2043, normalized size of antiderivative = 5.72

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(4*
a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c
^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 +
 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((
a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^
8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*
a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))
/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30
*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a
)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4))
 - (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3
 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*
a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*
c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*
c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(
7/4)*b^(17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(8*(-a)^(7/4)*b^(17/4))
 + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*
c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^
6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d
 + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36
*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/
```

```
(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d
 - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a^8*d^8 + 9*b^8*c^8 -
 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5
*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4
*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) - (((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*
(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(17/4))
```

3.168 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 317

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc-ad)^3x}{4ab^3(a+bx^4)} - \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a)
+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)
/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/
a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/32*(-a*d+b*c)^2*(3*a*d+b*c)*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+3/32*(-a*d
+b*c)^2*(3*a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/
4)/b^(13/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{4ab^3(a+bx^4)} + \frac{d^3x^5}{5b^2}$$

```
[In] Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2(3bc-2ad)}{b^3} + \frac{d^3x^4}{b^2} + \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\,(a+bx^4)^2} \right) dx$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{\int\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{(a+bx^4)^2}\,dx}{b^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\,(a+bx^4)}+\frac{(3(bc-ad)^2(bc+3ad))\int\frac{1}{a+bx^4}\,dx}{4ab^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\,(a+bx^4)}+\frac{(3(bc-ad)^2(bc+3ad))\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^3}+\frac{(3(bc-ad)^2(bc+3ad))\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\,(a+bx^4)}+\frac{(3(bc-ad)^2(bc+3ad))\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{7/2}}+\frac{(3(bc-ad)^2(bc+3ad))\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{7/2}}-\frac{(3(bc-ad)^2(bc+3ad))\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{(3(bc-ad)^2(bc+3ad))\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{13/4}}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\,(a+bx^4)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{(3(bc-ad)^2(bc+3ad))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{(3(bc-ad)^2(bc+3ad))\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc-ad)^3x}{4ab^3(a+bx^4)} - \frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \frac{160\sqrt[4]{b}d^2(3bc-2ad)x + 32b^{5/4}d^3x^5 + \frac{40\sqrt[4]{b}(bc-ad)^3x}{a(a+bx^4)} - \frac{30\sqrt{2}(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{30\sqrt{2}(bc-ad)^2(bc\cdots}{\cdots}}{1\cdots}$$

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
- a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*
c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(
b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.48

method	result
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4a\,b^3(b\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\ln\left(x-\right.}{\_R^3}\right.}{16b^4a}$
default	$-\frac{d^2\left(-\frac{1}{5}bd\,x^5+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{4a\left(b\,x^4+a\right)}+\frac{3\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}\right.\right.}{32a^2}}{b^3}$

[In] int((d*x^4+c)^3/(b*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/5*d^3*x^5/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^4+a)+3/16/b^4/a*sum((3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1741, normalized size of antiderivative = 5.49

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")

[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 15*(a*b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15*(-I*a*b^4*x^4 - I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*I*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15

```
*(I*a*b^4*x^4 + I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^1
0*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 64
4*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b
^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^
7*b^13))^(1/4)*log(-3*I*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^1
0*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*
a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12
)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)
*x) - 15*(a*b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
- 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a
^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)
/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b
^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d
^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 193
2*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^
12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^
3)*x) + 20*(b^3*c^3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 9*a^3*d^3)*x)/(a*b^4
*x^4 + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 86.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{4a^2b^3+4ab^4x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}-156492a^9b^3c^3d^9+84159a\right.$$
$$+\frac{d^3x^5}{5b^2}$$

```
[In] integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t**
4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b**2
*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 + 2656
8*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c**7*d*
*5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2*b**10
*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*log(16*_t
*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3)
 + x))) + d**3*x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{4\,(ab^4x^4+a^2b^3)} + \frac{bd^3x^5+5\,(3\,bcd^2-2\,ad^3)x}{5\,b^3}$$

$$+ \frac{3\left(\frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{}$$

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2*
b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*(b
^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^
3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2*c^2*d
- 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2
 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)))/(a*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$
$$= \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^4}$$
$$+ \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^4}$$
$$+ \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}$$
$$- \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^4}$$
$$+ \frac{b^3c^3x - 3ab^2c^2dx + 3a^2bcd^2x - a^3d^3x}{4(bx^4+a)ab^3} + \frac{b^8d^3x^5 + 15b^8cd^2x - 10ab^7d^3x}{5b^{10}}$$

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
- 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
+ (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)
*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/4*(b^3
*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3)
 + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 1616, normalized size of antiderivative = 5.10

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x^4)^3/(a + b*x^4)^2,x)
```

```
[Out] (d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a*d
 - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*
a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*
a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b
^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^
(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b
^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^
5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b
^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(1
6*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 +
b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*
b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(
36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*
b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x
*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*
c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(
3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(
16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d +
 b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d - b*c)^2*(3*a*d + b*c)
*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^
4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - ((a*d - b*c)
^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2
)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) + (3*(a*d - b*c)^
2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*
c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3)
 + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d -
 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)))/(
((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2
 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5)
)/(4*a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*
a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7
/4)*b^(13/4)) - ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9
*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d -
 30*a^5*b*c*d^5))/(4*a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 +
12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))
*3i)/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*
b^(13/4))
```

3.169 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 291

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} - \frac{(bc-ad)(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$

```
[Out] d^2*x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b*c
)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)
-1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {398, 393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{x(bc-ad)^2}{4ab^2(a+bx^4)} + \frac{d^2x}{b^2}$$

```
[In] Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2}{b^2} + \frac{b^2c^2 - a^2d^2 + 2bd(bc-ad)x^4}{b^2\left(a+bx^4\right)^2}\right) dx$$

$$= \frac{d^2x}{b^2} + \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{(a+bx^4)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{1}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^2} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}} + \frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}} - \frac{((bc-ad)(3bc+5ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{9/4}} - \frac{((bc-ad)(3bc+5ad))\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{9/4}}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} - \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{((bc-ad)(3bc+5ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} - \frac{((bc-ad)(3bc+5ad))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2 x}{4ab^2(a+bx^4)} - \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$

$$= \frac{32\sqrt[4]{b}d^2x + \frac{8\sqrt[4]{b}(bc-ad)^2x}{a(a+bx^4)} + \frac{2\sqrt{2}(-3b^2c^2-2abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\sqrt{2}(3b^2c^2+2abcd-5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}}{32b^{9/4}}$$

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[2
]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*
a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) +
(Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

method	result
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{4ab^2(bx^4+a)}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\frac{(5a^2d^2-2abcd-3b^2c^2)\ln(x-\_R)}{\_R^3}}{16b^3a}$
default	$\frac{d^2x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{4a(bx^4+a)}+\frac{(5a^2d^2-2abcd-3b^2c^2)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}}{b^2}$

```
[In] int((d*x^4+c)^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^4+a)-1/16/b^3/a*sum(
(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1210, normalized size of antiderivative = 4.16

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*
d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*
a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^
7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6
*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 9
00*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*
b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (I*a*b^3*x^4 + I*a^2*b^2)*(-(81*b^8*c
^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*
b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7
 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(I*a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(
a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (-I*a*b^3*x^4 -
I*a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*
b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*
d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(-I*a^2*b^2*(-(81
*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 64
6*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2
)*x) + (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*
```

```
c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5
- 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*lo
g(-a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*
b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*
d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b
*c*d - 5*a^2*d^2)*x) + 4*(b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^4 +
a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{4a^2b^2+4ab^3x^4} + \text{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b^4c^4d^4-984a^3\right. + \frac{d^2x}{b^2}$$

```
[In] integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + RootS
um(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*b*
*2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*a**
3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c*
*8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c*
*2) + x))) + d**2*x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(ab^3x^4+a^2b^2)} + \frac{d^2x}{b^2} + \frac{\frac{2\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3\,b^2c^2}{}}{32\,ab^2}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 + 1
/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
```

```
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sq
rt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$= \frac{d^2x}{b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$+ \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(bx^4+a)ab^2}$$

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d
 - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^
(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2
 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 +
 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((b*x^4 + a)*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1254, normalized size of antiderivative = 4.31

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4)) +
 (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*
a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9
/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^
4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/
(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b
^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a*d - b*c
)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^
3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/
4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26
*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*
c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4
)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d + 3*b*c)*1i)/(8*(
-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 +
9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b
) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*
1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d
 + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d
 - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 -
20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^
(9/4)))/(((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*
b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5
*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(2
5*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^
3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8
*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4))))*(a*d
 - b*c)*(5*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(9/4))
```

3.170 $\int \frac{c+dx^4}{(a+bx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 245

$$\int \frac{c+dx^4}{(a+bx^4)^2} \, dx = \frac{(bc-ad)x}{4ab(a+bx^4)} - \frac{(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

```
[Out] 1/4*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)
/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-1/32*(a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*(a*d+3*b*c)*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {393, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(ad+3bc)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(ad+3bc)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

```
[In] Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{1}{a+bx^4}\,dx}{4ab}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b} + \frac{(3bc+ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b}$$

$$= \frac{(bc-ad)x}{4ab\,(a+bx^4)} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}$$

$$- \frac{(3bc+ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{(bc-ad)x}{4ab(a+bx^4)} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+ad)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{(bc-ad)x}{4ab(a+bx^4)} - \frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{-\frac{8a^{3/4}\sqrt[4]{b}(-bc+ad)x}{a+bx^4} - 2\sqrt{2}(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}(3}{32a^{7/4}b^{5/4}}$$

```
[In] Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.27

method	result	size
risch	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(ad+3bc)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$	65
default	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{(ad+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2b}$	140

```
[In] int((d*x^4+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/4*(a*d-b*c)/b/a*x/(b*x^4+a)+1/16/a/b^2*sum((a*d+3*b*c)/_R^3*ln(x-_R),_R=
RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.64

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{(ab^2x^4+a^2b)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right.\right.}{}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
[Out] 1/16*((a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*
d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4 +
108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))
^(1/4) + (3*b*c + a*d)*x) - (-I*a*b^2*x^4 - I*a^2*b)*(-(81*b^4*c^4 + 108*a*
b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)
*log(I*a^2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*
b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) - (I*a*b^2*x^4 + I*a
^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3
 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-I*a^2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c +
 a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2
*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*b*(-(81*b^4*
c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7
*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{x(-ad+bc)}{4a^2b+4ab^2x^4} + \mathrm{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}+\right.\right.\right.$$

```
[In] integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*d
+ 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{(bc-ad)x}{4(ab^2x^4+a^2b)} + \frac{\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc+ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} + \frac{bcx-adx}{4\,(bx^4+a)ab}$$

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^3)
^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt
(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.02

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}\right)(a\,d+3\,b\,c)}{16\,(-a)^{7/4}\,b^{5/4}}+\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}\right)\cdots}{16\,(-a)^{7/4}\,b^{5/4}}}{\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}\right)(a\,d+3\,b\,c)\,1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}-\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}\right)\cdots}{16\,(-a)^{7/4}\,b^{5/4}}}\right)\cdots}{8\,(-a)^{7/4}\,b^{5/4}}$$

$$-\frac{x\,(a\,d-b\,c)}{4\,a\,b\,\left(b\,x^4+a\right)}$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,(-a)^{7/4}\,b^{5/4}}\right)(a\,d+3\,b\,c)\,1\mathrm{i}}{16\,(-a)^{7/4}\,b^{5/4}}+\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,(-a)^{7/4}\,b^{5/4}}\right)\cdots}{16\,(-a)^{7/4}\,b^{5/4}}}{\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}-\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,(-a)^{7/4}\,b^{5/4}}\right)(a\,d+3\,b\,c)}{16\,(-a)^{7/4}\,b^{5/4}}-\frac{\left(\frac{x\left(a^2\,b\,d^2+6\,a\,b^2\,c\,d+9\,b^3\,c^2\right)}{4\,a^2}+\frac{(a\,d+3\,b\,c)\left(12\,c\,b^3+4\,a\,d\,b^2\right)}{16\,(-a)^{7/4}\,b^{5/4}}\right)\cdots}{16\,(-a)^{7/4}\,b^{5/4}}}\right)\cdots}{8\,(-a)^{7/4}\,b^{5/4}}$$

```
[In] int((c + d*x^4)/(a + b*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)
*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((
a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)
*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/
(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(
a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b
^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^
(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)/(8*(-a)^
(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2)
 - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d
+ 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(
5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-
a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*
c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4))
))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c))/(4*a*b*(a + b*x^
4))
```

3.171 $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 513

$$\begin{aligned}\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx &= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} \\
&+ \frac{b^{3/4}(3bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} \\
&- \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} \\
&- \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&+ \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&- \frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} \\
&+ \frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}\end{aligned}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctan(-1+b^(1/4
)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/16*b^(3/4)*(-7*a*d+3*b*
c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/4*d^(
7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*
```

```
d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/
32*b^(3/4)*(-7*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/32*b^(3/4)*(-7*a*d+3*b*c)*ln(a^(1/4)*b^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(7/4)*
ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)^2*2^(
1/2)+1/8*d^(7/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/
(-a*d+b*c)^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {425, 536, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(3bc-7ad)}{8\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(3bc-7ad)}{8\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} - \frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{bx}{4a(a+bx^4)(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
```

```
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
 (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)}-\frac{\int \frac{-3bc+4ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4a(bc-ad)}\\
&= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)}+\frac{d^2\int \frac{1}{c+dx^4}\,dx}{(bc-ad)^2}+\frac{(b(3bc-7ad))\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^2}\\
&= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)}+\frac{d^2\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2}+\frac{d^2\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2}\\
&\quad+\frac{(b(3bc-7ad))\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^2}+\frac{(b(3bc-7ad))\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^2}
\end{aligned}
$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2}\,dx}{4\sqrt{c}(bc-ad)^2} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2}\,dx}{4\sqrt{c}(bc-ad)^2}$$

$$- \frac{d^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} + 2x}{-\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2}\,dx}{4\sqrt{2}c^{3/4}(bc-ad)^2} - \frac{d^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} - 2x}{-\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2}\,dx}{4\sqrt{2}c^{3/4}(bc-ad)^2}$$

$$+ \frac{\left(\sqrt{b}(3bc-7ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{16a^{3/2}(bc-ad)^2} + \frac{\left(\sqrt{b}(3bc-7ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{16a^{3/2}(bc-ad)^2}$$

$$- \frac{\left(b^{3/4}(3bc-7ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{16\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{\left(b^{3/4}(3bc-7ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{16\sqrt{2}a^{7/4}(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}$$

$$+ \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}$$

$$- \frac{d^{7/4}\log\left(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\log\left(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}$$

$$+ \frac{d^{7/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} - \frac{d^{7/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}$$

$$+ \frac{\left(b^{3/4}(3bc-7ad)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}$$

$$- \frac{\left(b^{3/4}(3bc-7ad)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}$$

$$
\begin{aligned}
&= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} \\
&+ \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} \\
&+ \frac{d^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&+ \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&- \frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.97

$$
\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx
$$

$$
= \frac{8a^{3/4}bc^{3/4}(bc-ad)x - 2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad)}{}
$$

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a
*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/4)
*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*
b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*
d)^2*(a + b*x^4))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.51

method	result
default	$-\frac{b\left(\frac{(ad-bc)x}{4a\left(b\,x^{4}+a\right)}+\frac{(7ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^{2}}\right)}{(ad-bc)^{2}}+\frac{d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\ldots\right.\right.}{\ldots}$
risch	Expression too large to display

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*d-b*c)^2*b*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(7*a*d-3*b*c)/a^2*(a/b)^
(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)))+1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)
^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.46 (sec) , antiderivative size = 2955, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+bx^4)^2\,(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*
c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*
a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^
3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56
*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5
*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2)) - 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c -
 a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 -
 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*
c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^
2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^
3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^
```

6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(-I*(a*b^2*c - a^2*b*d)*x^4 - I*a
^2*b*c + I*a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(I*b^2*c^3 - 2*I*
a*b*c^2*d + I*a^2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(I*(a*b^2*c - a^2
*b*d)*x^4 + I*a^2*b*c - I*a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10
*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*
b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(-
I*b^2*c^3 + 2*I*a*b*c^2*d - I*a^2*c*d^2)) - ((a*b^2*c - a^2*b*d)*x^4 + a^2*
b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*
a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b
^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^
5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c -
7*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^7*c^4 - 756*a*b
^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a
^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 7
0*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c
*d^7 + a^15*d^8))^(1/4)) + ((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(8
1*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a
^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c
^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x - (a^
2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a
^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^
8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^
4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))
^(1/4)) + (I*(a*b^2*c - a^2*b*d)*x^4 + I*a^2*b*c - I*a^3*d)*(-(81*b^7*c^4 -
756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3
*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5
*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*
a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x - (I*a^2*b^2*c^2
- 2*I*a^3*b*c*d + I*a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^
5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7
*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 5
6*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4
)) + (-I*(a*b^2*c - a^2*b*d)*x^4 - I*a^2*b*c + I*a^3*d)*(-(81*b^7*c^4 - 756
*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4
)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3
+ 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14
*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x - (-I*a^2*b^2*c^2 +
2*I*a^3*b*c*d - I*a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c
^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^
7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a

```
^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4))
+ 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx = \frac{\frac{2\sqrt{2}(3\,bc-7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3\,bc-7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3\,bc-7\,ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x\cdots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots}{32\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)} + \frac{bx}{4\left((ab^2c-a^2bd)x^4+a^2bc-a^3d\right)} + \frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x\cdots\right.}{\cdots}}{8\left(b^2c^2-2\,abcd+a^2d^2\right)}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
[Out] 1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*
sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3
*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
+ 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt(2)*d^2*
```

```
arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(
2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \frac{(cd^3)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} - \frac{(cd^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^2c^3-2\sqrt{2}abc^2d+\sqrt{2}a^2cd^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}bc-7\,(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}bc-7\,(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{\left(3\,(ab^3)^{\frac{1}{4}}bc-7\,(ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} - \frac{\left(3\,(ab^3)^{\frac{1}{4}}bc-7\,(ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)} + \frac{bx}{4\,(bx^4+a)(abc-a^2d)}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c*d
^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4
)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(
2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*
a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2
)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1
/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/16*(3*(a
*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/
4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 21975, normalized size of antiderivative = 42.84

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

```
[Out] 2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b
^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*
a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458752
0*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 -
524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*
c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*
b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 183
5008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4
- 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7
))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^1
4 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8
*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^
```

5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5
*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*
(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 4
66944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10
*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8
*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i) + (x*(3
185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d
^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
 + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
d^5)))(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4) - ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b
^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*
a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10
*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183500
8*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b
^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971
*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*
b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3
*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 6553
6*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^
10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835
008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4
 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6
 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^
9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*
d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d -
 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*
a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 -
8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c
^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*
a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 +
 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5)))*1i) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^
10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*

c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) + ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^

9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 +
2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^
3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a
^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670
016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/
4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c
^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*
b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^
11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5242
88*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28
672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 +
329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10
+ 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d
^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536
*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*
a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +
17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*
c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856
*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)
1i)/(64(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2
0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i - (x*(3185
*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12
+ 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
+ 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
d^5)))(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^
4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a
^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*
b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008
*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - atan((((-(81*b^7*c^4 + 24
01*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*
d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9
*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 367001
6*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)
*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*
a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*
a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^
3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 655
36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a
^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183
5008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*

a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/
(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^
6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a
^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3
072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 11
4688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 -
229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11
- 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c
^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^
14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 260915
2*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8
- 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^
10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 34693
12*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*
c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^
2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9
- 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(6
4*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b
^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4
*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65
536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c
^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12
*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - ((-
(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2
- 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7
*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*
c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14
*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*
d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b
^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(6553
6*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6
*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b
^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81
*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 7
56*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d
+ 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4
*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*
c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^
13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^
8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a
^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7
*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*
d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c
^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*

a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*

b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*
a^14*b*c*d^7))^(1/4) + ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d
^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^
8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c
^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13
*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) +
(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^
2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c
^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^
4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^
14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3
+ 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c
^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*
d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^
2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14
*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^
7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a
^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 2867
2*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*
d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^
13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5
*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24
190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^
5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^
13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) + (
x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8
*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c
^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9
*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^
2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 5242
88*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458
7520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
 - 524288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^
3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 655
36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a
^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183
5008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*2i - atan(((-d^7/(256*b
^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 143
36*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a
^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c
^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3
+ 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 20

48*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b *c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7 *d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^ 2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b ^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^1 4*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^ 12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9 *b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^ 12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5* b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^ 16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15* c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512 *a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7 *c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5 *b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^ 3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4*b^7*d^1 3 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2* b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^ 4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) - (- d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9 *d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^ 5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^ 5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^ 5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a ^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7* c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 1 4336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(30 72*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114 688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^ 3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^1 4*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152 *a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^1 0*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 346931 2*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c ^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2

*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a
^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 +
2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d +
15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d
^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a
^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*
b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*
b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14
336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*
a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*
c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3
+ 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2
048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7
*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c
^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))
^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10
*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*
c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7
*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 -
524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5
+ 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^1
3*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 145162
24*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^1
3 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 +
15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7
)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^
2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) - (x*
(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c
*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5))) + (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 71
68*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*
a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(
256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2
- 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7
168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*
a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8
*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6
- 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048
*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b
^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10

d))^(1/4)(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*
c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b
^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11
*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^
3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^1
7 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12
*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6
*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14
516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4
*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d
^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^
3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) +
(x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b
^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5
*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a
^9*b*c*d^5)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7
+ 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14
336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*2i +
2*atan(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*
a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5
*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256
*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 1
4336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168
*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8
*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^
3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 -
2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^
7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*
c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d)
)^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^1
0*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10
*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^
7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 -
524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^
5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^
13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516
224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^
13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d
^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^
3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10
*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^
2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c

d^2)) + (x(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 47
88*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6
*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d
^4 - 6*a^9*b*c*d^5))) - (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*
c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*
d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1
/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*
b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3
*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8
*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336
*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6
*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^
3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 +
 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 204
8*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 286
72*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 +
329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10
+ 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^
13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*
a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a
^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +
17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c
^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*
a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*
1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20
*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*
d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10
)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^
2*d - 3*a^6*b*c*d^2)) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^1
0*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^
8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 179
20*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*
b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*
d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4
- 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*
((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*
c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6
*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^
11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^
3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^
2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b
^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*
b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^1

0*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*
a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^
2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13
*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b
^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 2419
0976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*
d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13
*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b
^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i
+ ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145
*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3
 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c
^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)
1i)/(64(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2
0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (-d^7/(256*b^8*
c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*
a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*
b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*
d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 1
7920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*
a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^
4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^
4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4
)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b
^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*
c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c
^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*
c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^
9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*
b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2
*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16
+ 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^1
1*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^
7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 +
3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^
3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5
*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -
 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^
9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4
*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2
790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 1
5*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5
)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2
*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^

```
3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4) - (b*x)/(4*a*(
a + b*x^4)*(a*d - b*c))
```

3.172 $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 596

$$\begin{aligned}\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx &= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)}+\frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} \\ &\quad -\frac{b^{7/4}(3bc-11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\ &\quad +\frac{b^{7/4}(3bc-11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\ &\quad -\frac{d^{7/4}(11bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} \\ &\quad +\frac{d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} \\ &\quad -\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} \\ &\quad +\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} \\ &\quad -\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3} \\ &\quad +\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}\end{aligned}$$

```
[Out] 1/4*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^4+c)+1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)
/(d*x^4+c)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4)
)/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(1/4
)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*a*d+11*b
*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*
d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b
*c)^3*2^(1/2)-1/32*b^(7/4)*(-11*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*b^(7/4)*(-11*a*d+3*b*c
)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^
(1/2)-1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^
2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(c^(
1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{425, 541, 536, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = -\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(3bc-11ad)}{8\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(3bc-11ad)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)} + \frac{dx(ad+bc)}{4ac(c+dx^4)(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d
)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c -
11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*
(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3
) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx}{4a(bc-ad)} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} \\
&\quad - \frac{\int \frac{-4\left(3b^2c^2-8abcd+3a^2d^2\right)-12bd(bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{16ac(bc-ad)^2} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} \\
&\quad + \frac{\left(b^2(3bc-11ad)\right)\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^3} + \frac{\left(d^2(11bc-3ad)\right)\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} \\
&\quad + \frac{\left(b^2(3bc-11ad)\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^3} + \frac{\left(b^2(3bc-11ad)\right)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^3} \\
&\quad + \frac{\left(d^2(11bc-3ad)\right)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}(bc-ad)^3} + \frac{\left(d^2(11bc-3ad)\right)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}(bc-ad)^3}
\end{aligned}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)}$$

$$+ \frac{\left(b^{3/2}(3bc-11ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16a^{3/2}(bc-ad)^3} + \frac{\left(b^{3/2}(3bc-11ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16a^{3/2}(bc-ad)^3}$$

$$- \frac{\left(b^{7/4}(3bc-11ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{16\sqrt{2}a^{7/4}(bc-ad)^3}$$

$$- \frac{\left(b^{7/4}(3bc-11ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{16\sqrt{2}a^{7/4}(bc-ad)^3}$$

$$+ \frac{\left(d^{3/2}(11bc-3ad)\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{16c^{3/2}(bc-ad)^3} + \frac{\left(d^{3/2}(11bc-3ad)\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{16c^{3/2}(bc-ad)^3}$$

$$- \frac{\left(d^{7/4}(11bc-3ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{16\sqrt{2}c^{7/4}(bc-ad)^3}$$

$$- \frac{\left(d^{7/4}(11bc-3ad)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{16\sqrt{2}c^{7/4}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)}$$
$$- \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}$$
$$+ \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}$$
$$- \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}$$
$$+ \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}$$
$$+ \frac{\left(b^{7/4}(3bc-11ad)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}$$
$$- \frac{\left(b^{7/4}(3bc-11ad)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}$$
$$+ \frac{\left(d^{7/4}(11bc-3ad)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}$$
$$- \frac{\left(d^{7/4}(11bc-3ad)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}$$

$$
\begin{aligned}
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} \\
&\quad - \frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad - \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} \\
&\quad - \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad + \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad - \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3} \\
&\quad + \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \frac{1}{32}\left(\frac{8b^2x}{a(bc-ad)^2(a+bx^4)} + \frac{8d^2x}{c(bc-ad)^2(c+dx^4)} + \frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^3} + \frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(-bc+ad)^3} + \frac{2\sqrt{2}d^{7/4}(-11bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3} + \frac{2\sqrt{2}d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(bc-ad)^3}\right)$$

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] ((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^4)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) + (2*S
qrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(
c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b
*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/
```

```
4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d
^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x
^2])/(c^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(b*c - a*d)^3))/3
2
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.50

method	result
default	$\frac{b^2\left(\frac{(ad-bc)x}{4a\left(b\,x^4+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\cdots\right.}{}$
risch	Expression too large to display

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(11*a*d-3*b*c)/a^2*(a/b)^
(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)))+d^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d
-11*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)
)/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1
)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 165.51 (sec) , antiderivative size = 5234, normalized size of antiderivative = 8.78

$$\int \frac{1}{(a+bx^4)^2\,(c+dx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \frac{\left(\frac{2\sqrt{2}(3\,bc-11\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3\,bc-11\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3\,bc-11\,ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}\ldots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\ldots\right.}{32\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)} + \frac{(b^2cd+abd^2)x^5+(b^2c^2+a^2d^2)x}{4\left((ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^8+a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)\ldots\right.} + \frac{\frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(11\,bcd^2-3\,ad^3)\log\left(\ldots\right.}{c^{\frac{3}{4}}\ldots}\ldots}{32\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*c^2*d +
 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2 + a^2*d
^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 +
 a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2
```

```
)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*s
qrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sq
rt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 + sqr
t(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 -
 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d
^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(462) = 924.

Time = 0.45 (sec) , antiderivative size = 967, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*
b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1/4)
*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*
a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*s
qrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/16*
(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(
2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3
)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^
3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3)
+ 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 + sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt
(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3
*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^
2*d^3) + 1/4*(b^2*c*d*x^5 + a*b*d^2*x^5 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^8
+ b*c*x^4 + a*d*x^4 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 37266, normalized size of antiderivative = 62.53

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^5
*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d + b*
c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d
^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^1
2*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*
a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^
11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b
^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*
a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32
 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5
*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)
+ (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c
^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432
*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 +
32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^
13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a
^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^1
5*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 49
```

5*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^1
1*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c
^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 653
4*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^
12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*
c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264
*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 -
14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18
d))^(1/4)(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b
^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483
072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^1
2*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448307
2*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16
+ 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d
^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*
c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8
*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c
^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a
^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^
16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^
2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 92
4*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^1
3*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7
+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7
- 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b
^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*
c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512
*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 +
32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c
^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*
c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*
a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d
^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*
b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*
c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 2
8*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +

60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*
d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096
*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^1
5*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 -
3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 900772659
2*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^
10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 -
510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b
^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^
12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a
^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^
6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7
*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972
*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19
+ 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7
*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 3244032
0*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10
- 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^1
8*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*
b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1
993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^1
0*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 11151
36*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 +
3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d -
8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c
^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*1i - (x*(9801*a^8*b^9*
d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16
+ 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*
c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024
*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 +
66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a
^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b
^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*a^4*d^1
1 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a
^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^1
1 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^
8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 519045
12*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11
+ 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^
3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11
+ 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8

*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5190451
2*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d
^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*
c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (
60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9
*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c
^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56
*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7
+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 114
03264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c
^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 779
6490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^
10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c
^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16
+ 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096
*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^
21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^1
2*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b
^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^
11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^
8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641
*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^
10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43253
76*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^
4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^
5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432537
6*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4
- 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^1
6*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^1
0*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^1
2 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*
b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056
*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8
- 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^
9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) +
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^1

5*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^
12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^
7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) +
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((
(891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 -
(3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^
5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 +
(31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*
d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^
4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14
641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c
*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43
25376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15
*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7
*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432
5376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*
c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394
368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*
c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 +
9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 36354785
28*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*
b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^1
8 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a
^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/
(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d
^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 -
792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a
^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^
4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1
188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^
8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a
^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5
1904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^1
0*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*

b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 111513
6*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9
 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12
*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 +
 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^
6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 +
 a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -
 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2
*c^6*d^6))) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7
*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^
14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 10015
20*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c
^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a
^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2
*c^6*d^10)))))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6
534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*
d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^
9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 605552
64*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8
 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^
18*d))^(1/4)*2i + 2*atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*
c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 6553
6*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 1441
7920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d
^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b
^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 78643
2*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (
3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^
9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (3306
9*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 +
a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -
56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*
c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*
a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12
 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^
16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a
^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 1
4417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d

))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762
752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c
^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10
168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*
a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12
*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17
 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*
a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22
 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*
a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c
^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d
^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 6
6*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 +
292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^1
5*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a
^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d
^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*
b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^
19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*
b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) - (x*(9801*a^8*b^9*d^17 + 9
801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 100152
0*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13
 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*
c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^1
0*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11
*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8
 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b
^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10
)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376
*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4
 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*
c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*
a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^
4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)
/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*
a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 -
 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c
^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a

^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^

7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/
((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((
((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 -
(3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c
^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 +
(31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*
c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a
^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11
+ 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^
3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11
+ 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8
*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5190451
2*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*
b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 5
10394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*
b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d
^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 363
5478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*
a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^
7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762
752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^
23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15
*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^1
2*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7
+ 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + (
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3
072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6
- 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*

c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993 728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^ 9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^1 7*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^ 8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^ 10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28* a^10*b^2*c^6*d^6))*1i)*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149 094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^ 3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^1 2 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^ 7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6 *c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7* d^9 + 66*a^14*b^2*c^6*d^10))) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972* a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7* c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320 *a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a *b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c ^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320* a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/ 64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13* c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c ^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10* d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^1 0*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^ 7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3* b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 6055 5264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d ^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11* c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6* b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^1 0 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 36354 78528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^1 3*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^ 8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 5103 94368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^

3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7 *b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6* c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d ^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a *b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c ^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320* a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18* d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^ 15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 199 3728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10* c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136 *a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3 072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8 *a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8 *d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i)*1i + (x*(9801*a^8*b^9 *d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13 *c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(102 4*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792* a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12* b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*a^4*d^ 11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188* a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^ 11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b ^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904 512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^ 9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4) - atan(((-(81* b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^1 1*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320 *a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3* c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b ^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11 *c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320* a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c ^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^ 11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1

188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*
c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a
^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 5
1904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^
3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(58982
4*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d
^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 434484019
2*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*
c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13
 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 101683
69152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*
b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 +
 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*
c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a
^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*
c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*
d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
 + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*
a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d
^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^
14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1
993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9
*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*
a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4
*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6
*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 +
28*a^10*b^2*c^6*d^6)) + ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3
105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9
)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069
*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*
c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^
7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d
^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^1
0 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c
^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a
^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^
15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 4
95*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^
11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*
c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 65

34*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31

509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c
^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(
a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*
b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5
 + 28*a^10*b^2*c^6*d^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 1
49094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*
c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d
^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*
a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b
^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^
7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 1597
2*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^1
2 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^
2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b
^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 324403
20*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972
*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^
7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 3244032
0*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*
a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7
*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320
*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 -
 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*
b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 +
1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^
8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^
13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 +
 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 77964902
40*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*
c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403
264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a
^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d
^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 9
24*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^
13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d
^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536
*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^1
0*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519045

12*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 900772659

2*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*2i + 2*atan(((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 3244

0320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d
^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*
b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(
81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^
2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*
b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440
320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^
6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b
^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(8
1*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2
 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b
^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 324403
20*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6
 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^
3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(5
89824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^
19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 43448
40192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b
^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*
d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10
168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a
^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^
20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*
b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d
 - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a
^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b
^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*
d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a
^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12
 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^
9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^
13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14
417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11)
)^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17
*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072
*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d
^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a
^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 +
292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19
))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*
a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7
*d^5 + 28*a^10*b^2*c^6*d^6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d
^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b
^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/
32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b

^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28
*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*
b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 348460
2*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14
 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c
*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a
^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 1441792
0*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5
+ 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4
*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a
^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*
d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^
7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920
*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 +
 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*
c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^
18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d
^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7
*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*
a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 +
60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c
^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^1
8*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d
^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*
a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15
*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3
635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592
*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^1
0*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 -
510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^
5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^
12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d
^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*
b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440
320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^1

0 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d

^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 176691
6096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^
4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(
a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 6
6*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9
*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4
*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4
+ 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b
^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d
+ 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8
*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512
*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 +
4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^
19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b
^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595
712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c
^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 274534
4*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17
- 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c
^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7
*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^
6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7
*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*
a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*
d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 -
8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*
d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i -
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
) - (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9
*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 7864
32*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3
+ 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6
*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920
*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4
)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*

a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c
^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432
*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c
^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a
^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*
c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*
a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^
2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 +
6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12
*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*
b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555
264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^
8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c
d^11))^(1/4)(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^
6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4
483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*
c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448
3072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d
^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^
4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
+ 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d^15)/64 + (891*b^15
*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509
*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*
d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/
(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6
*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^
5 + 28*a^10*b^2*c^6*d^6))*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 -
149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
- 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12
*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*

```
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^
12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d
^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*
b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440
320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^1
0 - 786432*a^18*b*c*d^11))^(1/4)
```

3.173 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 321

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d}$$
$$+\frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21d^3\sqrt{a-bx^4}}$$
$$-\frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$
$$-\frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$

```
[Out] 1/7*b*x*(-b*x^4+a)^(3/2)/d-1/21*b*(-13*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/d^2+1/
21*a^(1/4)*b^(3/4)*(47*a^2*d^2-56*a*b*c*d+21*b^2*c^2)*EllipticF(b^(1/4)*x/a
^(1/4),I)*(1-b*x^4/a)^(1/2)/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*EllipticPi(b
^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/
4)/c/d^3/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {427, 542, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21d^3\sqrt{a-bx^4}}$$
$$-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$
$$-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$
$$-\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d}$$

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] -1/21*(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/d^2 + (b*x*(a - b*x^4)^(3/2))/
(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*d^3*Sqrt[a - b*x^4
]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(S
qrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(
1/4)*c*d^3*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
```

```
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{\int \frac{\sqrt{a-bx^4}\,(a(bc-7ad)-b(7bc-13ad)x^4)}{c-dx^4}\,dx}{7d}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\int \frac{a(7b^2c^2-16abcd+21a^2d^2)-b(21b^2c^2-56abcd+47a^2d^2)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{21d^2}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d^3} + \frac{(b(21b^2c^2-56abcd+47a^2d^2))\int \frac{1}{\sqrt{a-bx^4}}dx}{21d^3}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2cd^3} - \frac{(bc-ad)^3\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2cd^3} + \frac{\left(b(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{21d^3\sqrt{a-bx^4}}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}} - \frac{\left((bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2cd^3\sqrt{a-bx^4}} - \frac{\left((bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2cd^3\sqrt{a-bx^4}}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.90

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{x\left(5b(-a+bx^4)\left(7bc-16ad+3bdx^4\right)-\frac{b(21b^2c^2-56abcd+47a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx}{a}\right.}{c}\right.}{1}$$

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] (x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*b
*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^
4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Appel
lF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2,
 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*
x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.41 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.09

method	result
risch	$\frac{bx(-3bd\,x^4+16ad-7bc)\sqrt{-b\,x^4+a}}{21d^2}+\frac{\frac{b\left(47a^2d^2-56abcd+21b^2c^2\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{\left(-21a^3d^3+63a^2bc\,d^2-63a\,b^2c^2d+21\right.}{}}{}$
default	$-\frac{b^2x^5\sqrt{-b\,x^4+a}}{7d}-\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}+\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{}}{\sqrt{}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
elliptic	$-\frac{b^2x^5\sqrt{-b\,x^4+a}}{7d}-\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}+\frac{\left(-\frac{b^2(3ad-bc)}{d^2}+\frac{5b^2a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{}}{\sqrt{}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b*x*(-3*b*d*x^4+16*a*d-7*b*c)*(-b*x^4+a)^(1/2)/d^2+1/21/d^2*(b*(47*a^2
*d^2-56*a*b*c*d+21*b^2*c^2)/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1
/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^
(1/2)*b^(1/2))^(1/2),I)+1/8*(-21*a^3*d^3+63*a^2*b*c*d^2-63*a*b^2*c^2*d+21*b
^3*c^3)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha
^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = -\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt
(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c
+ d*x**4), x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a-bx^4\right)^{5/2}}{c-dx^4}\,dx=\int \frac{\left(a-b\,x^4\right)^{5/2}}{c-d\,x^4}\,dx$$

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4),x)
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4), x)
```

3.174 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 277

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4} dx = \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}$$

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-1/3*a^(1/4)*b^(3/4)*(-5*a*d+3*b*c)*EllipticF(b^(
1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+
b*c)^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-
b*x^4/a)^(1/2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+b*c)^2*Elli
pticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/
2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {427, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = -\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}} + \frac{bx\sqrt{a-bx^4}}{3d}$$

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*d^2*Sqrt[a - b*x^4])
 + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sq
rt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/
4)*c*d^2*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{3d} \\
&= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{d^2} \\
&= \frac{bx\sqrt{a-bx^4}}{3d} + \frac{(bc-ad)^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} \\
&\quad + \frac{(bc-ad)^2\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} - \frac{\left(b(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{3d^2\sqrt{a-bx^4}}
\end{aligned}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}$$
$$+ \frac{\left((bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2cd^2\sqrt{a-bx^4}}$$
$$+ \frac{\left((bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2cd^2\sqrt{a-bx^4}}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}$$
$$+ \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}$$
$$+ \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.23

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx =$$
$$-\frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(3a^2d-abdx^4+b^2x^4(-c+dx^4))\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4(a-bx^4)\right.}{(-c+dx^4)\left(5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\operatorname{Appel}\right.\right.}\right.}{15d\sqrt{a-bx^4}}$$

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] -1/15*(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1,
 9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4*(
-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(a
- b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x^4)*(
5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, (b*x^4)/a, (d*x^4)/c])))))/(d*Sqrt[a - b*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.26 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.10

method	result
risch	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{\left(-3a^2d^2+6abcd-3b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2}{2\sqrt{\frac{ad-bc}{d}}\sqrt{}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{}\right.}{}}{3d}$
default	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{-}{2}\right)}{}\right.}{}$
elliptic	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{-}{2}\right)}{}\right.}{}$

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d+1/3/d*(b*(5*a*d-3*b*c)/d/(1/a^(1/2)*b^(1/2))^(1/
2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(
1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/8*(-3*a^2*d^2+6*a*b*c*d-3*b
^2*c^2)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha
^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx=\text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx=-\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)\,dx$$

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
[Out] -Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx=\int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx=\int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx=\int \frac{{\left(a-b\,x^4\right)}^{3/2}}{c-d\,x^4}\,dx$$

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4),x)
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4), x)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

```
[Out] a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d/(-b*x^4+
a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/
2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)-1/2*a^
(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/
2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

= {415, 230, 227, 418, 1233, 1232}

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{d}+\frac{(-bc+ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{d}\\
&= \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd}\\
&\quad+\frac{(-bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd}+\frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{d\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}\\
&\quad+\frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2cd\sqrt{a-bx^4}}\\
&\quad+\frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2cd\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}\\
&\quad-\frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}\\
&\quad-\frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx =$$

$$-\frac{5acx\sqrt{a-bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{Appe}\cdots\right.\right.}$$

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
)/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08

method	result
default	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}} - \frac{\sum_{\_\alpha=\text{RootOf}\left(d\_Z^4-c\right)}\frac{(ad-bc)\left(-\frac{\text{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2}{\cdots}}}{\sqrt{\cdots}}\cdots\right)}{\_\alpha^3}}{8d^2}\cdots$
elliptic	$\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}} - \frac{\sum_{\_\alpha=\text{RootOf}\left(d\_Z^4-c\right)}\frac{(ad-bc)\left(-\frac{\text{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2}{\cdots}}}{\sqrt{\cdots}}\cdots\right)}{\_\alpha^3}}{8d^2}\cdots$

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/
8/d^2*sum((a*d-b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alp
```

```
ha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2
))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1
/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = -\int \frac{\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
[Out] -Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int \frac{\sqrt{a-b\,x^4}}{c-d\,x^4}\,dx$$

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4),x)
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4), x)
```

3.176 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

```
[Out] 1/2*a^(1/4)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {418, 1233, 1232}

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c}+\frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c}\\
&= \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\,(-c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{A}\right.\right.}$$

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^4
]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*
x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2 b x^2\_\alpha^2+2 a}{2\sqrt{\frac{a d-b c}{d}}\sqrt{-b x^4+a}}\right)}{\sqrt{\frac{a d-b c}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b x^4+a}}}{\_\alpha^3}}{8d}$	183
elliptic	$-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2 b x^2\_\alpha^2+2 a}{2\sqrt{\frac{a d-b c}{d}}\sqrt{-b x^4+a}}\right)}{\sqrt{\frac{a d-b c}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b x^4+a}}}{\_\alpha^3}}{8d}$	183

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*
x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/
(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_al
pha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = -\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
[Out] -Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int \frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)}\,dx$$

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)
```

3.177 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}}$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1/2*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/
4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {425, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt{a-bx^4}(bc-ad)} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)} + \frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c - a*d)*Sqrt[a - b*x
^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sq
rt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c -
a*d)*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}}+\frac{\int \frac{bc-2ad-bdx^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{2a(bc-ad)}\\
&= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}}+\frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{2a(bc-ad)}-\frac{d\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{bc-ad}\\
&= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}}-\frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)}\\
&\quad -\frac{d\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)}+\frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{2a(bc-ad)\sqrt{a-bx^4}}
\end{aligned}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\left(d\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c(bc-ad)\sqrt{a-bx^4}} - \frac{\left(d\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c(bc-ad)\sqrt{a-bx^4}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{5acx\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(-2bc+2ad+bdx^4)+bdx^4\sqrt{1-\frac{b}{}}\right.}{10ac(-bc+ad)\sqrt{a}}$$

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c + 2
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqrt[1
 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*Appe
llF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.07

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\ldots}{\ldots}\right)}{8\ldots}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\ldots}{\ldots}\right)}{8\ldots}$

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*x/a/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(1/a^(1/2)*b^(1
/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/(a*d-b*c)/_
alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*
(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c
*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/
2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/
a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}\,(c-dx^4)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = -\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
[Out] -Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*s
qrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)}\,dx$$

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)),x)
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)), x)
```

3.178 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 334

$$
\begin{aligned}
\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx &= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} \\
&+ \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}} \\
&+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}} \\
&+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}}
\end{aligned}
$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(3/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*
c)^2/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-11*a*d+5*b*c)*EllipticF(b^(1/4)*x/a^(1
/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*
d^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x
^4/a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*d^2*Ellipti
cPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}\sqrt{a-bx^4}(bc-ad)^2} + \frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}(bc-ad)^2} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2} + \frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7/4)*(b*c - a*d)^2*S
qrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(
b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi
[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{\int \frac{5bc-6ad-5bdx^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2-bd(5bc-11ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{(bc-ad)^2} + \frac{(b(5bc-11ad))\int\frac{1}{\sqrt{a-bx^4}}dx}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2c(bc-ad)^2} + \frac{d^2\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2c(bc-ad)^2} + \frac{\left(b(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{12a^2(bc-ad)^2\sqrt{a-bx^4}}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}} + \frac{\left(d^2\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2c(bc-ad)^2\sqrt{a-bx^4}} + \frac{\left(d^2\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2c(bc-ad)^2\sqrt{a-bx^4}}$$

$$= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.72 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}dx = \frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac(12a^3d^2+a^2bd(-24c+dx^4)+5b^3\right.}{}\right.}{}$$

[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]

[Out] (x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*x^4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4)*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4])

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.08

method	result
default	$-\frac{x\sqrt{-b\,x^4+a}}{6ab(ad-bc)\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_\right.}\right.}{}$
elliptic	$-\frac{x\sqrt{-b\,x^4+a}}{6ab(ad-bc)\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_\right.}\right.}{}$

[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)

[Out] -1/6*x/a/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b*x/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1

```
/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)
*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx =$$
$$-\int \frac{1}{-a^2c\sqrt{a-bx^4}+a^2dx^4\sqrt{a-bx^4}+2abcx^4\sqrt{a-bx^4}-2abdx^8\sqrt{a-bx^4}-b^2cx^8\sqrt{a-bx^4}+b^2dx^{12}\sqrt{a}$$

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
[Out] -Integral(1/(-a**2*c*sqrt(a - b*x**4) + a**2*d*x**4*sqrt(a - b*x**4) + 2*a*
b*c*x**4*sqrt(a - b*x**4) - 2*a*b*d*x**8*sqrt(a - b*x**4) - b**2*c*x**8*sqr
t(a - b*x**4) + b**2*d*x**12*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
[Out] -integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx=\int -\frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx=\int \frac{1}{{\left(a-b\,x^4\right)}^{5/2}\,\left(c-d\,x^4\right)}\,dx$$

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)),x)
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)), x)
```

3.179 $\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 926

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}$$

$$-\frac{(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}$$

$$-\frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}}$$

$$+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{a+bx^4}}$$

$$+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{a+bx^4}}$$

$$+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}}$$

$$+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(3/4)/d^(7/4)-1/4*(a*d-b*c)^(3/2)*arctan(x*(a*d-b*c)^(1/2)/(-
c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/d^(7/4)+1/3*b*x*(b*x^4+a)^(1/2
)/d-1/6*b^(3/4)*(-5*a*d+3*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/a^(1/4)/d^2/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1
/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d^2/
(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/
b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)
^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1
/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a
^(1/4)/d^2/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticP
i(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2)
)^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(1/4)/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {427, 537, 226, 418, 1231, 1721}

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)(}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)(bc}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)(bc-}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{bx^4+a}}$$

$$-\frac{\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)(bc-ad)^{3/2}}{4(-c)^{3/4}d^{7/4}}-\frac{(ad-bc)^{3/2}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}d^{7/4}}$$

$$-\frac{b^{3/4}(3bc-5ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{bx^4+a}}$$

$$+\frac{bx\sqrt{bx^4+a}}{3d}$$

```
[In] Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(7/4)) - ((-(b*c) +
 a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(4*(-c)^(3/4)*d^(7/4)) - (b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*
Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sq
rt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sq
```

```
rt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{bx\sqrt{a+bx^4}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}} + \frac{(bc-ad)^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd^2} + \frac{(bc-ad)^2\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd^2}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{-c}d^2(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2d^2(bc+ad)} - \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd^{3/2}(bc+ad)} + \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd^{3/2}(bc+ad)}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}$$

$$- \frac{(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}$$

$$- \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{a+bx^4}}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}d^2(bc+ad)\sqrt{a+bx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd^2(bc+ad)\sqrt{a+bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.37

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1+\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} + \frac{5\left(-5ac\left(3a^2d+abdx^4+b^2x^4(c+dx^4)\right)\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2}\right.\right.}{(c+dx^4)\left(-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1\right.\right.}\right.}{15d\sqrt{}}$$

```
[In] Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^4*
(c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x
^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
```

```
b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))))/(15*d*Sqrt
[a + b*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.34

method	result
risch	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\left(3a^2d^2-6abcd+3b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2}{2\sqrt{\frac{ad-bc}{d}}\sqrt{}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{}\right.}{}}{3d}$
default	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\right.}{}\right.}{}$
elliptic	$\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-\frac{\operatorname{arctanh}\left(\right.}{}\right.}{}$

```
[In] int((b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x*(b*x^4+a)^(1/2)/d+1/3/d*(b*(5*a*d-3*b*c)/d/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*(3*a^2*d^2-6*a*b*c*d+3*
b^2*c^2)/d^2*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha
^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^
(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{3}{2}}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx=\int \frac{{\left(b\,x^4+a\right)}^{3/2}}{d\,x^4+c} \,d x$$

```
[In] int((a + b*x^4)^(3/2)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(3/2)/(c + d*x^4), x)
```

3.180 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 881

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} - \frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}}$$
$$+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$
$$- \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d(bc+ad)\sqrt{a+bx^4}}$$
$$- \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}d(bc+ad)\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}}$$

```
[Out] 1/4*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))*(-a*d+b*c
)^(1/2)/(-c)^(3/4)/d^(3/4)-1/4*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/
```

```
(b*x^4+a)^(1/2))*(a*d-b*c)^(1/2)/(-c)^(3/4)/d^(3/4)+1/2*b^(3/4)*(cos(2*arct
an(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(b*x^4+a)^(1/2)-1/4*b^(1/4)*(-
a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x
^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a
^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4
)*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{415, 226, 418, 1231, 1721\}$

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx =$$

$$-\frac{(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{bx^4+a}}$$

$$-\frac{\sqrt[4]{b}(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)}{4\sqrt[4]{a}\sqrt{-c}d(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}d^{3/4}}-\frac{\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}d^{3/4}}$$

$$+\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}d(bc+ad)\sqrt{bx^4+a}}$$

$$-\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{bx^4+a}}$$

```
[In] Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(4*(-c)^(3/4)*d^(3/4)) - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(3/4)) + (b^
(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4])
 - (b^(1/4)*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d*(b*c + a*d)*Sqrt[a + b*x^4]
) - (b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(b*c - a*d)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c
] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) -
((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)
```

```
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] +
 Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{d}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{(bc-ad)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd} - \frac{(bc-ad)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{-c}d(bc+ad)} - \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{2d(bc+ad)} + \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c\sqrt{d}(bc+ad)} - \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c\sqrt{d}(bc+ad)}$$

$$= \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} - \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{-c}d(bc+ad)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}d(bc+ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{a+bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{5acx\sqrt{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,\text{Ap}\right.\right.}$$

```
[In] Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.31

method	result
default	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}(d\_Z^4+c)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1-}}{}\right.}{\_\alpha^3}}{8d^2}$
elliptic	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}(d\_Z^4+c)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1-}}{}\right.}{\_\alpha^3}}{8d^2}$

```
[In] int((b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_
```

```
alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/
2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4} \, dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c} \, dx$$

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4} \, dx = \int \frac{\sqrt{b\,x^4+a}}{d\,x^4+c} \, dx$$

```
[In] int((a + b*x^4)^(1/2)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

3.181 $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx$

Optimal result

Integrand size = 21, antiderivative size = 742

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx =& -\frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}}\\
&+\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(bc+ad)\sqrt{a+bx^4}}\\
&+\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc+ad)\sqrt{a+bx^4}}\\
&+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}}\\
&+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}}\end{aligned}$$

```
[Out] -1/4*d^(1/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/
(-c)^(3/4)/(-a*d+b*c)^(1/2)-1/4*d^(1/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)
/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(1/2)+1/8*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/
2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(
-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
```

```
/4)/b^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4
)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(
-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/
c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(a*d+b*c)/(b*x^4+a)
^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {418, 1231, 226, 1721}

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = &-\frac{\sqrt[4]{d}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\arctan\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}}\\ &+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\\ &+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c\sqrt{a+bx^4}(ad+bc)}\\ &+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}(ad+bc)}\\ &+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}(ad+bc)}\end{aligned}$$

```
[In] Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] -1/4*(d^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/((-c)^(3/4)*Sqrt[b*c - a*d]) - (d^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/
((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*Sqrt[-(b*c) + a*d]) +
```

```
(b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(4*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^
2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]
*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c +
 a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d)*Sqrt[a +
b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c}$$

$$= \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2c(bc+ad)}$$
$$- \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc+ad)}$$
$$+ \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc+ad)}$$

$$= -\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}}$$
$$+ \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(bc+ad)\sqrt{a+bx^4}}$$
$$+ \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc+ad)\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc+ad)\sqrt{a+bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\,(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.\right.}$$

```
[In] Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a +
 b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.26

method	result	size
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{4}+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^{3}d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}\,\_\alpha^{2}d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^{4}+a}}}{\_\alpha^{3}}}{8d}$	191
elliptic	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{4}+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^{3}d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}\,\_\alpha^{2}d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^{4}+a}}}{\_\alpha^{3}}}{8d}$	191

```
[In] int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*sum(1/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^
2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_
alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_
alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)}\,dx$$

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)
```

3.182 $\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)} dx$

Optimal result

Integrand size = 21, antiderivative size = 913

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)} dx = \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c\left(b^2c^2-a^2d^2\right)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}}$$

```
[Out] 1/4*d^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(
-c)^(3/4)/(-a*d+b*c)^(3/2)-1/4*d^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/
```

```
d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(3/2)+1/2*b*x/a/(-a*d+b*c)/(b
*x^4+a)^(1/2)+1/4*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
5/4)/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/
2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d
^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2
*d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1
/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2
)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^
(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2*
d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c
)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(-a*d+b*c)/(a*d+
b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(-a^2*d^2+b^2*c^2)/(
b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{425, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx =$$

$$-\frac{d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{d^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}}-\frac{d^{5/4}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}(ad-bc)^{3/2}}$$

$$+\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{bx^4+a}}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{bx^4+a}}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c\left(b^2c^2-a^2d^2\right)\sqrt{bx^4+a}}$$

$$-\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}}$$

$$+\frac{bx}{2a(bc-ad)\sqrt{bx^4+a}}$$

```
[In] Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + (d^(5/4)*ArcTan[(Sqrt[b*c - a*d]*
x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(3/2))
- (d^(5/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/(4*(-c)^(3/4)*(-(b*c) + a*d)^(3/2)) + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b
] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b^2*c^2 - a^2*d^2)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b
```

```
]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4
)*c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]
*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)
*c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} - \frac{\int \frac{-bc+2ad-bdx^4}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)} - \frac{d\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc-ad)(bc+ad)} - \frac{\left(\sqrt{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2c\left(b^2c^2-a^2d^2\right)} + \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d^{3/2}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c\left(b^2c^2-a^2d^2\right)} - \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d^{3/2}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c\left(b^2c^2-a^2d^2\right)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}c\left(b^2c^2-a^2d^2\right)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(b^2c^2-a^2d^2\right)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(b^2c^2-a^2d^2\right)\sqrt{a+bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.36

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx = \frac{x\left(-\frac{bdx^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} + \frac{5\left(5ac\left(2ad-b\left(2c+dx^4\right)\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\right.\right.}{\left(c+dx^4\right)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\right.\right.}\right.}{10a(-bc+}$$

```
[In] Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/2,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.34

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_}{}}{}\right.}{}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_}{}}{}\right.}{}$

```
[In] int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*x/a/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*sum(1/(a*d-b*c)
/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/(1/d
*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*d
,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*
d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{2}}(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/2}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)), x)
```

3.183 $\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 976

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx &= \frac{bx}{6a(bc-ad)(a+bx^4)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} \\
&- \frac{d^{9/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}} - \frac{d^{9/4}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{5/2}} \\
&+ \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2 d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2 d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}
\end{aligned}
$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/2)-1/4*d^(9/4)*arctan(x*(-a*d+b*c)^(1/2)/
(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(5/2)-1/4*d^(9/4)
```

```
*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a
*d-b*c)^(5/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/2
4*b^(3/4)*(-11*a*d+5*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
9/4)/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)
^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/
2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(
-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)
/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+
a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)-a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx = -\frac{\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}}$$
$$-\frac{\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$
$$+\frac{b^{3/4}(5bc-11ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{bx^4+a}}$$
$$+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{bx^4+a}}+\frac{bx}{6a(bc-ad)\left(bx^4+a\right)^{3/2}}$$

```
[In] Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a + b*x^4]) - (d^(9/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(5/2)) - (d^(9/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(
-c)^(3/4)*(-(b*c) + a*d)^(5/2)) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqr
```

```
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) +
 ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a
]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}-\frac{\int \frac{-5bc+6ad-5bdx^4}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx}{6a(bc-ad)}\\
&= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}+\frac{\int \frac{5b^2c^2-11abcd+12a^2d^2+bd(5bc-11ad)x^4}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{12a^2(bc-ad)^2}\\
&= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}\\
&\quad+\frac{d^2\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2}+\frac{(b(5bc-11ad))\int \frac{1}{\sqrt{a+bx^4}}\,dx}{12a^2(bc-ad)^2}\\
&= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}\\
&\quad+\frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{a+bx^4}}\\
&\quad+\frac{d^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)^2}+\frac{d^2\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)^2}
\end{aligned}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}$$
$$+\frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d^2\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc-ad)^2(bc+ad)}+\frac{\left(\sqrt{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\right)\int\frac{1}{\sqrt{a+bx^4}}\,dx}{2c(bc-ad)^2(bc+ad)}$$
$$-\frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d^{5/2}\right)\int\frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)^2(bc+ad)}$$
$$+\frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d^{5/2}\right)\int\frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c(bc-ad)^2(bc+ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}}+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}}$$
$$-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{5/2}}$$
$$+\frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{a+bx^4}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)^2(bc+ad)\sqrt{a+bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.44

$$\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx =$$

$$-\frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(12a^3d^2+5b^3cx^4(2c+dx^4)-a^2bd(24c+dx^4)+ab^2(12c^2-15cdx^4-11d^2\right.}{(a+bx^4)(c+dx^4)\left(-5ac\,\text{A}\right.}}{60a}$$

```
[In] Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] -1/60*(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c*x
^4*(2*c + d*x^4) - a^2*b*d*(24*c + d*x^4) + a*b^2*(12*c^2 - 15*c*d*x^4 - 11
*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4
*(c + d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Appe
llF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(a^2*(b*c - a*d)^2*Sqrt[a + b*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.38

method	result
default	$-\frac{x\sqrt{b\,x^4+a}}{6ab(ad-bc)\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}(\ldots)}\right.}{}$
elliptic	$-\frac{x\sqrt{b\,x^4+a}}{6ab(ad-bc)\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\left(\sum_{\_\alpha=\mathrm{RootOf}(\ldots)}\right.}{}$

```
[In] int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x/a/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b*x/a^2*(11*a*d-5*b*c
)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*
sum(1/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(2*_alpha^
2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(
1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(
1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_a
lpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{2}}\,(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/2}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/2}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)), x)
```

3.184 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 426

$$\begin{aligned}\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2} dx =& -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} \\ &+\frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2}-\frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} \\ &+\frac{\sqrt[4]{a}b^{3/4}(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{84cd^4\sqrt{a-bx^4}} \\ &-\frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} \\ &-\frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}\end{aligned}$$

```
[Out] 1/28*b*(-7*a*d+11*b*c)*x*(-b*x^4+a)^(3/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a)
^(5/2)/c/d/(-d*x^4+c)-1/84*b*(21*a^2*d^2-122*a*b*c*d+77*b^2*c^2)*x*(-b*x^4+
a)^(1/2)/c/d^3+1/84*a^(1/4)*b^(3/4)*(21*a^3*d^3+349*a^2*b*c*d^2-553*a*b^2*c
^2*d+231*b^3*c^3)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^4/(-
b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(1/4)*x
/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/
d^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(
1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)
/c^2/d^4/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {424, 542, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = -\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{84cd^3} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{84cd^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} + \frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{28cd^2} - \frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

```
[In] Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/84*(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(c*d^3)
 + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) - ((b*c - a*d)*x*(a
- b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a
*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c
- a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd(c-dx^4)} - \frac{\int \frac{(a-bx^4)^{3/2}(-a(bc+3ad)+b(11bc-7ad)x^4)}{c-dx^4}\,dx}{4cd}$$

$$= \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd(c-dx^4)} + \frac{\int \frac{\sqrt{a-bx^4}(-a(11b^2c^2-14abcd-21a^2d^2)+b(77b^2c^2-122abcd+21a^2d^2)x^4)}{c-dx^4}\,dx}{28cd^2}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd(c-dx^4)} - \frac{\int \frac{-a(77b^3c^3-155ab^2c^2d+63a^2bcd^2+63a^3d^3)+b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)x^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{84cd^3}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd(c-dx^4)} - \frac{((bc-ad)^3(11bc+3ad))\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4cd^4} + \frac{(b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{84cd^4}$$

$$= -\frac{b(77b^2c^2-122abcd+21a^2d^2)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd(c-dx^4)} - \frac{((bc-ad)^3(11bc+3ad))\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d^4} - \frac{((bc-ad)^3(11bc+3ad))\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d^4} + \frac{\left(b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{84cd^4\sqrt{a-bx^4}}$$

$$= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(231b^3c^3 - 553ab^2c^2d + 349a^2bcd^2 + 21a^3d^3)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{84cd^4\sqrt{a-bx^4}} - \frac{\left((bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2d^4\sqrt{a-bx^4}} - \frac{\left((bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2d^4\sqrt{a-bx^4}}$$

$$= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(231b^3c^3 - 553ab^2c^2d + 349a^2bcd^2 + 21a^3d^3)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{84cd^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.12

$$\int\frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(231b^3c^3 - 553ab^2c^2d + 349a^2bcd^2 + 21a^3d^3)\,x^5\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right) + \frac{5c\left(5acx(-84a^4d^3+\right.}{}}{}$$

```
[In] Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/420*(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x^
5*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5
*c*(5*a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b^3*
c*d*x^4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*x^8)
)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4)*(-6
3*a^2*b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-77*c^2
+ 44*c*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d
*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((c - d*
x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^3*Sqrt[a - b*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.94 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.27

method	result
default	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+3b\right.}{d^4}\right.}{}$
elliptic	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+3b\right.}{d^4}\right.}{}$
risch	$\frac{b^2x(-3bd\,x^4+23ad-14bc)\sqrt{-b\,x^4+a}}{21d^3}+\frac{\frac{b^2(103a^2d^2-154abcd+63b^2c^2)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{21b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2\right.}{}}{}$

```
[In] int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)/
(-d*x^4+c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7
*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1
/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*b/c+1/3*(-2*b^3/d^3*(2
*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1
/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^
(1/2)*b^(1/2))^(1/2),I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2*b^2*
c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arcta
nh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1
/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b
^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/
2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/
2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx=\int \frac{(a-b\,x^4)^{7/2}}{(c-d\,x^4)^2}\,dx$$

```
[In] int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
[Out] int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

3.185 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 365

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(3/2)/c/d/(-d*x^4+c)+1/12*b*(-3*a*d+7*b*c)*x*(
-b*x^4+a)^(1/2)/c/d^2-1/12*a^(1/4)*b^(3/4)*(-3*a^2*d^2-26*a*b*c*d+21*b^2*c^
2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^3/(-b*x^4+a)^(1/2)+
1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2
)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+a)^(
1/2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^
(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {424, 542, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} dx = -\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2-26abcd+21b^2c^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} + \frac{bx\sqrt{a-bx^4}(7bc-3ad)}{12cd^2} - \frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^
4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d -
 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c
 - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqr
t[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\int \frac{\sqrt{a-bx^4}\,(-a(bc+3ad)+b(7bc-3ad)x^4)}{c-dx^4}\,dx}{4cd}$$

$$=\frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2}-\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\left(c-dx^4\right)}+\frac{\int\frac{-a(7b^2c^2-6abcd-9a^2d^2)+b(21b^2c^2-26abcd-3a^2d^2)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{12cd^2}$$

$$=\frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2}-\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\left(c-dx^4\right)}+\frac{((bc-ad)^2(7bc+3ad))\int\frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd^3}-\frac{(b(21b^2c^2-26abcd-3a^2d^2))\int\frac{1}{\sqrt{a-bx^4}}dx}{12cd^3}$$

$$=\frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2}-\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\left(c-dx^4\right)}+\frac{((bc-ad)^2(7bc+3ad))\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{8c^2d^3}+\frac{((bc-ad)^2(7bc+3ad))\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{8c^2d^3}-\frac{\left(b(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{12cd^3\sqrt{a-bx^4}}$$

$$=\frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2}-\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\left(c-dx^4\right)}-\frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\left((bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{8c^2d^3\sqrt{a-bx^4}}+\frac{\left((bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{8c^2d^3\sqrt{a-bx^4}}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.58 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(-21b^2c^2+26abcd+3a^2d^2)\,x^5\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx(12a^3d^2+2ab^2cdx^4-3a^2bd^2x^4+b^3cx\right.}{(}}{6}$$

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/60*(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*App
ellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2 +
 2*a*b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d + 3*
a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/
a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-
c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4,
 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^2*Sqrt[a - b*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.16 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-bx^4+a}}{4d^2c(-dx^4+c)}+\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)b}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}$
elliptic	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-bx^4+a}}{4d^2c(-dx^4+c)}+\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)b}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}$
risch	$\frac{b^2x\sqrt{-bx^4+a}}{3d^2}+\frac{\frac{2b^2(4ad-3bc)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}-\frac{9b(a^2d^2-2abcd+b^2c^2)\left(\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^4-c)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2bx^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\cdots}\right)}{\sqrt{\frac{ad-b\cdots}{d}}}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$

[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/4/d^2/c*(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/3*b^2/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*b/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7*b^3*c^3)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha

```
^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{5}{2}}}{(-c+dx^4)^2}\,dx$$

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
[Out] Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx=\int \frac{(a-b\,x^4)^{5/2}}{(c-d\,x^4)^2}\,dx$$

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4)^2,x)
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4)^2, x)
```

3.186 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(1/2)/c/d/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(a*d+
3*b*c)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^2/(-b*x^4+a)^(1
/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*
d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/
2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^
(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {424, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*Sqrt[a - b*x^4])/(c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(
3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (
b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c -
 a*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]
*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*
x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
```

```
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} - \frac{\int \frac{-a(bc+3ad)+b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4c} + \frac{(b(3bc+ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2}$$
$$+ \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2} + \frac{\left(b(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{4cd^2\sqrt{a-bx^4}}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}$$
$$+ \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{8c^2\sqrt{a-bx^4}}$$
$$+ \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{8c^2\sqrt{a-bx^4}}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}$$
$$+ \frac{3\sqrt[4]{a}\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}}$$
$$+ \frac{3\sqrt[4]{a}\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.11

$$\int\frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}dx = \frac{x\left(-b(3bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\left(-c+dx^4\right)\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(-5ac\left(4a^2d+b^2c\right.\right.}{}\right.}{20c^2d}$$

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a*b
*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a*d)
*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF1[1/4
, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9
/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^
4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.03 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.06

method	result
default	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(-\frac{\mathrm{arctanh}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
elliptic	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(-\frac{\mathrm{arctanh}\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+(b^2/d^2+1/4*b/d^2*(a*d-b*c
)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3
/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(
1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^
(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1
/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),
a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))
^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2}\,dx = \int \frac{(a - bx^4)^{\frac{3}{2}}}{(-c + dx^4)^2}\,dx$$

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
[Out] Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2}\,dx = \int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2}\,dx = \int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx=\int \frac{\left(a-b\,x^4\right)^{3/2}}{\left(c-d\,x^4\right)^2}\,dx$$

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4)^2,x)
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4)^2, x)
```

3.187 $\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx = \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

```
[Out] 1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x
/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)
*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/
a)^(1/2)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)*EllipticPi
(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(
1/4)/c^2/d/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {423, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} + \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a
]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sqrt[a - b*x^4]) - (a^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqr
t[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a -
 b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2
*d*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
```

```
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{a-bx^4}}{4c\,(c-dx^4)}-\frac{\int \frac{-3a+bx^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4c}\\
&= \frac{x\sqrt{a-bx^4}}{4c\,(c-dx^4)}+\frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd}+\frac{(-bc+3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4cd}\\
&= \frac{x\sqrt{a-bx^4}}{4c\,(c-dx^4)}-\frac{(bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d}\\
&\quad -\frac{(bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d}+\frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{4cd\sqrt{a-bx^4}}
\end{aligned}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2d\sqrt{a-bx^4}} - \frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2d\sqrt{a-bx^4}}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int\frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx$$

$$= \frac{x\left(-\frac{5(a-bx^4)}{c}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}(c-dx^4)\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{75a^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\cdots\right.}{5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\cdots\right.\right.}\cdots\right)}{20\sqrt{a-bx^4}\left(-c+dx^4\right)}$$

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5/
4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1,
5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.06

method	result
default	$\frac{x\sqrt{-b\,x^{4}+a}}{4c(-d\,x^{4}+c)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}\cdots}{\cdots}\right)}{\cdots}}{32c\,d^{2}}$
elliptic	$\frac{x\sqrt{-b\,x^{4}+a}}{4c(-d\,x^{4}+c)}+\frac{b\sqrt{1-\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^{2}\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{4}-c\right)}\frac{(3ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}\cdots}{\cdots}\right)}{\cdots}}{32c\,d^{2}}$

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4/c/d*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-
x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((3*a*d-b*c)/_alpha^3
*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(1/d*(a*d-b*
c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2
*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Elli
pticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)
*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{a-bx^4}}{(-c+dx^4)^2}\,dx$$

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
[Out] Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{a-b\,x^4}}{(c-d\,x^4)^2}\,dx$$

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4)^2,x)
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4)^2, x)
```

3.188 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}}$$

```
[Out] -1/4*d*x*(-b*x^4+a)^(1/2)/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^(1/4)*b^(3/4)*Ellip
ticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(
1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1
/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {425, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = -\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)} - \frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*Sqrt[a - b*x^4])/(c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b^(3/4)*S
qrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*(b*c -
a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*
Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
```

```
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\int \frac{-4bc+3ad-bdx^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4c(bc-ad)} \\
&= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4c(bc-ad)} + \frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4c(bc-ad)} \\
&= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} + \frac{(5bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)} \\
&\quad + \frac{(5bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)} - \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{4c(bc-ad)\sqrt{a-bx^4}}
\end{aligned}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\left((5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)\sqrt{a-bx^4}} + \frac{\left((5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)\sqrt{a-bx^4}}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx$$

$$= \frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(4bc-4ad+bdx^4)+bdx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\right.\right.}{20c^2(bc-ad)\sqrt{a-bx^4}\left(-c+dx^4\right)\left(5ac\,\text{App}\right.}$$

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4*
a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt[1
- (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b*x^4]*
(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/
4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.48 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.04

method	result
default	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{32c}$
elliptic	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{32c}$

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d/c/(a*d-b*c)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/4*b/c/(a*d-b*c)/(1/a^(1/2
)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*sum((3
*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2
))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{a-bx^4}\,(-c+dx^4)^2}\,dx$$

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
[Out] Integral(1/(sqrt(a - b*x**4)*(-c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{a-b\,x^4}\,(c-d\,x^4)^2}\,dx$$

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2), x)
```

3.189 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx &= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} \\ &+ \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}}\end{aligned}$$

```
[Out] 1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)/
(-d*x^4+c)/(-b*x^4+a)^(1/2)+1/4*b^(3/4)*(a*d+2*b*c)*EllipticF(b^(1/4)*x/a^(
1/4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(1/
4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(
1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(
1/4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^
(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4a^{3/4}c\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} + \frac{bx(ad+2bc)}{4ac\sqrt{a-bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c
 - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c + a*d)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*a^(3/4)*c*(b*c - a*d)
^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]
)
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
```

```
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{\int \frac{-4bc+3ad-5bdx^4}{(a-bx^4)^{3/2}(c-dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{\int \frac{-2(2b^2c^2-8abcd+3a^2d^2)+2bd(2bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{8ac(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{(3d(3bc-ad))\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx}{4c(bc-ad)^2} + \frac{(b(2bc+ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} - \frac{(3d(3bc-ad))\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)^2} - \frac{(3d(3bc-ad))\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)^2} + \frac{\left(b(2bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{4ac(bc-ad)^2\sqrt{a-bx^4}}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}} - \frac{\left(3d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)^2\sqrt{a-bx^4}} - \frac{\left(3d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)^2\sqrt{a-bx^4}}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \frac{x\left(-bd(2bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{c\left(25ac\left(4a^2d^2+2b\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
(b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) - a*
b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10*x^
4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a,
 (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2*(b*c -
a*d)^2*Sqrt[a - b*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.75 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

method	result
default	$\frac{d^2 x\sqrt{-b\,x^4+a}}{4c(ad-bc)^2(-d\,x^4+c)}+\frac{b^2 x}{2a(ad-bc)^2\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\left(\frac{bd}{4(ad-bc)^2 c}+\frac{b^2}{2a(ad-bc)^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-$ 3 $\_\alpha$=R
elliptic	$\frac{d^2 x\sqrt{-b\,x^4+a}}{4c(ad-bc)^2(-d\,x^4+c)}+\frac{b^2 x}{2a(ad-bc)^2\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\left(\frac{bd}{4(ad-bc)^2 c}+\frac{b^2}{2a(ad-bc)^2}\right)\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-$ 3 $\_\alpha$=R

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2/c/(a*d-b*c)^2*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/2*b^2*x/a/(a*d-b*c)^2
/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d/(a*d-b*c)^2/c+1/2*b^2/a/(a*d-b*c)^2)/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*sum((a
*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2
))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}\,(c-dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx=\int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)^2}\,dx$$

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2), x)
```

3.190 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 439

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx &= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2(a-bx^4)^{3/2}} \\ &+ \frac{b(5b^2c^2-17abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\,(a-bx^4)^{3/2}\,(c-dx^4)} \\ &+ \frac{b^{3/4}(5b^2c^2-17abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}}\end{aligned}$$

```
[Out] 1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(3/2)-1/4*d*x/c/(-a*d+b*
c)/(-b*x^4+a)^(3/2)/(-d*x^4+c)+1/12*b*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*x/a
^2/c/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-3*a^2*d^2-17*a*b*c*d+5*b^
2*c^2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/c/(-a*d+b*c
)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1/4)*x/a
^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-
a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1
/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/
c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {425, 541, 537, 230, 227, 418, 1233, 1232}

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \frac{bx(-3a^2d^2-17abcd+5b^2c^2)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2-17abcd+5b^2c^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3} - \frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)} + \frac{bx(3ad+2bc)}{12ac(a-bx^4)^{3/2}(bc-ad)^2}$$

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) + (b*(5*b^2*
c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) -
 (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + (b^(3/4)*(5*b^2*c^
2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(12*a^(7/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^
2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a - b*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-9bdx^4}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-2\left(10b^2c^2-24abcd+9a^2d^2\right)+10bd(2bc+3ad)x^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{24ac(bc-ad)^2}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-4\left(5b^3c^3-17ab^2c^2d+36a^2bcd^2-9a^3d^3\right)+4bd\left(5b^2c^2-17abcd-3a^2d^2\right)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{48a^2c(bc-ad)^3}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} + \frac{\left(d^2(13bc-3ad)\right)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c(bc-ad)^3} + \frac{\left(b\left(5b^2c^2-17abcd-3a^2d^2\right)\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{12a^2c(bc-ad)^3}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} + \frac{\left(d^2(13bc-3ad)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)^3} + \frac{\left(d^2(13bc-3ad)\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)^3} + \frac{\left(b\left(5b^2c^2-17abcd-3a^2d^2\right)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{12a^2c(bc-ad)^3\sqrt{a-bx^4}}$$

$$
\begin{aligned}
&= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b(5b^2c^2-17abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} \\
&\quad + \frac{b^{3/4}(5b^2c^2-17abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad + \frac{\left(d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad + \frac{\left(d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2(bc-ad)^3\sqrt{a-bx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b(5b^2c^2-17abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} \\
&\quad + \frac{b^{3/4}(5b^2c^2-17abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad + \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}} \\
&\quad + \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.87

$$
\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx = \frac{x\left(\frac{bd(-5b^2c^2+17abcd+3a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2} + 5\left(\frac{5b^3c}{a^2} - \frac{17b^2d}{a} - \frac{2b}{a-}\right.\right.}{}
$$

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*Appe
llF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^2
```

```
- (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*a*d
^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 17*a*
b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/
a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c]
+ b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(60*(b*c - a*d
)^3*Sqrt[a - b*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.43 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.10

method	result
default	$-\frac{b\,d^3 x\sqrt{-b\,x^4+a}}{4(ad-bc)c(a^2d^2-2abcd+b^2c^2)(bd\,x^4-bc)}+\frac{x\sqrt{-b\,x^4+a}}{6(ad-bc)^2 a\left(x^4-\frac{a}{b}\right)^2}+\frac{b^2 x(17ad-5bc)}{12a^2(ad-bc)^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^2}{4(ad-bc)c\left(a^2d^2-2abcd+\right.}\right.}{}$
elliptic	$-\frac{b\,d^3 x\sqrt{-b\,x^4+a}}{4(ad-bc)c(a^2d^2-2abcd+b^2c^2)(bd\,x^4-bc)}+\frac{x\sqrt{-b\,x^4+a}}{6(ad-bc)^2 a\left(x^4-\frac{a}{b}\right)^2}+\frac{b^2 x(17ad-5bc)}{12a^2(ad-bc)^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^2}{4(ad-bc)c\left(a^2d^2-2abcd+\right.}\right.}{}$

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*d^3/(a*d-b*c)/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(b*d*
x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)^2+1/12*b^2*x/a^2*(1
7*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d^2/(a*d-b*c)/c/(a^2*d
^2-2*a*b*c*d+b^2*c^2)+1/12*b^2/a^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/2)*b
^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32*d/c*sum((3*a*
d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/(1/d*(a*d-b*c))^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/(1/d*(a*d-b*c))^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
```

```
/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2
))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-bx^4)^{\frac{5}{2}}(-c+dx^4)^2}\,dx$$

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
[Out] Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}\,(c-dx^4)^2}\,dx = \int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)^2}\,dx$$

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)
```

3.191 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] 1/4*arctan(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c*2^(
1/2)+1/4*arctanh(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)
/c*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {413, 218, 212, 209}

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[In] Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 413

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[In] Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
[Out] (ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]] + ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\left(-2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)+\ln\left(\frac{-\sqrt{2}\,(ab)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}{\sqrt{2}\,(ab)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}\right)\right)}{8c(ab)^{\frac{1}{4}}}$	89
default	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c(ab)^{\frac{1}{4}}}$	94
elliptic	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c(ab)^{\frac{1}{4}}}$	94

```
[In] int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x,method=_RETURNVERBOSE)
[Out] 1/8*2^(1/2)*(-2*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4))+ln((-2^(1
/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2))/(2^(1/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2)))
)/c/(a*b)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 437, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$$
$$=\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*
c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(a*
c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^
(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a
*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))
/(b*x^4 - a)) - 1/4*I*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^(3/4)*
a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) - 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4)
 - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a)) + 1/4*I*(1
/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4
))^(3/4) + 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4 + a)*(a*c
^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = -\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

```
[In] integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

```
[In] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
[Out] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

3.192 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] 1/2*arctan(b^(1/4)*x*(a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^(1/2))/a^(1/4
)/b^(1/4)/c+1/2*arctanh(b^(1/4)*x*(a^(1/2)-x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^
(1/2))/a^(1/4)/b^(1/4)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {414}

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[In] Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rule 414

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
```

```
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(\arctan\left(\frac{(1+i)\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a-bx^4}}\right) - i\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-bx^4}}{\sqrt[4]{a}\sqrt[4]{b}x}\right)\right)}{\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[In] Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
[Out] ((1/4 - I/4)*(ArcTan[((1 + I)*a^(1/4)*b^(1/4)*x)/Sqrt[a - b*x^4]] - I*ArcTa
n[((1/2 + I/2)*Sqrt[a - b*x^4])/(a^(1/4)*b^(1/4)*x)]))/(a^(1/4)*b^(1/4)*c)
```

Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141
elliptic	$-\frac{\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141
pseudoelliptic	$-\frac{\ln\left(\frac{-b\,x^4+2x^2\sqrt{ab}-2(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}\,x+a}{-b\,x^4+2(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}\,x+2x^2\sqrt{ab}+a}\right)+2\arctan\left(\frac{x(ab)^{\frac{1}{4}}+\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{-x(ab)^{\frac{1}{4}}+\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}\right)}{8(ab)^{\frac{1}{4}}c}$	149

```
[In] int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c/(a*b)^(1/4)*(ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(
a*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a*b)^(1/2))
)+2*arctan((-b*x^4+a)^(1/2)/x/(a*b)^(1/4)+1)+2*arctan((-b*x^4+a)^(1/2)/x/(a
*b)^(1/4)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{\ }}{bx^4+a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}-\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{-}}{bx^4+a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-}{bx^4+a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-}{bx^4+a}\right.$$

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(
a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/4)
*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*(1/4
)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^
(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1
/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) - 1/4*I*(1/4)^(1/4)*
(-1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4)
+ sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) + 2*I*(1/4)^(1/4)*a*c*x*(-1/(a*
b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*I*(1/4)^(1/4)*(-1/
(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + s
qrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*I*(1/4)^(1/4)*a*c*x*(-1/(a*b*c
^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a))
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

```
[In] integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

```
[In] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
[Out] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

3.193 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}$$

```
[Out] 1/4*b*x*(b*x^4+a)^(3/4)/d-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x
^4+a)^(1/4))/c^(3/4)/d^2-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*
x^4+a)^(1/4))/c^(3/4)/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {427, 544, 246, 218, 212, 209, 385, 214, 211}

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = -\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)(4bc-7ad)}{8d^2} + \frac{(bc-ad)^{7/4}\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)(4bc-7ad)}{8d^2} + \frac{(bc-ad)^{7/4}\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{bx(a+bx^4)^{3/4}}{4d}$$

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(a+bx^4)^{3/4}}{4d}+\frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4d}\\
&= \frac{bx(a+bx^4)^{3/4}}{4d}-\frac{(b(4bc-7ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^2}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{d^2}\\
&= \frac{bx(a+bx^4)^{3/4}}{4d}-\frac{(b(4bc-7ad))\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2}\\
&\quad+\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2}
\end{aligned}
$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{(b(4bc-7ad))\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{(b(4bc-7ad))\text{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}d^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}d^2}$$

$$= \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{2bdx(a+bx^4)^{3/4} - b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{(2+2i)(bc-ad)^{7/4}\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc}}{\sqrt[4]{c}\sqrt[4]{a}}}{}\right)}{c^{3/}}}{}$$

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (2*b*d*x*(a + b*x^4)^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTan[(((1 - I)*(b*c - a*d)
^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4
))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTanh[(b^
(1/4)*x)/(a + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTanh[(((1 - I
)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a
+ b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4))/(8*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(167) = 334$.

Time = 8.93 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{2}\,(ad-bc)^{2} \ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)}{2}+\sqrt{2}\,(ad-bc)^{2} \arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-}{}$

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-1/2*2^(1/2)*(a*d-b*c)^2*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1
/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+
a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+2^(1/2)*(a*d-b
*c)^2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^
(1/4)/x)-2^(1/2)*(a*d-b*c)^2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a
)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-7/2*((a*d-b*c)/c)^(1/4)*c*((-1/2*a*d*b^(3/4
)+2/7*b^(7/4)*c)*ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)
))+(a*d*b^(3/4)-4/7*b^(7/4)*c)*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-2/7*(b*x
^4+a)^(3/4)*x*b*d))/((a*d-b*c)/c)^(1/4)/d^2/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 1962, normalized size of antiderivative = 9.30

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*
c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6
*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*
a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5 - 5*
a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5
*d^5)*(b*x^4 + a)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^
5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^
6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^
6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5
*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*b
```

```
^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^
5)*(b*x^4 + a)^(1/4))/x) + 4*I*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5
*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6
*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((I*c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c
^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^
5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*(b*x^4 + a)^(1/4))/x) - 4*I*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^
5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^
6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((-I*c^2*d^6*x*((b^7*c^7 - 7*a*b^6
*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*
a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*
a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5
*d^5)*(b*x^4 + a)^(1/4))/x) - d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2
*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*
x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*
d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^
2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d
^4)/d^8)^(1/4)*log((d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c
^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 -
336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x
) - I*d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*
b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log((I*d^6*x*((256*b^7*c^4 - 1792*
a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)
/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a^3*b
^2*d^3)*(b*x^4 + a)^(1/4))/x) + I*d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704
*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log((-
I*d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*
b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d +
588*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx=\int \frac{(a+bx^4)^{\frac{7}{4}}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{7/4}}{d\,x^4+c}\,dx$$

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

3.194 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

```
[Out] 1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctan
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d+1/2*b^(3/4)*arctanh(
b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctanh((-a*d+b*c)^(1/4)*
x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {399, 246, 218, 212, 209, 385, 214, 211}

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b\text{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}d}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx =$$

$$-\frac{-2b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{(1+i)\left((-1+i)b^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+(bc-ad)^{3/4}\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}}{\sqrt[4]{b}}}{2x}\right)}{c^{3/4}}}{4d}$$

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] -1/4*(-2*b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((1 + I)*((-1 + I
)*b^(3/4)*c^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (b*c - a*d)^(3/4)
*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1
+ I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (b*c - a*d)^(3/
4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/c^(3/4))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(133) = 266$.

Time = 5.58 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.95

method	result
pseudoelliptic	$-\frac{2\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} c\left(2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right)-\ln\left(\frac{-b^{\frac{1}{4}} x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}}+\sqrt{2}\,(ad-bc)\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{ad}}\right.\right.}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} dc}$

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((a*d-b*c)/c)^(1/4)*(2*((a*d-b*c)/c)^(1/4)*c*(2*arctan(1/b^(1/4)/x*(b*
x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))))
*b^(3/4)+2^(1/2)*(a*d-b*c)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2
)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)
^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((((a*d-
b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan(((
(a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)))/d/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.24

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx =$$

$$-\frac{1}{4}\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,abcd\right.$$

$$+\frac{1}{4}\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(-\frac{c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,abc\right.$$

$$+\frac{1}{4}i\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{i\,c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,abc\right.$$

$$-\frac{1}{4}i\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\log\left(\frac{-i\,c^2d^3x\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}(b^2c^2-2\,a\right.$$

$$+\frac{1}{4}\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)-\frac{1}{4}\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(-\frac{d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

$$-\frac{1}{4}i\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{i\,d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

$$+\frac{1}{4}i\left(\frac{b^3}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{-i\,d^3x\left(\frac{b^3}{d^4}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^2}{x}\right)$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*
log((c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^
4))^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^
2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/
4) - (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*I*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((I*c^2*d^
3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) +
 (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) - 1/4*I*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((-I*c^2*d^3*x
*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) + (b
```

```
*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1/4)*l
og((d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b^3/d^4)^(1/4)
*log(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*I*(b^3/d^4)^
(1/4)*log((I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) + 1/4*I*(b^3
/d^4)^(1/4)*log((-I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x)
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{3}{4}}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{d\,x^4+c}\,dx$$

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

3.195 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[Out] 1/2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(
1/4)+1/2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+
b*c)^(1/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {385, 218, 214, 211}

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx=\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+\text{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)\right)}{c^{3/4}\sqrt[4]{bc-ad}}$$

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ((1/4 + I/4)*(ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^
(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + Ar
cTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 +
I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/(c^(3/4)*(b*c - a
*d)^(1/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(81) = 162$.

Time = 4.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)+2\arctan\left(\frac{\left(\frac{ad-b}{c}\right.}{}\right.}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c}$

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((a*d-b*c)/c)^(1/4)*2^(1/2)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*
2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*
x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((
((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arc
tan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x))
/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

3.196 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/4)-1/2*d*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)-1/2*d*arctanh((-a*d+b*c)^(1/4)*x/c^
(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {390, 385, 218, 214, 211}

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = -\frac{d\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\,\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} + \frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}$$

$$=\frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}}-\frac{d\text{Subst}\left(\int\frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)}-\frac{d\text{Subst}\left(\int\frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)}$$

$$=\frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int\frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx=\frac{1}{4}\left(\frac{4bx}{(abc-a^2d)\sqrt[4]{a+bx^4}}-\frac{(1+i)d\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}}-\frac{(1+i)d\text{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}}\right)$$

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] ((4*b*x)/((a*b*c - a^2*d)*(a + b*x^4)^(1/4)) - ((1 + I)*d*ArcTan[(((1 - I)*
(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a +
b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d)^(5/4)) - ((1
+ I)*d*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4))
```

```
 + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*
(b*c - a*d)^(5/4)))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(108) = 216$.

Time = 4.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.43

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right) a d\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x+\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right) a d\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\ln\left(\frac{-\left(\frac{ad-b}{c}\right.}{\left(\frac{ad-bc}{c}\right.}\right.}{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}(ad-bc)ca}$

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(1/4)*(arctan((((a*d-b*c)/c)^(1/4)*x-2^(1
/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)*a*d*2^(1/2)*(b*x^4+a)^(1/4)-arc
tan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)*
a*d*2^(1/2)*(b*x^4+a)^(1/4)-1/2*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^
(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/((((a*d-b*c)/c)^(1/4)*(b*x^
4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))*a*d*2^(1/2)*
(b*x^4+a)^(1/4)-4*b*x*c*((a*d-b*c)/c)^(1/4))/(a*d-b*c)/c/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{4}}(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

3.197 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 180

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}$$

```
[Out] 1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^(5/4)+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^4+a)^(1/4)+1/2*d^2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4)
)/c^(3/4)/(-a*d+b*c)^(9/4)+1/2*d^2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^
4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(9/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {425, 541, 12, 385, 218, 214, 211}

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2} + \frac{d^2\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} - \frac{\int \frac{-4bc+5ad-4bdx^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{5a(bc-ad)} \\
&= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{\int \frac{5a^2d^2}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{5a^2(bc-ad)^2} \\
&= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{(bc-ad)^2} \\
&= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{(bc-ad)^2} \\
&= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)^2} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)^2} \\
&= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} \\
&\quad + \frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 12.12 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.45

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{-585c^4(bc-ad)x^4(a+bx^4)^2 - 936c^3d(bc-ad)x^8(a+bx^4)^2 - 416c^2d^2(bc-}{}$$

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b*
x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4)^
3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925*c^
5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*
x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c
- a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c - a*d)^
3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*
x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric2F1[2, 13/
4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)^3*x^12*Hyp
ergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*HypergeometricP
FQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(325*c^4*(b
*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(148) = 296.

Time = 4.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.83

method	result
pseudoelliptic	$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}} a^2 d^2 \left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^4+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^4+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)+2\,\mathrm{arc}}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^4+a\right)^{\frac{5}{4}} (ad-bc)^2}$

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((b*x^4+a)^(5/4)*a^2*d^2*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4
+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((((a
*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan
```

```
((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x))*2^
(1/2)+16*x*((a*d-b*c)/c)^(1/4)*b*c*(a^2*d-1/2*b*(-9/5*d*x^4+c)*a-2/5*b^2*c*
x^4))/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(5/4)/(a*d-b*c)^2/c/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,\left(d\,x^4+c\right)}\,dx$$

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

3.198 $\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} \\ &+ \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)\,x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} \\ &- \frac{d^3\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}\end{aligned}$$

```
[Out] 1/9*b*x/a/(-a*d+b*c)/(b*x^4+a)^(9/4)+1/45*b*(-17*a*d+8*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(5/4)+1/45*b*(113*a^2*d^2-100*a*b*c*d+32*b^2*c^2)*x/a^3/(-a*d
+b*c)^3/(b*x^4+a)^(1/4)-1/2*d^3*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)
^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)-1/2*d^3*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4
)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {425, 541, 12, 385, 218, 214, 211}

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \frac{bx(8bc-17ad)}{45a^2(a+bx^4)^{5/4}(bc-ad)^2} + \frac{bx(113a^2d^2-100abcd+32b^2c^2)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3} - \frac{d^3\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} + \frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{(a+bx^4)^{9/4}(c+dx^4)}\,dx}{9a(bc-ad)} \\
&= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} \\
&\quad + \frac{\int \frac{32b^2c^2-68abcd+45a^2d^2+4bd(8bc-17ad)x^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{45a^2(bc-ad)^2} \\
&= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} \\
&\quad + \frac{b\left(32b^2c^2-100abcd+113a^2d^2\right)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\int \frac{45a^3d^3}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{45a^3(bc-ad)^3} \\
&= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} \\
&\quad + \frac{b\left(32b^2c^2-100abcd+113a^2d^2\right)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{d^3\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{(bc-ad)^3}
\end{aligned}
$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{d^3\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{(bc-ad)^3}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)^3} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)^3}$$

$$= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{d^3\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.50 (sec) , antiderivative size = 1172, normalized size of antiderivative = 5.03

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx =$$

$$-\frac{-16575c^5(bc-ad)^2x^8(a+bx^4)^2 - 39780c^4d(bc-ad)^2x^{12}(a+bx^4)^2 - 35360c^3d^2(bc-ad)^2x^{16}(a+bx^4)^2}{}$$

```
[In] Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] -1/11475*(-16575*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 - 39780*c^4*d*(b*c - a
*d)^2*x^12*(a + b*x^4)^2 - 35360*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^2 -
 10880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 - 29835*c^6*(b*c - a*d)*x^4
*(a + b*x^4)^3 - 71604*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 - 63648*c^4*d^2*
(b*c - a*d)*x^12*(a + b*x^4)^3 - 19584*c^3*d^3*(b*c - a*d)*x^16*(a + b*x^4)
^3 - 149175*c^7*(a + b*x^4)^4 - 358020*c^6*d*x^4*(a + b*x^4)^4 - 318240*c^5
*d^2*x^8*(a + b*x^4)^4 - 97920*c^4*d^3*x^12*(a + b*x^4)^4 + 149175*c^7*(a +
 b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
 + 358020*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*
d)*x^4)/(c*(a + b*x^4))] + 318240*c^5*d^2*x^8*(a + b*x^4)^4*Hypergeometric2
F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 97920*c^4*d^3*x^12*(a
```

```
+ b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))
] + 13620*c^3*(b*c - a*d)^4*x^16*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a
*d)*x^4)/(c*(a + b*x^4))] + 36900*c^2*d*(b*c - a*d)^4*x^20*Hypergeometric2F
1[2, 17/4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 33840*c*d^2*(b*c - a*
d)^4*x^24*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4)
)] + 10560*d^3*(b*c - a*d)^4*x^28*Hypergeometric2F1[2, 17/4, 21/4, ((b*c -
a*d)*x^4)/(c*(a + b*x^4))] + 6480*c^3*(b*c - a*d)^4*x^16*HypergeometricPFQ[
{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 18720*c^2*d*(
b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x
^4)/(c*(a + b*x^4))] + 18000*c*d^2*(b*c - a*d)^4*x^24*HypergeometricPFQ[{2,
 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 5760*d^3*(b*c -
a*d)^4*x^28*HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c
*(a + b*x^4))] + 960*c^3*(b*c - a*d)^4*x^16*HypergeometricPFQ[{2, 2, 2, 17/
4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 2880*c^2*d*(b*c - a*
d)^4*x^20*HypergeometricPFQ[{2, 2, 2, 17/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4
)/(c*(a + b*x^4))] + 2880*c*d^2*(b*c - a*d)^4*x^24*HypergeometricPFQ[{2, 2,
 2, 17/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 960*d^3*(b*c
- a*d)^4*x^28*HypergeometricPFQ[{2, 2, 2, 17/4}, {1, 1, 21/4}, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))])/(c^5*(-(b*c) + a*d)^3*x^11*(a + b*x^4)^(17/4))
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$-\frac{3\left(\frac{a^3 d^3 \left(b x^4+a\right)^{\frac{9}{4}}\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b x^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b x^4+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x-\sqrt{2}\left(b x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}\right)+2\arctan}{24}\right.}{\left(\frac{ad-b}{c}\right.}$

```
[In] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -3/((a*d-b*c)/c)^(1/4)*(1/24*a^3*d^3*(b*x^4+a)^(9/4)*(ln((-((a*d-b*c)/c)^(1
/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a
*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a
)^(1/2)))-2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*
c)/c)^(1/4)/x)+2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a
*d-b*c)/c)^(1/4)/x))*2^(1/2)+x*((a*d-b*c)/c)^(1/4)*b*(a^4*d^2-b*(-9/5*d*x^4
+c)*d*a^3+1/3*b^2*(113/45*d^2*x^8-5*c*d*x^4+c^2)*a^2+8/15*x^4*b^3*(-25/18*d
*x^4+c)*c*a+32/135*b^4*c^2*x^8)*c)/(b*x^4+a)^(9/4)/(a*d-b*c)^3/c/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{13}{4}}(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx=\int \frac{1}{(b\,x^4+a)^{13/4}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

3.199 $\int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 316

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx = -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\sqrt{a}b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12d^2(a+bx^4)^{3/4}} + \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2} + \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2}$$

```
[Out] -1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/d^2+1/6*b*x*(b*x^4+a)^(5/4)/d+1/1
2*b^(3/2)*(-11*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d^2/(b*x^4+a)^(3/4)+1/2*(-a*d
+b*c)^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1
/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2+1/2*(-a*d+b*c)^2*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/
(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {427, 542, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}cd^2}$$
$$+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}cd^2}$$
$$+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12d^2\left(a+bx^4\right)^{3/4}}$$
$$-\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}$$

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] -1/12*(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/d^2 + (b*x*(a + b*x^4)^(5/4)
)/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*d^2*(a + b*x^4)^(3/4)) + ((b*
c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d^2) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(2*b^(1/4)*c*d^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(a+bx^4)^{5/4}}{6d}+\frac{\int \frac{\sqrt[4]{a+bx^4}(-a(bc-6ad)-b(6bc-11ad)x^4)}{c+dx^4}\,dx}{6d}\\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}\\
&\quad+\frac{\int \frac{a(6b^2c^2-13abcd+12a^2d^2)+b(12b^2c^2-30abcd+23a^2d^2)x^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{12d^2}\\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}\\
&\quad-\frac{(ab(6bc-11ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{12d^2}+\frac{(bc-ad)^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{d^2}\\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}\\
&\quad-\frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12d^2(a+bx^4)^{3/4}}\\
&\quad+\frac{\left((bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}(c-(bc-ad)x^4)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2}\\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}\\
&\quad+\frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{12d^2(a+bx^4)^{3/4}}\\
&\quad+\frac{\left((bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2cd^2}\\
&\quad+\frac{\left((bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2cd^2}
\end{aligned}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d}$$
$$+ \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}$$
$$+ \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}$$
$$+ \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}dx, x, \frac{1}{x^2}\right)}{24d^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d}$$
$$+ \frac{\sqrt{a}b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}$$
$$+ \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}$$
$$+ \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.65 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.93

$$\int\frac{(a+bx^4)^{9/4}}{c+dx^4}dx = \frac{x\left(5b(a+bx^4)(-6bc+13ad+2bdx^4)+\frac{b(12b^2c^2-30abcd+23a^2d^2)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4}\cdots\right.}{c}\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] (x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*b
*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2*d^
2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*Ap
pellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4,
7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3/4))
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}}{d\,x^{4}+c}dx$$

[In] int((b*x^4+a)^(9/4)/(d*x^4+c),x)

[Out] int((b*x^4+a)^(9/4)/(d*x^4+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \text{Timed out}$$

[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{9}{4}}}{c+dx^4}\,dx$$

[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)

[Out] Integral((a + b*x**4)**(9/4)/(c + d*x**4), x)

Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)

Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{9/4}}{d\,x^4+c}\,dx$$

[In] int((a + b*x^4)^(9/4)/(c + d*x^4),x)

[Out] int((a + b*x^4)^(9/4)/(c + d*x^4), x)

3.200 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4} dx = \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2d(a+bx^4)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd}$$

```
[Out] 1/2*b*x*(b*x^4+a)^(1/4)/d-1/2*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot
(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Ellipt
icF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d/(b*x^4+a)^(3/4)
-1/2*(-a*d+b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1
/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d-1/2*(-a*d+b*
c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)
*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {417, 201, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = -\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}cd} - \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}cd} - \frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2d\,(a+bx^4)^{3/4}} + \frac{bx\sqrt[4]{a+bx^4}}{2d}$$

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*
EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a + b*x^4)^(3/4)) - ((
b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^4)^(5/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/
d, Int[(a + b*x^4)^(1/4), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(1/4
)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \sqrt[4]{a+bx^4}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{d}$$

$$=\frac{bx\sqrt[4]{a+bx^4}}{2d}+\frac{(ab)\int\frac{1}{(a+bx^4)^{3/4}}\,dx}{2d}-\frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-bx^4}(c-(bc-ad)x^4)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$=\frac{bx\sqrt[4]{a+bx^4}}{2d}+\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int\frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2d\,(a+bx^4)^{3/4}}-\frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2cd}-\frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2cd}$$

$$=\frac{bx\sqrt[4]{a+bx^4}}{2d}-\frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{2d\,(a+bx^4)^{3/4}}$$

$$=\frac{bx\sqrt[4]{a+bx^4}}{2d}-\frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{4d\,(a+bx^4)^{3/4}}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} + \frac{5\left(-5ac\left(2a^2d+abdx^4+b^2x^4(c+dx^4)\right)\text{AppellF1}\left(\right.\right.}{(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\right.\right.}\right.}{10d\,(}$$

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4
, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*x^
4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x
^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)
])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(10*d*(a
 + b*x^4)^(3/4))
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{d\,x^4+c}dx$$

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx=\int \frac{\left(a+bx^4\right)^{\frac{5}{4}}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx=\int \frac{(b\,x^4+a)^{5/4}}{d\,x^4+c}\,dx$$

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4), x)
```

3.201 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c}$$

```
[Out] 1/2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),
I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c+1/2*EllipticPi(b^(1/4)*x/(
b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x
^4+a)^(1/2)/b^(1/4)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {416, 418, 1232}

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}$$

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*
Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c) + (Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\,(c-(bc-ad)x^4)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\
&= \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c}\\
&\quad+\frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c}\\
&= \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}\\
&\quad+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{5acx\sqrt[4]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(-4ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,\text{App}\right.\right.}$$

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{d\,x^{4}+c}dx$$

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{d\,x^4+c}\,dx$$

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4), x)
```

3.202 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(bc-ad)\,(a+bx^4)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

```
[Out] -b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2
)/a^(1/2))),2^(1/2))/(-a*d+b*c)/(b*x^4+a)^(3/4)/a^(1/2)-1/2*d*EllipticPi(b^
(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^
(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)-1/2*d*EllipticPi(b^(1/4)*x/(b*x^
4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a
)^(1/2)/b^(1/4)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {419, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}dx = -\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^4)^{3/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)]*
Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a
+ b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 419

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{bc-ad}-\frac{d\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{bc-ad}\\
&= \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{(bc-ad)\left(a+bx^4\right)^{3/4}}\\
&\quad-\frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}
\end{aligned}
$$

$$= -\frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{(bc-ad)\,(a+bx^4)^{3/4}}$$

$$-\frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)}$$

$$-\frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)}$$

$$= -\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

$$-\frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{2(bc-ad)\,(a+bx^4)^{3/4}}$$

$$= -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\,(a+bx^4)^{3/4}}$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int\frac{1}{(a+bx^4)^{3/4}\,(c+dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^{3/4}\,(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\right.\right.}$$

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*x
^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(d\,x^{4}+c\right)}d x$$

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{\left(a+bx^{4}\right)^{\frac{3}{4}}\left(c+dx^{4}\right)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{{\left(bx^{4}+a\right)}^{\frac{3}{4}}{\left(dx^{4}+c\right)}}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/4}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)
```

3.203 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 304

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}}\\ &-\frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(bc-ad)^2(a+bx^4)^{3/4}}\\ &+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}\\ &+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}\end{aligned}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/3*b^(3/2)*(-5*a*d+2*b*c)*(1+a/b/x^4)
^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^
2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))
/a^(3/2)/(-a*d+b*c)^2/(b*x^4+a)^(3/4)+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a
)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^
(1/2)/b^(1/4)/c/(-a*d+b*c)^2+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(
-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1
/4)/c/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = -\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a*d)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/
2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(
1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\int \frac{-2bc+3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\left(d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}(c-(bc-ad)x^4)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\left(d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)^2} + \frac{\left(d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{6a(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$
$$+ \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}$$
$$+ \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx = \frac{x\left(-\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} + \frac{5\left(5ac\left(3ad-b\left(3c+dx^4\right)\right)\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1\ldots\right.}{\left(c+dx^4\right)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1\ldots\right.\right.}\right.}{15a(-bc+\ldots}$$

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(d\,x^4+c\right)}dx$$

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{7}{4}}(c+dx^4)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{7/4}\,(d\,x^4+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)), x)
```

3.204 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx &= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} \\
&+ \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} \\
&- \frac{b^{3/2}(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}(bc-ad)^3(a+bx^4)^{3/4}} \\
&- \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3} \\
&- \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}
\end{aligned}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^4+a)^(7/4)+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(3/4)-1/21*b^(3/2)*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/b/
x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1
/2))/a^(5/2)/(-a*d+b*c)^3/(b*x^4+a)^(3/4)-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x
^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4
+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^3-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {425, 541, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{bx(6bc-13ad)}{21a^2(a+bx^4)^{3/4}(bc-ad)^2} - \frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(47a^2d^2-38abcd+12b^2c^2)\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}(a+bx^4)^{3/4}(bc-ad)^3} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3} + \frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*
(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*a
^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2
, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a + b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[
(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
```

```
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} - \frac{\int \frac{-6bc+7ad-6bdx^4}{(a+bx^4)^{7/4}(c+dx^4)}\,dx}{7a(bc-ad)} \\
&= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} \\
&\quad + \frac{\int \frac{12b^2c^2-26abcd+21a^2d^2+2bd(6bc-13ad)x^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{21a^2(bc-ad)^2} \\
&= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} \\
&\quad - \frac{d^3\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^3} + \frac{\left(b\left(12b^2c^2-38abcd+47a^2d^2\right)\right)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{21a^2(bc-ad)^3} \\
&= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} \\
&\quad + \frac{\left(b\left(12b^2c^2-38abcd+47a^2d^2\right)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{21a^2(bc-ad)^3\left(a+bx^4\right)^{3/4}} \\
&\quad - \frac{\left(d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{(bc-ad)^3}
\end{aligned}$$

$$=\frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}}+\frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{\left(b(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{21a^2(bc-ad)^3\left(a+bx^4\right)^{3/4}}-\frac{\left(d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)^3}-\frac{\left(d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c(bc-ad)^3}$$

$$=\frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}}+\frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{\left(b(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{42a^2(bc-ad)^3\left(a+bx^4\right)^{3/4}}$$

$$=\frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}}+\frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{b^{3/2}(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}(bc-ad)^3\left(a+bx^4\right)^{3/4}}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.71 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{x\left(-\frac{2bd(-6bc+13ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac(21a^3d^2+6b^3cx^4(3c \cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*x^
4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4 -
13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4
*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(a + b*x^4)^(
3/4))
```

Maple [F]

$$\int \frac{1}{(b\,x^4+a)^{\frac{11}{4}}\,(d\,x^4+c)}dx$$

```
[In] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx=\int \frac{1}{\left(a+bx^4\right)^{\frac{11}{4}}\left(c+dx^4\right)}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(11/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}\left(dx^4+c\right)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}\left(dx^4+c\right)}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx=\int \frac{1}{{\left(b\,x^4+a\right)}^{11/4}\,\left(d\,x^4+c\right)}\,dx$$

```
[In] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)), x)
```

3.205 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\begin{aligned}\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx = & \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} \\ & - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{b^{7/4}(8bc-11ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\ & + \frac{(bc-ad)^{7/4}(8bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ & - \frac{b^{7/4}(8bc-11ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\ & + \frac{(bc-ad)^{7/4}(8bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}\end{aligned}$$

```
[Out] 1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^(3/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(7/4)
/c/d/(d*x^4+c)-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4)
)/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/
(b*x^4+a)^(1/4))/c^(7/4)/d^3-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctanh(b^(1/4)*x/
(b*x^4+a)^(1/4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctanh((-a*d+b*c)^
(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {424, 542, 544, 246, 218, 212, 209, 385, 214, 211}

$$\begin{aligned}\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = &-\frac{b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)(8bc-11ad)}{8d^3} \\ &+\frac{(bc-ad)^{7/4}(3ad+8bc)\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ &-\frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)(8bc-11ad)}{8d^3} \\ &+\frac{(bc-ad)^{7/4}(3ad+8bc)\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ &+\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{4cd^2}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}\end{aligned}$$

```
[In] Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} + \frac{\int \frac{(a+bx^4)^{3/4}(a(bc+3ad)+4b(2bc-ad)x^4)}{c+dx^4}\,dx}{4cd}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} + \frac{\int \frac{-4a(2b^2c^2-2abcd-3a^2d^2)-4b^2c(8bc-11ad)x^4}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{16cd^2}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^3} + \frac{((bc-ad)^2(8bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4cd^3}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad))\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^3} + \frac{((bc-ad)^2(8bc+3ad))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4cd^3}$$

$$= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad))\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} - \frac{(b^2(8bc-11ad))\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{((bc-ad)^2(8bc+3ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}d^3} + \frac{((bc-ad)^2(8bc+3ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}d^3}$$

$$
\begin{aligned}
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} \\
&\quad - \frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\
&\quad + \frac{(bc-ad)^{7/4}(8bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\
&\quad - \frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\
&\quad + \frac{(bc-ad)^{7/4}(8bc+3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.25

$$
\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)dx(a+bx^4)^{3/4}(-2abcd+a^2d^2+b^2c(2c+dx^4))}{c(c+dx^4)} - (1-i)b^{7/4}(8bc-11ad)\arctan\left(\right.\right.}{}
$$

```
[In] Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((1/16 + I/16)*(((2 - 2*I)*d*x*(a + b*x^4)^(3/4)*(-2*a*b*c*d + a^2*d^2 + b^
2*c*(2*c + d*x^4)))/(c*(c + d*x^4)) - (1 - I)*b^(7/4)*(8*b*c - 11*a*d)*ArcT
an[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcT
an[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*
c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(7/4) - (1 - I)*b^(
7/4)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)
^(7/4)*(8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a
 + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(
2*x)])/c^(7/4)))/d^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(232) = 464$.

Time = 5.36 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.81

method	result
pseudoelliptic	$-\frac{-8\left(2b^2c^2-2b\left(-\frac{bx^4}{2}+a\right)dc+a^2d^2\right)xd\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c\left(bx^4+a\right)^{\frac{3}{4}}+\left(16\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^3\left(\ln\left(\frac{-b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{(b x \cdots}{}\right.\right.\right.}{}$

```
[In] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-8*(2*b^2*c^2-2*b*(-1/2*b*x^4+a)*d*c+a^2*d^2)*x*d*((a*d-b*c)/c)^(1/4
)*c*(b*x^4+a)^(3/4)+(16*((a*d-b*c)/c)^(1/4)*c^3*(ln((-b^(1/4)*x-(b*x^4+a)^(
1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b
^(11/4)-22*((a*d-b*c)/c)^(1/4)*a*c^2*d*(ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^
(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b^(7/4)+2^
(1/2)*(3*a*d+8*b*c)*(a*d-b*c)^2*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2
^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x
^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan(((
(a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arct
an((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)))
*(d*x^4+c))/((a*d-b*c)/c)^(1/4)/d^3/c^2/(d*x^4+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 2764, normalized size of antiderivative = 9.87

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*
a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*
b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^
3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))
^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b
^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c
^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8
 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4
) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^
5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 -
```

81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2)*
((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*
b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c
^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9
- 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log((c^5*d^9*x*((4096
*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^
8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6
 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*
a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) - (512*b^8*c^8 - 1984*a*b^7
*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4
+ 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*
(b*x^4 + a)^(1/4))/x) - (-I*c*d^3*x^4 - I*c^2*d^2)*((4096*b^11*c^11 - 22528
*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b
^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^
4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81
*a^11*d^11)/(c^7*d^12))^(1/4)*log((I*c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b
^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c
^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^
7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^1
1*d^11)/(c^7*d^12))^(3/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*
c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5
+ 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x)
- (I*c*d^3*x^4 + I*c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*
a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*
b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^
3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))
^(1/4)*log((-I*c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2
*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6
*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d
^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3
/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*
c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6
- 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2
)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*log(-(d^9*x*((4096*b^11*c^4 - 22
528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*
b^7*d^4)/d^12)^(3/4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2
 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b
^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3
+ 14641*a^4*b^7*d^4)/d^12)^(1/4)*log((d^9*x*((4096*b^11*c^4 - 22528*a*b^10*
c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^
12)^(3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3
*b^5*d^3)*(b*x^4 + a)^(1/4))/x) - (I*c*d^3*x^4 + I*c^2*d^2)*((4096*b^11*c^4
 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641
*a^4*b^7*d^4)/d^12)^(1/4)*log((I*d^9*x*((4096*b^11*c^4 - 22528*a*b^10*c^3*d

```
 + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(
3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3*b^5*
d^3)*(b*x^4 + a)^(1/4))/x) - (-I*c*d^3*x^4 - I*c^2*d^2)*((4096*b^11*c^4 - 2
2528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4
*b^7*d^4)/d^12)^(1/4)*log((-I*d^9*x*((4096*b^11*c^4 - 22528*a*b^10*c^3*d +
46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(3/4
) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3*b^5*d^3
)*(b*x^4 + a)^(1/4))/x) + 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*x)*(b*x^4 + a)^(3/4))/(c*d^3*x^4 + c^2*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{11/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

3.206 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx = -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd(c+dx^4)} + \frac{b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} + \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(3/4)/c/d/(d*x^4+c)+1/2*b^(7/4)*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctan((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2+1/2*b^(7/4)*arctanh(b^(1/4)*x
/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctanh((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{424, 544, 246, 218, 212, 209, 385, 214, 211\}$

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \frac{b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc)\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} + \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc)\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{x(a+bx^4)^{3/4}(bc-ad)}{4cd(c+dx^4)}$$

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[
(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d
)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2
) + (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)
^(3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(
1/4))])/(8*c^(7/4)*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)}+\frac{\int \frac{a(bc+3ad)+4b^2cx^4}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4cd}\\
&= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d^2}-\frac{((bc-ad)(4bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4cd^2}\\
&= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd\,(c+dx^4)}+\frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2}\\
&\quad -\frac{((bc-ad)(4bc+3ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4cd^2}
\end{aligned}
$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd(c+dx^4)} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}$$
$$- \frac{((bc-ad)(4bc+3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}d^2}$$
$$- \frac{((bc-ad)(4bc+3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd(c+dx^4)} + \frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}$$
$$- \frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} + \frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}$$
$$- \frac{(bc-ad)^{3/4}(4bc+3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)d(bc-ad)x(a+bx^4)^{3/4}}{c(c+dx^4)} + (4-4i)b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - \frac{(4b^2c^2-abcd-3a\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((1/16 + I/16)*(((-2 + 2*I)*d*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*(c + d*x^
4)) + (4 - 4*I)*b^(7/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c^2
 - a*b*c*d - 3*a^2*d^2)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a
 + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(
```

```
2*x)])/(c^(7/4)*(b*c - a*d)^(1/4)) + (4 - 4*I)*b^(7/4)*ArcTanh[(b^(1/4)*x)/
(a + b*x^4)^(1/4)] - ((4*b^2*c^2 - a*b*c*d - 3*a^2*d^2)*ArcTanh[(((1 - I)*(
b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b
*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4))))/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(186) = 372$.

Time = 4.51 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.77

method	result
pseudoelliptic	$-\frac{-\left(b x^4+a\right)^{\frac{3}{4}} x(ad-bc)dc\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}+\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} c^2\left(2\arctan\left(\frac{\left(b x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right)-\ln\left(\frac{-b^{\frac{1}{4}} x-\left(b x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x-\left(b x^4+a\right)^{\frac{1}{4}}}\right)\right) b^{\frac{7}{4}}+\frac{\sqrt{2}\left(3a^2d^2+\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-(b*x^4+a)^(3/4)*x*(a*d-b*c)*d*c*((a*d-b*c)/c)^(1/4)+(((a*d-b*c)/c)^(
1/4)*c^2*(2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1
/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))))*b^(7/4)+1/8*2^(1/2)*(3*a^2*d^2+a*b*c*d-4
*b^2*c^2)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)
^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+
((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((((a*d-b*c)/c)^(1/4)*x-2
^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan((((a*d-b*c)/c)^(1/4
)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)))*(d*x^4+c))/((a*d-b*c)
/c)^(1/4)/d^2/c^2/(d*x^4+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 1440, normalized size of antiderivative = 6.26

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + (c*d^2*x^4 + c^2*d)*((256*b^7*c^
7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a
^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((c^5*d^
6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3
```

```
*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(
3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*
d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) - (c*d^2*x^4 + c^2
*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3
*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(
1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*
d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*
d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2
- 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) +
 (-I*c*d^2*x^4 - I*c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4
*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81
*a^7*d^7)/(c^7*d^8))^(1/4)*log((I*c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5
*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 18
9*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d
 - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*
(b*x^4 + a)^(1/4))/x) + (I*c*d^2*x^4 + I*c^2*d)*((256*b^7*c^7 - 672*a^2*b^5
*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5
- 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((-I*c^5*d^6*x*((256*b^
7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 1
89*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) + (64*b
^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4
*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d
^8)^(1/4)*log((d^6*x*(b^7/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2
*x^4 + c^2*d)*(b^7/d^8)^(1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4 + a)^(1/
4)*b^5)/x) + 4*(I*c*d^2*x^4 + I*c^2*d)*(b^7/d^8)^(1/4)*log((I*d^6*x*(b^7/d^
8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(-I*c*d^2*x^4 - I*c^2*d)*(b^7/d^8)
^(1/4)*log((-I*d^6*x*(b^7/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x))/(c*d^2*x^
4 + c^2*d)
```

Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx=\int \frac{(a+bx^4)^{\frac{7}{4}}}{(c+dx^4)^2}\,dx$$

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{7/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

3.207 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)}+\frac{3a\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

```
[Out] 1/4*x*(b*x^4+a)^(3/4)/c/(d*x^4+c)+3/8*a*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(
b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)+3/8*a*arctanh((-a*d+b*c)^(1/4)*x/c
^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {386, 385, 218, 214, 211}

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\frac{3a\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)}$$

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)} + \frac{(3a)\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4c}$$

$$= \frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)} + \frac{(3a)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c}$$

$$= \frac{x(a+bx^4)^{3/4}}{4c\,(c+dx^4)} + \frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}} + \frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}}$$

$$= \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \frac{4c^{3/4}\sqrt[4]{bc-ad}x(a+bx^4)^{3/4} + (3+3i)a(c+dx^4)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a}}{\sqrt[4]{bc-}}}{2x}\right.}{16c^{7/4}\sqrt[4]{bc-ad}\,(c}$$

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (4*c^(3/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + (3 + 3*I)*a*(c + d*x^4)*
ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 +
 I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (3 + 3*I)*a*(c +
 d*x^4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)
) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(16*c^(7
/4)*(b*c - a*d)^(1/4)*(c + d*x^4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(107) = 214.

Time = 4.43 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.17

method	result
pseudoelliptic	$\frac{\frac{\left(b\,x^4+a\right)^{\frac{3}{4}} x c \left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}}{4} + \frac{3\sqrt{2}\,a\left(d\,x^4+c\right)\left(2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x+\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-\ln\left(\right.\right.}{32}}{c^2\left(d\,x^4+c\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}}$

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16/((a*d-b*c)/c)^(1/4)*(4/3*(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/4)+1/2*2
^(1/2)*a*(d*x^4+c)*(2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4)
)/((a*d-b*c)/c)^(1/4)/x)-2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^
(1/4))/((a*d-b*c)/c)^(1/4)/x)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1
/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+
a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))))/c^2/(d*x^4+c
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{3}{4}}}{(c+dx^4)^2}\,dx$$

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(b\,x^4+a)^{3/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

3.208 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx=-\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)\,(c+dx^4)}+\frac{(4bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

```
[Out] -1/4*d*x*(b*x^4+a)^(3/4)/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*arctan((
-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(5/4)+1/8*(-3
*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a
*d+b*c)^(5/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {390, 385, 218, 214, 211}

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx=\frac{(4bc-3ad)\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx(a+bx^4)^{3/4}}{4c\,(c+dx^4)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*(a + b*x^4)^(3/4))/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)
*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c
 - a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a
 + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$=-\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}$$

$$= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)} + \frac{(4bc-3ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)} + \frac{(4bc-3ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)}$$

$$= -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)} + \frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2}\,dx$$

$$= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)c^{3/4}dx(a+bx^4)^{3/4}}{(bc-ad)(c+dx^4)} + \frac{(4bc-3ad)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{5/4}} + \frac{(4bc-3ad)\mathrm{arctanh}\left(\right.}{}\right.}{c^{7/4}}$$

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] ((1/16 + I/16)*(((-2 + 2*I)*c^(3/4)*d*x*(a + b*x^4)^(3/4))/((b*c - a*d)*(c
+ d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/
4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/
4))/(2*x)])/(b*c - a*d)^(5/4) + ((4*b*c - 3*a*d)*ArcTanh[(((1 - I)*(b*c - a
*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(
1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(5/4)))/c^(7/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(134) = 268$.

Time = 4.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.12

method	result
pseudoelliptic	$-\frac{3\left(ad-\frac{4bc}{3}\right)\sqrt{2}\left(dx^4+c\right)\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(bx^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{bx^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(bx^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{bx^4+a}}\right)}{32}+\frac{3\left(ad-\frac{4bc}{3}\right)\sqrt{2}\left(dx^4+c\right)\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x \cdots}{\left(\frac{ad \cdots}{}\right)}\right)}{16} \cdots$ $c^2(ad-bc)(dx^4+c)\left(\frac{ad- \cdots}{c}\right.$

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*(-1/2*(a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+
a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^
(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+(
a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*arctan(((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4
+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-(a*d-4/3*b*c)*2^(1/2)*(d*x^4+c)*arctan(((
(a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+4/3*d*
(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/4))/((a*d-b*c)/c)^(1/4)/c^2/(a*d-b*c)/
(d*x^4+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)^2}\,dx$$

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

3.209 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{d(8bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}$$

```
[Out] 1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d+b*c)/(
b*x^4+a)^(1/4)/(d*x^4+c)-1/8*d*(-3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(
1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)-1/8*d*(-3*a*d+8*b*c)*arctanh
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {425, 541, 12, 385, 218, 214, 211}

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx = -\frac{d(8bc-3ad)\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc-3ad)\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} + \frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} + \frac{\int \frac{4bc-3ad-4bdx^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4c(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)^2} - \frac{(d(8bc-3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{d(8bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)c^{3/4}x(a^2d^2+abd^2x^4+4b^2c(c+dx^4))}{a(bc-ad)^2\sqrt[4]{a+bx^4}(c+dx^4)} + \frac{d(-8bc+3ad)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}}{}\right.}{(bc-ad)^{9/4}}\right.}{}$$

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] ((1/16 + I/16)*(((2 - 2*I)*c^(3/4)*x*(a^2*d^2 + a*b*d^2*x^4 + 4*b^2*c*(c +
d*x^4)))/(a*(b*c - a*d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) + (d*(-8*b*c + 3*a
*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]])/(b*c - a*d)^(
9/4) + (d*(-8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4
)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4
))/(2*x)]])/(b*c - a*d)^(9/4)))/c^(7/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(173) = 346.

Time = 4.48 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.73

method	result
pseudoelliptic	$\frac{\frac{\sqrt{2}\, da \left(d\,x^{4}+c\right)\left(3ad-8bc\right)\left(2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x+\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)-\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4} \cdots}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4} \cdots}\right.\right.}{8}}{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}} c^{2}\left(d\,x^{4}+c\right)\left(a \cdots\right.}$

[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/4/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(1/4)*(1/8*2^(1/2)*d*a*(d*x^4+c)*(3*a*d-8*b*c)*(2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))))*(b*x^4+a)^(1/4)+x*((a*d-b*c)/c)^(1/4)*c*(4*b^2*c^2+4*b^2*c*d*x^4+d^2*a*(b*x^4+a)))/c^2/(d*x^4+c)/(a*d-b*c)^2/a

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx=\text{Timed out}$$

[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)^2}\,dx$$

[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)

[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)^2}\,dx$$

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

3.210 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)\,x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{5/4}(c+dx^4)} + \frac{3d^2(4bc-ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}$$

```
[Out] 1/20*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(5/4)+1/20*b*(-5*a^2*d^2-
56*a*b*c*d+16*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d
+b*c)/(b*x^4+a)^(5/4)/(d*x^4+c)+3/8*d^2*(-a*d+4*b*c)*arctan((-a*d+b*c)^(1/4
)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13/4)+3/8*d^2*(-a*d+4*b*c)
*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13
/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{425, 541, 12, 385, 218, 214, 211\}$

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{3d^2(4bc-ad)\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} - \frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)} + \frac{bx(5ad+4bc)}{20ac(a+bx^4)^{5/4}(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)} \\
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}-\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} \\
&\quad -\frac{\int \frac{-16b^2c^2+40abcd-15a^2d^2-4bd(4bc+5ad)x^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{20ac(bc-ad)^2} \\
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} \\
&\quad -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}+\frac{\int \frac{15a^2d^2(4bc-ad)}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{20a^2c(bc-ad)^3} \\
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} \\
&\quad -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}+\frac{\left(3d^2(4bc-ad)\right)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} \\
&\quad -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} \\
&\quad +\frac{\left(3d^2(4bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)^3} \\
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} \\
&\quad -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} \\
&\quad +\frac{\left(3d^2(4bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)^3} \\
&\quad +\frac{\left(3d^2(4bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)^3} \\
&= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}}+\frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} \\
&\quad -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} \\
&\quad +\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$
\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = \frac{\left(\frac{1}{80}+\frac{i}{80}\right)\left(\frac{(2-2i)c^{3/4}x\left(5a^4d^3+10a^3bd^3x^4-16b^4c^2x^4\left(c+dx^4\right)+5a^2b^2d\left(12c^2+12cdx^4+d^2x^8\right)+4ab^3c\right.}{a^2(-bc+ad)^3\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\right.}{}
$$

[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]

[Out] ((1/80 + I/80)*(((2 - 2*I)*c^(3/4)*x*(5*a^4*d^3 + 10*a^3*b*d^3*x^4 - 16*b^4*c^2*x^4*(c + d*x^4) + 5*a^2*b^2*d*(12*c^2 + 12*c*d*x^4 + d^2*x^8) + 4*a*b^3*c*(-5*c^2 + 9*c*d*x^4 + 14*d^2*x^8)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^4)^(5/4)*(c + d*x^4)) + (15*d^2*(4*b*c - a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13/4) + (15*d^2*(4*b*c - a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(13/4)))/c^(7/4)

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$-\frac{3\left(\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2} d^{2} \left(d\,x^{4}+c\right)\left(ad-4bc\right)\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)-2\arctan\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x-\cdots}{\left(\frac{ad-b\cdots}{c}\right.}\right.\right.\right.}{\cdots}$

[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)

[Out] -3/32/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(5/4)*((b*x^4+a)^(5/4)*a^2*d^2*(d*x^4+c)*(a*d-4*b*c)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))-2*arctan((((a*d-b*c)/c)^(1/4)*x-2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x)+2*arctan((((a*d-b*c)/c)^(1/4)*x+2^(1/2)*(b*x^4+a)^(1/4))/((a*d-b*c)/c)^(1/4)/x))*2^(1/2)-8/15*(5*a^2*b^2*d^3*x^8+56*a*b^3*c*d^2*x^8-16*b^4*c^2*d*x^8+10*a^3*b*d^3*x^4+60*a^2*b^2*c*d^2*x^4+36*a*b^3*c^2*d*x^4-16*b^4*c^3*x^4+5*a^4*d^3+60*a^2*b^2*c^2*d-20*a*b^3*c^3)*x*c*((a*d-b*c)/c)^(1/4))/c^2/(d*x^4+c)/(a*d-b*c)^3/a^2

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)^2}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,(d\,x^4+c)^2}\,dx$$

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

3.211 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 353

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}$$
$$-\frac{\sqrt{a}b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd^2\,(a+bx^4)^{3/4}}$$
$$-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2}$$
$$-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2}$$

```
[Out] 1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^(1/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(5/4)
/c/d/(d*x^4+c)-1/4*b^(3/2)*(-a*d+3*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/d^2/(b*x^4+a)
^(3/4)-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(
-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1
/4)/c^2/d^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {424, 542, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = -\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2} - \frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2} - \frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd^2(a+bx^4)^{3/4}} + \frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{4cd^2} - \frac{x(a+bx^4)^{5/4}(bc-ad)}{4cd(c+dx^4)}$$

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4
))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*d^2*(a + b
*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*S
qrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
```

```
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}+\frac{\int \frac{\sqrt[4]{a+bx^4}\left(a(bc+3ad)+2b(3bc-ad)x^4\right)}{c+dx^4}\,dx}{4cd}\\
&= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}\\
&\quad+\frac{\int \frac{-2a(3b^2c^2-2abcd-3a^2d^2)-4b(3b^2c^2-3abcd-a^2d^2)x^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{8cd^2}\\
&= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}\\
&\quad+\frac{(ab(3bc-ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4cd^2}-\frac{(3(bc-ad)(2bc+ad))\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4cd^2}\\
&= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}\\
&\quad+\frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4cd^2\,(a+bx^4)^{3/4}}\\
&\quad-\frac{\left(3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\,(c-(bc-ad)x^4)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4cd^2}
\end{aligned}$$

$$=\frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}-\frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{4cd^2\,(a+bx^4)^{3/4}}-\frac{\left(3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2d^2}-\frac{\left(3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2d^2}$$

$$=\frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}-\frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{8cd^2\,(a+bx^4)^{3/4}}$$

$$=\frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2}-\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd\,(c+dx^4)}-\frac{\sqrt{a}b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\,(a+bx^4)^{3/4}}-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}-\frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.48 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{2b(-3b^2c^2+3abcd+a^2d^2)\,x^5\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c(-5acx(4}{}}{}$$

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1[
5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2 +
 a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b
*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*
c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*
x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appell
F1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(a + b*x^4
)^(3/4))
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{9}{4}}}{(d\,x^4+c)^2}dx$$

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx=\int \frac{(a+bx^4)^{\frac{9}{4}}}{(c+dx^4)^2}\,dx$$

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(9/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx=\int \frac{(b\,x^4+a)^{9/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(9/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(9/4)/(c + d*x^4)^2, x)
```

3.212 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 298

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd(a+bx^4)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(1/4)/c/d/(d*x^4+c)+1/4*b^(3/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(
1/2)/c/d/(b*x^4+a)^(3/4)+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/
2)/b^(1/4)/c^2/d+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {424, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx=\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2d}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd\,(a+bx^4)^{3/4}}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^4)^(1/4))/(c*d*(c + d*x^4)) + (Sqrt[a]*b^(3/2)
*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
4*c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x
^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(
b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)}+\frac{\int\frac{a(bc+3ad)+2b(bc+ad)x^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} - \frac{(ab)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4cd} - \frac{(-2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4cd\,(a+bx^4)^{3/4}} - \frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}(c-(bc-ad)x^4)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4cd\,(a+bx^4)^{3/4}} - \frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2d} - \frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2d}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{8cd\,(a+bx^4)^{3/4}}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\,(c+dx^4)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\,(a+bx^4)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \frac{x\left(2b(bc+ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d-b^2cx^4+abdx^4\right.}{(c+dx^4)\left(-5\right.}\right.}{20}$$

[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]

[Out] (x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*x^4*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*d*(a + b*x^4)^(3/4))

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{(d\,x^4+c)^2}dx$$

[In] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)

[Out] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{5}{4}}}{(c+dx^4)^2}\,dx$$

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4)^2, x)
```

3.213 $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 308

$$\begin{aligned}
&\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} \\
&+ \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)} \\
&+ \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}
\end{aligned}$$

```
[Out] 1/4*x*(b*x^4+a)^(1/4)/c/(d*x^4+c)-1/4*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/
2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2))
)*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/(-a*d+b
*c)/(b*x^4+a)^(3/4)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4)
,-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b
^(1/4)/c^2/(-a*d+b*c)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {423, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$
$$= \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$
$$+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$
$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4c\,(a+bx^4)^{3/4}\,(bc-ad)}+\frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)}$$

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*(b*c - a*d)*(
a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*El
lipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^
4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a +
b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)} - \frac{\int \frac{-3a-2bx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4c}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}+\frac{(ab)\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4c(bc-ad)}+\frac{(2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}+\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}}$$

$$+\frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}(c-(bc-ad)x^4)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}-\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}}$$

$$+\frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)}$$

$$+\frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}+\frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$

$$+\frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$

$$-\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{8c(bc-ad)\left(a+bx^4\right)^{3/4}}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}-\frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}}$$

$$+\frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$

$$+\frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$
$$= \frac{x\left(\frac{2bx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right.}{-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx}{c}\right.\right.}\right.}{c+dx^4}\right.}{20\left(a+bx^4\right)^{3/4}}$$

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
)))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{(d\,x^4+c)^2}dx$$

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{(d\,x^4+c)^2}\,dx$$

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4)^2, x)
```

3.214 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 330

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} \, dx = & -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} \\
& -\frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4\sqrt{a}c(bc-ad)^2(a+bx^4)^{3/4}} \\
& -\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2} \\
& -\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}
\end{aligned}
$$

```
[Out] -1/4*d*x*(b*x^4+a)^(1/4)/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^(3/2)*(-a*d+4*b*c)*(1
+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*
arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2)))
,2^(1/2))/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)/a^(1/2)-3/8*d*(-a*d+2*b*c)*Ellipti
cPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^
4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*c)*Ell
ipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b
*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = -\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2} - \frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4\sqrt{a}c\,(a+bx^4)^{3/4}(bc-ad)^2} - \frac{dx\sqrt[4]{a+bx^4}}{4c\,(c+dx^4)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*(a + b*x^4)^(1/4))/(c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)*(4*b*c
- a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (3*d*(2*b*c - a*d)*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*
d)^2) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[S
qrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1
])/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\int\frac{4bc-3ad-2bdx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$=-\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{(3d(2bc-ad))\int\frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)^2}+\frac{(b(4bc-ad))\int\frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4c(bc-ad)^2}$$

$$=-\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int\frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$
$$-\frac{\left(3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)^2}$$

$$=-\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{4c(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$
$$-\frac{\left(3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)^2}$$
$$-\frac{\left(3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)^2}$$

$$=-\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}$$
$$-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$
$$-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$
$$-\frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{x^2}\right)}{8c(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)} - \frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}c(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$
$$-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$
$$-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}\,dx = \frac{x\left(\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad} + \frac{c\left(25ac\left(-4bc+4ad+bdx^4\right)\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\right.\right.}{(bc-ad)\left(c+dx^4\right)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\right.\right.}\right.}{20c^2\left(\right.}$$

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a
), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^4)*
(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d*x^
4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*
a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*(a + b*x^4)^(3/
4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(d\,x^4+c\right)^2}dx$$

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{4}}(c+dx^4)^2}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{3/4}\,(d\,x^4+c)^2}\,dx$$

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x)
```

3.215 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 390

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx &= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} \\ &- \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} \\ &- \frac{b^{3/2}(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}c(bc-ad)^3(a+bx^4)^{3/4}} \\ &+ \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} \\ &+ \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}\end{aligned}$$

```
[Out] 1/12*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-1/4*d*x/c/(-a*d+b*c
)/(b*x^4+a)^(3/4)/(d*x^4+c)-1/12*b^(3/2)*(3*a^2*d^2-32*a*b*c*d+8*b^2*c^2)*(
1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))
),2^(1/2))/a^(3/2)/c/(-a*d+b*c)^3/(b*x^4+a)^(3/4)+1/8*d^2*(-3*a*d+10*b*c)*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a
/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3+1/8*d^2*(-3*a*d+
10*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/
2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {425, 541, 543, 243, 342, 281, 237, 416, 418, 1232}

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = -\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3a^2d^2-32abcd+8b^2c^2)\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}c\,(a+bx^4)^{3/4}\,(bc-ad)^3}$$
$$+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$
$$+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$
$$+\frac{bx(3ad+4bc)}{12ac\,(a+bx^4)^{3/4}\,(bc-ad)^2}-\frac{dx}{4c\,(a+bx^4)^{3/4}\,(c+dx^4)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (d*x)/(4*c
*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*(8*b^2*c^2 - 32*a*b*
c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*
c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 543

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
```

```
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-6bdx^4}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$=\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}-\frac{\int \frac{-8b^2c^2+24abcd-9a^2d^2-2bd(4bc+3ad)x^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{12ac(bc-ad)^2}$$

$$=\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}+\frac{\left(d^2(10bc-3ad)\right)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)^3}+\frac{\left(b\left(8b^2c^2-32abcd+3a^2d^2\right)\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{12ac(bc-ad)^3}$$

$$=\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}+\frac{\left(b\left(8b^2c^2-32abcd+3a^2d^2\right)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12ac(bc-ad)^3\left(a+bx^4\right)^{3/4}}+\frac{\left(d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)^3}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} - \frac{\left(b(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}dx, x, \frac{1}{x}\right)}{12ac(bc-ad)^3(a+bx^4)^{3/4}} + \frac{\left(d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)^3} + \frac{\left(d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^2(bc-ad)^3}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} - \frac{\left(b(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}dx, x, \frac{1}{x^2}\right)}{24ac(bc-ad)^3(a+bx^4)^{3/4}}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} - \frac{b^{3/2}(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c(bc-ad)^3(a+bx^4)^{3/4}} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.54 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)^2}\,dx = \frac{x\left(2bd(4bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c\left(25ac(12\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c +
d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(4*a*
d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5
/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF
1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4,
3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)
)
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(d\,x^{4}+c\right)^{2}}dx$$

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(a+bx^4\right)^{\frac{7}{4}}\left(c+dx^4\right)^2}\,dx$$

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)^2}\,dx$$

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{7/4}\,\left(d\,x^4+c\right)^2}\,dx$$

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x)
```

3.216 $\int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

```
[Out] 1/4*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+1/4*arctanh(1/2*x*2^(3/4)/(
x^4+1)^(1/4))*2^(1/4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {385, 218, 212, 209}

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

```
[In] Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}$$

```
[In] Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
[Out] (ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(2*2^(3/4))
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{4}}\left(2\arctan\left(\frac{2^{\frac{1}{4}}\left(x^{4}+1\right)^{\frac{1}{4}}}{x}\right)-\ln\left(\frac{2^{\frac{3}{4}}x+2\left(x^{4}+1\right)^{\frac{1}{4}}}{-2^{\frac{3}{4}}x+2\left(x^{4}+1\right)^{\frac{1}{4}}}\right)\right)}{8}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{4}-2\right)\ln\left(\frac{2\sqrt{x^{4}+1}\operatorname{RootOf}\left(\_Z^{4}-2\right)^{3}x^{2}+2\left(x^{4}+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}x^{3}+3\operatorname{RootOf}\left(\_Z^{4}-2\right)x^{4}+4\left(x^{4}+1\right)^{\frac{3}{4}}x+2}{x^{4}+2}\right.}{8}$

[In] int(1/(x^4+1)^(1/4)/(x^4+2),x,method=_RETURNVERBOSE)

[Out] -1/8*2^(1/4)*(2*arctan(1/x*2^(1/4)*(x^4+1)^(1/4))-ln((2^(3/4)*x+2*(x^4+1)^(1/4))/(-2^(3/4)*x+2*(x^4+1)^(1/4))))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 5.09

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx$$
$$= \frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2+8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)+\frac{1}{64}i$$
$$\cdot 8^{\frac{3}{4}}\log\left(-\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8i\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(3i\,x^4+2i)-16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$
$$-\frac{1}{64}i$$
$$\cdot 8^{\frac{3}{4}}\log\left(-\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8i\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(-3i\,x^4-2i)-16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$
$$-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(3x^4+2)+16(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$

[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")

[Out] 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) + 1/64*I*8^(3/4)*log(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*I*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*I*x^4 + 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64*I*8^(3/4)*lo

```
g(-(8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*I*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)
*(-3*I*x^4 - 2*I) - 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64*8^(3/4)*log((8*
sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4
+ 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}\,dx$$

```
[In] integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\int \frac{1}{(x^4+1)^{1/4}\,(x^4+2)}\,dx$$

```
[In] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
[Out] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

3.217 $\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[Out] 1/2*arctan(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)+1/2*arctanh(a^(1/4)*x/(b*x^4+
a)^(1/4))/a^(5/4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 218, 212, 209}

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[In] Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{a-(ab-a(-a+b))x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}+\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[In] Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
[Out] (ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1
/4)])/(2*a^(5/4))
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x}\right)+\ln\left(\frac{-a^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{4a^{\frac{5}{4}}}$	65

```
[In] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
[Out] 1/4*(-2*arctan(1/a^(1/4)/x*(b*x^4+a)^(1/4))+ln((-a^(1/4)*x-(b*x^4+a)^(1/4))
/(a^(1/4)*x-(b*x^4+a)^(1/4))))/a^(5/4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx=-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

```
[In] integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a
 + b*x**4)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx=\int -\frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(a-x^4\,(a-b))}\,dx$$

```
[In] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
[Out] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

3.218 $\int (a+bx^4)^p (c+dx^4)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

```
[Out] x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^4/a,-d*x^4/c)/((1+b*x
^4/a)^p)/((1+d*x^4/c)^q)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} (c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

```
[In] Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((
d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p(c+dx^4)^q\,dx$$
$$=\left((a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int\left(1+\frac{bx^4}{a}\right)^p\left(1+\frac{dx^4}{c}\right)^q\,dx$$
$$=x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int(a+bx^4)^p(c+dx^4)^q\,dx$$
$$=\frac{5acx(a+bx^4)^p(c+dx^4)^q\,\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ac\,\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(bcp\,\text{AppellF1}\left(\frac{5}{4},1-p,-q,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adq\,\text{AppellF}\right.}$$

```
[In] Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a
), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)
/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)^{q}dx$$

```
[In] int((b*x^4+a)^p*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx = \int \left(bx^{4}+a\right)^{p}\left(dx^{4}+c\right)^{q}dx$$

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx = \text{Timed out}$$

```
[In] integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx = \int \left(bx^{4}+a\right)^{p}\left(dx^{4}+c\right)^{q}dx$$

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(b\,x^4+a\right)^p\,\left(d\,x^4+c\right)^q \, dx$$

```
[In] int((a + b*x^4)^p*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)^p*(c + d*x^4)^q, x)
```

3.219 $\int (a+bx^4)^2 (c+dx^4)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^4)^2 (c+dx^4)^q \, dx = -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)}+\frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+16q^2))\,x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4}\right.}{d^2(5+4q)(9+4q)}$$

```
[Out] -b*(5*b*c-a*d*(13+4*q))*x*(d*x^4+c)^(1+q)/d^2/(16*q^2+56*q+45)+b*x*(b*x^4+a
)*(d*x^4+c)^(1+q)/d/(9+4*q)+(5*b^2*c^2-2*a*b*c*d*(9+4*q)+a^2*d^2*(16*q^2+56
*q+45))*x*(d*x^4+c)^q*hypergeom([1/4, -q],[5/4],-d*x^4/c)/d^2/(16*q^2+56*q+
45)/((1+d*x^4/c)^q)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 396, 252, 251}

$$\int (a+bx^4)^2 (c+dx^4)^q \, dx$$
$$= \frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}(a^2d^2(16q^2+56q+45)-2abcd(4q+9)+5b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\right.}{d^2(4q+5)(4q+9)} - \frac{bx(c+dx^4)^{q+1}(5bc-ad(4q+13))}{d^2(4q+5)(4q+9)}+\frac{bx(a+bx^4)(c+dx^4)^{q+1}}{d(4q+9)}$$

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*
q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 -
2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hyperge
ometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^
4)/c)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{bx(a+bx^4)\left(c+dx^4\right)^{1+q}}{d(9+4q)} + \frac{\int \left(c+dx^4\right)^q \left(-a(bc-ad(9+4q))-b(5bc-ad(13+4q))x^4\right)\,dx}{d(9+4q)}$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+16q^2))\int (c+dx^4)^q\,dx}{d^2(5+4q)(9+4q)}$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{\left((5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+16q^2))(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{dx^4}{c}\right)^q dx}{d^2(5+4q)(9+4q)}$$

$$= -\frac{b(5bc-ad(13+4q))x(c+dx^4)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx(a+bx^4)(c+dx^4)^{1+q}}{d(9+4q)} + \frac{(5b^2c^2-2abcd(9+4q)+a^2d^2(45+56q+16q^2))x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q} {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(5+4q)(9+4q)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int (a+bx^4)^2 (c+dx^4)^q\,dx = \frac{1}{45}x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\left(45a^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)+bx^4\left(18a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-q,\frac{9}{4},-\frac{dx^4}{c}\right)+5bx^4\,\text{Hypergeometric2F1}\left(\frac{9}{4},-q,\frac{13}{4},-\frac{dx^4}{c}\right)\right)\right)$$

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)] + b*
x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)] + 5*b*x^4*Hypergeom
etric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*(1 + (d*x^4)/c)^q)
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{2} \left(d\,x^{4}+c\right)^{q}dx$$

[In] int((b*x^4+a)^2*(d*x^4+c)^q,x)

[Out] int((b*x^4+a)^2*(d*x^4+c)^q,x)

Fricas [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q\,dx = \int \left(bx^{4}+a\right)^{2}\left(dx^{4}+c\right)^{q}dx$$

[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="fricas")

[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.69 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q\,dx = \frac{a^{2}c^{q}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},-q\\\frac{5}{4}\end{matrix}\right|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{abc^{q}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4},-q\\\frac{9}{4}\end{matrix}\right|\frac{dx^{4}e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{b^{2}c^{q}x^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{9}{4},-q\\\frac{13}{4}\end{matrix}\right|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

[In] integrate((b*x**4+a)**2*(d*x**4+c)**q,x)

[Out] a**2*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*gamma(5/4)) + a*b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_polar(I*pi)/c)/(2*gamma(9/4)) + b**2*c**q*x**9*gamma(9/4)*hyper((9/4, -q), (13/4,), d*x**4*exp_polar(I*pi)/c)/(4*gamma(13/4))

Maxima [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^2 \left(dx^4+c\right)^q \, dx$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^2 \left(dx^4+c\right)^q \, dx$$

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx = \int \left(b\,x^4+a\right)^2 \left(d\,x^4+c\right)^q \, dx$$

```
[In] int((a + b*x^4)^2*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)^2*(c + d*x^4)^q, x)
```

3.220 $\int (a+bx^4)(c+dx^4)^q \, dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^4)(c+dx^4)^q \, dx = \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \frac{(bc-ad(5+4q))x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right)}{d(5+4q)}$$

```
[Out] b*x*(d*x^4+c)^(1+q)/d/(5+4*q)-(b*c-a*d*(5+4*q))*x*(d*x^4+c)^q*hypergeom([1/
4, -q],[5/4],-d*x^4/c)/d/(5+4*q)/((1+d*x^4/c)^q)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$\int (a+bx^4)(c+dx^4)^q \, dx = x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a - \frac{bc}{4dq+5d}\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right) + \frac{bx(c+dx^4)^{q+1}}{d(4q+5)}$$

```
[In] Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*q))*x*(c +
 d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(1 + (d*x^4)/c)^q
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \left(-a+\frac{bc}{5d+4dq}\right)\int (c+dx^4)^q\,dx \\
&= \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} - \left(\left(-a+\frac{bc}{5d+4dq}\right)(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int\left(1+\frac{dx^4}{c}\right)^q dx \\
&= \frac{bx(c+dx^4)^{1+q}}{d(5+4q)} + \left(a-\frac{bc}{5d+4dq}\right)x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^4)(c+dx^4)^q\,dx$$

$$= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\left(b(c+dx^4)\left(1+\frac{dx^4}{c}\right)^q+(-bc+ad(5+4q))\,\text{Hypergeometric2F1}\left(\frac{1}{4},-q,\frac{5}{4},-\frac{dx^4}{c}\right.\right.}{d(5+4q)}$$

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(b*(c + d*x^4)*(1 + (d*x^4)/c)^q + (-(b*c) + a*d*(5 + 4*q)
)*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]))/(d*(5 + 4*q)*(1 + (d*x^4)
/c)^q)
```

Maple [F]

$$\int \left(b\,x^{4}+a\right) \left(d\,x^{4}+c\right)^{q}\,dx$$

```
[In] int((b*x^4+a)*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)*(d*x^4+c)^q,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx = \int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx = \frac{ac^q x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},-q\\ \frac{5}{4}\end{matrix}\middle|\,\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{bc^q x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},-q\\ \frac{9}{4}\end{matrix}\middle|\,\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

```
[Out] a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*g
amma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\int \left(b\,x^4+a\right)\,\left(d\,x^4+c\right)^q\,dx$$

```
[In] int((a + b*x^4)*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)*(c + d*x^4)^q, x)
```

3.221 $\int \frac{(c+dx^4)^q}{a+bx^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^4)^q}{a+bx^4} \, dx = \frac{x(c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{4},1,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,1,-q,5/4,-b*x^4/a,-d*x^4/c)/a/((1+d*x^4/c)^q)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(c+dx^4)^q}{a+bx^4} \, dx = \frac{x(c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{4},1,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[In] Int[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*
(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int\frac{\left(1+\frac{dx^4}{c}\right)^q}{a+bx^4}\,dx\\ &= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(c+dx^4)^q}{a+bx^4}\,dx$$
$$=\frac{5acx(c+dx^4)^q\,\text{AppellF1}\left(\frac{1}{4},-q,1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-q,1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+4x^4\left(adq\,\text{AppellF1}\left(\frac{5}{4},1-q,1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bc\,\text{App}\right.\right.}$$

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] -
 b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [F]

$$\int\frac{(d\,x^4+c)^q}{b\,x^4+a}dx$$

```
[In] int((d*x^4+c)^q/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**4+c)**q/(b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \int \frac{(d\,x^4+c)^q}{b\,x^4+a}\,dx$$

```
[In] int((c + d*x^4)^q/(a + b*x^4),x)
[Out] int((c + d*x^4)^q/(a + b*x^4), x)
```

3.222 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx = \frac{x(c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,2,-q,5/4,-b*x^4/a,-d*x^4/c)/a^2/((1+d*x^4/c)^q)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx = \frac{x(c+dx^4)^q \left(\frac{dx^4}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[In] Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^
2*(1 + (d*x^4)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int\frac{\left(1+\frac{dx^4}{c}\right)^q}{(a+bx^4)^2}\,dx$$

$$=\frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx$$

$$=\frac{5acx(c+dx^4)^q\,\text{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^2\left(5ac\,\text{AppellF1}\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(adq\,\text{AppellF1}\left(\frac{5}{4},2,1-q,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bc\,\text{A}\right.\right.}$$

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)
])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F]

$$\int\frac{(d\,x^4+c)^q}{(b\,x^4+a)^2}dx$$

```
[In] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="fricas")
[Out] integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="giac")
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \int \frac{(d\,x^4+c)^q}{(b\,x^4+a)^2}\,dx$$

```
[In] int((c + d*x^4)^q/(a + b*x^4)^2,x)
```

```
[Out] int((c + d*x^4)^q/(a + b*x^4)^2, x)
```

3.223 $\int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 545

$$\begin{aligned}\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx &= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\sqrt[5]{bc-ad}x}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ &+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}+\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sqrt[5]{bc-ad}x}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ &-\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ &+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}-\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+2c^{2/5}(a+bx^5)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\ &+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x\sqrt[5]{a+bx^5}+2c^{2/5}(a+bx^5)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}\end{aligned}$$

```
[Out] -1/5*ln(c^(1/5)-(-a*d+b*c)^(1/5)*x/(b*x^5+a)^(1/5))/c^(4/5)/(-a*d+b*c)^(1/5
)+1/20*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5
)+2*c^(2/5)*(b*x^5+a)^(2/5)-c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1
/2))/(b*x^5+a)^(2/5))*(-5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/20*ln((2*(-a*
d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)+2*c^(2/5)*(b*x^
5+a)^(2/5)+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1/2))/(b*x^5+a)^(2
/5))*(5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(1/5*(-a*d+b*c)^(1/5)*
x*(50+10*5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^(1/5)+1/5*(25+10*5^(1/2))^(1/2))*
```

```
(10-2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(-1/5*(25-10*5^(1/
2))^(1/2)+2*(-a*d+b*c)^(1/5)*x*2^(1/2)/(5+5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^
(1/5))*(10+2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {385, 208, 648, 632, 210, 642, 31}

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}+\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} - \frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

```
[In] Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 +
 Sqrt[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))]])/(c^(4/5)*(b*c
 - a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (S
qrt[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))]])
/(5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b
*x^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a
*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(
1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(
a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(
b*c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt
[5]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(
2/5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*
n))*Int[1/(r - s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^5}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)$$

$$=\frac{2\text{Subst}\left(\int\frac{\sqrt[5]{c}+\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}}+\frac{2\text{Subst}\left(\int\frac{\sqrt[5]{c}+\frac{1}{4}\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}x}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt[5]{c}-\sqrt[5]{bc-ad}x}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}}$$

$$=-\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{20c^{3/5}}+\frac{\left(5+\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{20c^{3/5}}+\frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}+2(bc-ad)^{2/5}x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\text{Subst}\left(\int\frac{\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}+2(bc-ad)^{2/5}x}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}dx,x,\frac{x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

$$=-\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}-\frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}+\frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}-\frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{2}\left(5-\sqrt{5}\right)c^{2/5}(bc-ad)^{2/5}-x^2}dx,x,\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}+\frac{2(bc-ad)^{2/5}x}{\sqrt[5]{a+bx^5}}\right)}{10c^{3/5}}-\frac{\left(5+\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{2}\left(5+\sqrt{5}\right)c^{2/5}(bc-ad)^{2/5}-x^2}dx,x,\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}+\frac{2(bc-ad)^{2/5}x}{\sqrt[5]{a+bx^5}}\right)}{10c^{3/5}}$$

$$= \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{2\sqrt{10}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} - \frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}-\frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1+\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}+\frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{5},1,\frac{6}{5},\frac{(bc-ad)x^5}{c(a+bx^5)}\right)}{c\sqrt[5]{a+bx^5}}$$

```
[In] Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
[Out] (x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(a
 + b*x^5)^(1/5))
```

Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{\left(\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}+1\right)\ln\left(\frac{2\left(\frac{ad-bc}{c}\right)^{\frac{2}{5}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{5}}\left(b\,x^{5}+a\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)x+2\left(b\,x^{5}+a\right)^{\frac{2}{5}}}{x^{2}}\right)}{2}-\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}-1\right)\ln\left(2\left(\frac{ad}{}\right.\right.}{}\right.}{}$

```
[In] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/100*(1/2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)*(5^(1/2)+1)*ln((2*((a*d-b*c
)/c)^(2/5)*x^2-((a*d-b*c)/c)^(1/5)*(b*x^5+a)^(1/5)*(5^(1/2)+1)*x+2*(b*x^5+a
)^(2/5))/x^2)-1/2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)*(5^(1/2)-1)*ln((2*((a
*d-b*c)/c)^(2/5)*x^2+((a*d-b*c)/c)^(1/5)*(b*x^5+a)^(1/5)*(5^(1/2)-1)*x+2*(b
*x^5+a)^(2/5))/x^2)+2^(1/2)*(5-5^(1/2))^(1/2)*(5+5^(1/2))*arctan(1/2*(((a*d
-b*c)/c)^(1/5)*(5^(1/2)-1)*x+4*(b*x^5+a)^(1/5))/((a*d-b*c)/c)^(1/5)/(5+5^(1
/2))^(1/2)*2^(1/2)/x)+(5+5^(1/2))^(1/2)*(-2*(5-5^(1/2))^(1/2)*ln((((a*d-b*c
)/c)^(1/5)*x+(b*x^5+a)^(1/5))/x)+arctan(1/2/((a*d-b*c)/c)^(1/5)*2^(1/2)*(((
a*d-b*c)/c)^(1/5)*(5^(1/2)+1)*x-4*(b*x^5+a)^(1/5))/(5-5^(1/2))^(1/2)/x)*2^(
1/2)*(5^(1/2)-5)))*5^(1/2)/((a*d-b*c)/c)^(1/5)/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

Sympy [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

```
[In] integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(b\,x^5+a)^{1/5}\,(d\,x^5+c)}\,dx$$

```
[In] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
[Out] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

3.224 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx = -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3x + \frac{c^2(bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] c^2*(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-7/5*d*(c+d/x)^2*(a+b
/x)^(1/2)-1/15*d*(-4*a^2*d^2+30*a*b*c*d+114*b^2*c^2+b*d*(2*a*d+33*b*c)/x)*(
a+b/x)^(1/2)/b^2+(c+d/x)^3*x*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 99, 158, 152, 65, 214}

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx = -\frac{d\sqrt{a+\frac{b}{x}}\left(2(-2a^2d^2+15abcd+57b^2c^2)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2} + \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)}{\sqrt{a}} + x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 - \frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2$$

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(c+dx)^2\left(\frac{1}{2}(bc+6ad)+\frac{7bdx}{2}\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x\\
&\quad - \frac{2\text{Subst}\left(\int \frac{(c+dx)\left(\frac{5}{4}bc(bc+6ad)+\frac{1}{4}bd(33bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{5b}\\
&= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2}\\
&\quad + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{1}{2}\left(c^2(bc+6ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2}\\
&\quad + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{\left(c^2(bc+6ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2(57b^2c^2+15abcd-2a^2d^2)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2}\\
&\quad + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x + \frac{c^2(bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx
$$
$$
=\frac{\sqrt{a+\frac{b}{x}}(4a^2d^3x^2-2abd^2x(d+15cx)-3b^2(2d^3+10cd^2x+30c^2dx^2-5c^3x^3))}{15b^2x^2}
$$
$$
+\frac{c^2(bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
$$

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 1
0*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\left(15 b^{2} c^{3} x^{3}+4 a^{2} d^{3} x^{2}-30 a b c \,d^{2} x^{2}-90 b^{2} c^{2} d \,x^{2}-2 a \,d^{3} x b -30 c \,d^{2} x \,b^{2}-6 b^{2} d^{3}\right) \sqrt{\frac{a x +b}{x}}}{15 x^{2} b^{2}}+\frac{\left(6 a d +b c \right) c^{2} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}}{2 \sqrt{a}\, \left(a x +b \right)}$
default	$\frac{\sqrt{\frac{a x +b}{x}}\, \left(180 a^{\frac{3}{2}} \sqrt{a \,x^{2}+b x}\, b \,c^{2} d \,x^{4}+30 \sqrt{a}\, \sqrt{a \,x^{2}+b x}\, b^{2} c^{3} x^{4}+90 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a \,b^{2} c^{2} d \,x^{4}+15 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right.}{30 x^{3} \sqrt{x \left(a x +b \right)}\, \sqrt{a}\, b^{2}}$

```
[In] int((c+d/x)^3*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*b^2*c^3*x^3+4*a^2*d^3*x^2-30*a*b*c*d^2*x^2-90*b^2*c^2*d*x^2-2*a*b*
d^3*x-30*b^2*c*d^2*x-6*b^2*d^3)/x^2/b^2*((a*x+b)/x)^(1/2)+1/2*(6*a*d+b*c)*c
^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(
a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.14

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$

$$= \left[\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2b}{30\,ab^2x^2}\right.$$

$$-\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2bcd^2-2\,a}{15\,ab^2x^2}$$

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d
 + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt
((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a)*x^
2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 -
2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a
^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.22

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx = \frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1} - 3c^2d\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + 3cd^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) + \frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[In] integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7
/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(b)*c**3*sqrt(x)*sqrt(
a*x/b + 1) - 3*c**2*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
+ 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 3*c*d**2*Piecewise
((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**3*asinh(
sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3 - 3\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)c^2d - \frac{2}{15}d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^2}\right) - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}cd^2}{b}$$

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/b^2
 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx = \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right)+\sqrt{a+\frac{b}{x}}\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)+c^3\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{5/2}}{5\,b^2}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,(6\,a\,d+b\,c)\,1\mathrm{i}}{\sqrt{a}}$$

[In] int((a + b/x)^(1/2)*(c + d/x)^3,x)

[Out] (a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a + b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)^(5/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i)/a^(1/2)

3.225 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx=-\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}+\frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2/3*d^2*(a+b/x)^(3/2)/b+c^2*(a+b/x)^(3/2)*x/a+c*(4*a*d+b*c)*arctanh((a+b/x
)^(1/2)/a^(1/2))/a^(1/2)-c*(4*a*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 91, 81, 52, 65, 214}

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx=\frac{c\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+bc)}{\sqrt{a}}+\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}(4ad+bc)}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}c(bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(c(bc+4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}(-2ad^2x+b(-2d^2-12cdx+3c^2x^2))}{3bx} + \frac{c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\left(-3b\,c^{2}x^{2}+2xa\,d^{2}+12bcdx+2b\,d^{2}\right)\sqrt{\frac{ax+b}{x}}}{3xb}+\frac{(4ad+bc)c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(24a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,cd\,x^{3}+6\sqrt{a}\,\sqrt{a\,x^{2}+bx}\,b\,c^{2}x^{3}+12\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abcd\,x^{3}+3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}c^{2}x^{3}-\right.}{6x^{2}\sqrt{x(ax+b)}\,\sqrt{a}\,b}$

[In] int((c+d/x)^2*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(-3*b*c^2*x^2+2*a*d^2*x+12*b*c*d*x+2*b*d^2)/x/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+b*c)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.10

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^{2}\,dx$$
$$=\left[\frac{3\,(b^{2}c^{2}+4\,abcd)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,abc^{2}x^{2}-2\,abd^{2}-2\,(6\,abcd+a^{2}d^{2})x)\sqrt{\frac{ax+b}{x}}}{6\,abx},\right.$$
$$\left.-\frac{3\,(b^{2}c^{2}+4\,abcd)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,abc^{2}x^{2}-2\,abd^{2}-2\,(6\,abcd+a^{2}d^{2})x)\sqrt{\frac{ax+b}{x}}}{3\,abx}\right]$$

[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]

Sympy [A] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1} - 2cd\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) + \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[In] integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*c*d*Piecewise((2*a*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True
)) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), Tru
e)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2 - 2\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)cd - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3b}$$

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx = \left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\sqrt{a+\frac{b}{x}}+c^2\,x\,\sqrt{a+\frac{b}{x}} - \frac{2\,d^2\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b}-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+b\,c\right)\,1\mathrm{i}}{\sqrt{a}}$$

```
[In] int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

```
[Out] ((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1/
2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*
(4*a*d + b*c)*1i)/a^(1/2)
```

3.226 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=-\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] c*(a+b/x)^(3/2)*x/a+(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-(2*a
*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 52, 65, 214}

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc)}{\sqrt{a}}-\frac{\sqrt{a+\frac{b}{x}}(2ad+bc)}{a}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

```
[In] Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\left(\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \sqrt{a+\frac{b}{x}}(-2d+cx) + \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x),x]
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/Sqrt[a]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

method	result
risch	$(cx-2d)\sqrt{\frac{ax+b}{x}}+\frac{\left(\frac{bc}{2}+ad\right)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(4a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,d\,x^{2}+2\sqrt{a}\,\sqrt{a\,x^{2}+bx}\,bc\,x^{2}+2\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd\,x^{2}+\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}c\,x^{2}-4\sqrt{a}\,(a\,x^{2}}{2x\sqrt{x(ax+b)}\,b\sqrt{a}}$

```
[In] int((c+d/x)*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (c*x-2*d)*((a*x+b)/x)^(1/2)+(1/2*b*c+a*d)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x
)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \left[\frac{(bc+2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a}, -\frac{(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{a}\right]$$

[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]

Sympy [A] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-d\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

[In] integrate((c+d/x)*(a+b/x)**(1/2),x)

[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c - \left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\sqrt{a+\frac{b}{x}}\right)d$$

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a))) + 2*sqrt(a + b/x))*d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = 2\sqrt{a}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - 2\,d\sqrt{a+\frac{b}{x}} + c\,x\sqrt{a\,x^2+b\,x}\sqrt{\frac{1}{x^2}} + \frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

```
[In] int((a + b/x)^(1/2)*(c + d/x),x)
```

```
[Out] 2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*x
 + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*x^
2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.227 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {248, 43, 65, 214}

$$\int \sqrt{a+\frac{b}{x}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}$$

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}x - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}x - \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= \sqrt{a+\frac{b}{x}}x + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{a+\frac{b}{x}}x + \frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[a + b/x],x]
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

method	result	size
risch	$x\sqrt{\frac{ax+b}{x}}+\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$	72
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\,\sqrt{a}}$	74

```
[In] int((a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*((a*x+b)/x)^(1/2)+1/2*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)
*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int\sqrt{a+\frac{b}{x}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a},\ \frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[In] integrate((a+b/x)**(1/2),x)
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
[Out] sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt
(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \sqrt{a+\frac{b}{x}}\,dx = -\frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\mathrm{sgn}(x)}{2\sqrt{a}} + \frac{b\log\left(|b|\right)\mathrm{sgn}(x)}{2\sqrt{a}}+\sqrt{ax^2+bx}\,\mathrm{sgn}(x)$$

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
[Out] -1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/sqrt(
a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \sqrt{a+\frac{b}{x}}\,dx = x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

```
[In] int((a + b/x)^(1/2),x)
```

```
[Out] x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.228 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x}{c}+\frac{2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

```
[Out] (-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^2/a^(1/2)+2*arctan(d^(1/2)*(a
+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)*(-a*d+b*c)^(1/2)/c^2+x*(a+b/x)^(1/2)/
c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 101, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{\sqrt{a}c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

```
[In] Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-\frac{bdx}{2}}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2d(bc-ad))\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} + \frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \frac{c\sqrt{a+\frac{b}{x}}x + 2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a
])/c^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(86) = 172$.

Time = 0.22 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.23

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c}-\frac{\left(\frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2(ad-bc)d\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}}{2c(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{3}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^2-2\sqrt{x(ax+b)}\,c^2\sqrt{a}\sqrt{\frac{(ad-bc)d}{c^2}}-2\sqrt{a}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2a}{cx+d}\right.\right.}{2\sqrt{x(ax+b)}\,c^3\sqrt{a}\sqrt{\frac{(ad-b}{c^2}}}$

```
[In] int((a+b/x)^(1/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*x*((a*x+b)/x)^(1/2)-1/2/c*((2*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+
b*x)^(1/2))/a^(1/2)+2*(a*d-b*c)*d/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*
c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*
d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)^(1/2)*(x*(a*
x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.63

$$\int\frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,\sqrt{-bcd+ad^2}a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+}}{cx+d}\right.}{2\,ac^2}\right.$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*
a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), 1
/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d -
 a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*
```

```
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d
+ a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d
^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c
- 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

```
[In] integrate((a+b/x)**(1/2)/(c+d/x),x)
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \frac{x\sqrt{a+\frac{b}{x}}}{c}+\frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\,\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2}-\frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)\,\left(2\,a\,d-b\,c\right)}{2\,\sqrt{a}\,c^2}-\frac{\mathrm{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\,\sqrt{a\,d^2-b\,c\,d}\,2\mathrm{i}}{c^2}$$

```
[In] int((a + b/x)^(1/2)/(c + d/x),x)
```

```
[Out] (x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2
)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((a
+ b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^(1/
2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

3.229 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx = \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

```
[Out] (-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^3/a^(1/2)+(-4*a*d+3*b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/c^3/(-a*d+b*c)^(1/2)+2*d
*(a+b/x)^(1/2)/c^2/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 101, 156, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx = \frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}c^3} + \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-4ad)-\frac{3bdx}{2}}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-4ad)(bc-ad)+bd(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc-4ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc-4ad))\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(2d+cx)}{d+cx}+\frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(127) = 254.

Time = 0.24 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.24

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^2}-\frac{\left(\frac{(4ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2d^2(ad-bc)\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)}-\frac{(2ad-bc)c\ln\left(\frac{2(ad-bc)d}{c^2}-\frac{(2a}{}\right.}{}\right.}{c^3}\right.}{}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}}{cx+d}\right.\right.}{}$

```
[In] int((a+b/x)^(1/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*((4*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))/a^(1/2)+2*d^2*(a*d-b*c)/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*
(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a
*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/
c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d
/c^2)^(1/2))/(x+d/c)))+2/c^2*d*(3*a*d-2*b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln((2*
```

```
(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^
2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)^(1/2)
*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.45

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \left[-\frac{(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(3\,abcd-4\,a^2d^2+(3\,abc^2-4\,a^2cd}{2\,(ac^4x+ac^3d)}\right.$$

$$-\frac{2\,(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(3\,abcd-4\,a^2d^2+(3\,abc^2-4\,a^2cd)x)\sqrt{-}}{2\,(ac^4x+ac^3d)}$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*
d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt(
(a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x
)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2
 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b*c*d -
 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c -
 a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*
x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d),
 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sq
rt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))
/(a*c^4*x + a*c^3*d)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(127) = 254.

Time = 0.34 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$
$$=\frac{\sqrt{ax^2+bx}\operatorname{sgn}(x)}{c^2}-\frac{\left(3\,bcd\operatorname{sgn}(x)-4\,ad^2\operatorname{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3}$$
$$-\frac{\left(bc\operatorname{sgn}(x)-4\,ad\operatorname{sgn}(x)\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\sqrt{a}c^3}$$
$$-\frac{\left(6\sqrt{a}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-8\,a^{\frac{3}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-\sqrt{bcd-ad^2}\,bc\log\left(|b|\right)+4\sqrt{bcd-ad^2}\,ad\log\left(|\right.\right.}{2\sqrt{bcd-ad^2}\sqrt{a}c^3}$$
$$-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd\operatorname{sgn}(x)-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^2\operatorname{sgn}(x)-\sqrt{a}bd^2\operatorname{sgn}(x)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d+bd\right)c^3}$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*sgn(x)/c^2 - (3*b*c*d*sgn(x) - 4*a*d^2*sgn(x))*arctan(-((
sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/(sqrt(b*
c*d - a*d^2)*c^3) - 1/2*(b*c*sgn(x) - 4*a*d*sgn(x))*log(abs(2*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^3) - 1/2*(6*sqrt(a)*b*c*d*arcta
n(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(3/2)*d^2*arctan(sqrt(a)*d/sqrt(b*c*
d - a*d^2)) - sqrt(b*c*d - a*d^2)*b*c*log(abs(b)) + 4*sqrt(b*c*d - a*d^2)*a
*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqr
t(a)*c^3) - ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d*sgn(x) - 2*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*a*d^2*sgn(x) - sqrt(a)*b*d^2*sgn(x))/(((sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)*c^3
)
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 1195, normalized size of antiderivative = 8.13

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b/x)^(1/2)/(c + d/x)^2,x)
```

```
[Out] - ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((a
 + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^3*
(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a +
b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(a^(1
/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^
5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^
6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d
- b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/
2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 -
a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*
b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3)
)/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c)
)^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*
a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*
d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/c^6 - ((d*(a*d
- b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^
3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7
*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*
c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c
^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(
1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))
/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*
a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*
```

```
(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^
3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d))))*(d*(a*d - b*c))^(1/2)*(4*a*
d - 3*b*c)*1i)/(b*c^4 - a*c^3*d)
```

3.230 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} dx = \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{(bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

```
[Out] (-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^4/a^(1/2)+1/4*(24*a^2*d^2-40*
a*b*c*d+15*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/
c^4/(-a*d+b*c)^(3/2)+3/2*d*(a+b/x)^(1/2)/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11*b*
c)*(a+b/x)^(1/2)/c^3/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {382, 101, 156, 162, 65, 214, 211}

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\sqrt{d}(24a^2d^2-40abcd+15b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)}{\sqrt{a}c^4} + \frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-6ad)-\frac{5bdx}{2}}{x\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-((bc-6ad)(bc-ad))+\frac{9}{2}bd(bc-ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}\\
&= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2}\\
&\quad - \frac{\text{Subst}\left(\int \frac{(bc-6ad)(bc-ad)^2-\frac{1}{4}bd(11bc-12ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)^2}
\end{aligned}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^4} + \frac{\left(d(15b^2c^2-40abcd+24a^2d^2)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{8c^4(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^4} + \frac{\left(d(15b^2c^2-40abcd+24a^2d^2)\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4bc^4(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

$$= \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-2ad(6d^2+9cdx+2c^2x^2)+bc(11d^2+17cdx+4c^2x^2))}{(bc-ad)(d+cx)^2} + \frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{4(bc-6ad)\text{arctanh}\left(\right.}{\sqrt{a}}}{4c^4}$$

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-2*a*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + b*c*(11*d^2 + 1
7*c*d*x + 4*c^2*x^2)))/((b*c - a*d)*(d + c*x)^2) + (Sqrt[d]*(15*b^2*c^2 - 4
0*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b
*c - a*d)^(3/2) + (4*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])
/(4*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(185) = 370$.

Time = 0.26 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.55

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^3} - \frac{\left(\frac{(6ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2d^2(4ad-3bc)\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}\cdots}{\cdots}\right)}{\cdots}\right)}{c^3}\cdots\right)}{\cdots}$
default	Expression too large to display

[In] int((a+b/x)^(1/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)

```
[Out] 1/c^3*x*((a*x+b)/x)^(1/2)-1/2/c^3*((6*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))/a^(1/2)+2/c^3*d^2*(4*a*d-3*b*c)*(-1/(a*d-b*c)/d*c^2/(x+d/c)
*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*
c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(
x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*
c)*d/c^2)^(1/2))/(x+d/c)))-2*d^3*(a*d-b*c)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c
)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*
c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+
d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)
^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2
)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2*
a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)
/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*
d-b*c)*d/c^2)^(1/2))/(x+d/c)))+6/c^2*d*(2*a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)*
```

```
ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x
+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)
^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(185) = 370$.

Time = 0.41 (sec) , antiderivative size = 1749, normalized size of antiderivative = 8.21

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
 (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
 - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2
)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
 - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
```

```
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{x^3\sqrt{a+\frac{b}{x}}}{\left(cx+d\right)^3}\,dx$$

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
[Out] Integral(x**3*sqrt(a + b/x)/(c*x + d)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(185) = 370$.

Time = 0.34 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx =$$

$$-\frac{\left(15\sqrt{a}b^2c^2d\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-40\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+24\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-2\sqrt{bcd-\right.}{4\left(\sqrt{bcd-\right.}$$

$$-\frac{(15\,b^2c^2d\text{sgn}(x)-40\,abcd^2\text{sgn}(x)+24\,a^2d^3\text{sgn}(x))\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\,(bc^5-ac^4d)\sqrt{bcd-ad^2}}$$

$$+\frac{\sqrt{ax^2+bx}\text{sgn}(x)}{c^3}$$

$$-\frac{9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3d\text{sgn}(x)-32\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d^2\text{sgn}(x)+24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^2}{}$$

$$-\frac{(bc\text{sgn}(x)-6\,ad\text{sgn}(x))\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\sqrt{a}c^4}$$

[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")

[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) + 14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d*sgn(x) - 32*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^4*sgn(x) + 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^4*sgn(x) - 9*sqrt(a)*b^3*c*d^3*sgn(x) + 10*a^(3/2)*b^2*d^4*sgn(x))/((b*c^5 - a*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 1895, normalized size of antiderivative = 8.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

```
[Out] (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c*
d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3*c
^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2)*(
12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*a*d^
3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2 + 24*
a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2 - 2*b
*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1
/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a^2*d^2 +
15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a
*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^
3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^
2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*
a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c
^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 -
256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2
)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*
c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4
*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*
(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^
2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2
 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(
1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d -
 b*c)*1i)/(2*a^(1/2)*c^4))/((216*a^4*b^3*d^7 + (165*b^7*c^4*d^3)/8 - (805*a
*b^6*c^3*d^4)/4 - 594*a^3*b^4*c*d^6 + 558*a^2*b^5*c^2*d^5)/(b^2*c^11 + a^2*
c^9*d^2 - 2*a*b*c^10*d) - ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
- b*c))/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
```

```
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
 - b*c))/(2*a^(1/2)*c^4)))*(6*a*d - b*c)*1i)/(a^(1/2)*c^4)
```

3.231 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx = -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2} \left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2} + \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 x + 3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -9/7*d*(a+b/x)^(3/2)*(c+d/x)^2-1/35*d*(a+b/x)^(3/2)*(2*(-a*d+13*b*c)*(2*a*d
+5*b*c)+3*b*d*(2*a*d+19*b*c)/x)/b^2+(a+b/x)^(3/2)*(c+d/x)^3*x+3*c^2*(2*a*d+
b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*c^2*(2*a*d+b*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 99, 158, 152, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx = 3\sqrt{a}c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc) - \frac{d\left(a+\frac{b}{x}\right)^{3/2} \left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2} - 3c^2\sqrt{a+\frac{b}{x}}(2ad+bc) + x\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2$$

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x-\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^2\left(\frac{3}{2}(bc+2ad)+\frac{9bdx}{2}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2+\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x\\
&\quad -\frac{2\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)\left(\frac{21}{4}bc(bc+2ad)+\frac{3}{4}bd(19bc+2ad)x\right)}{x}\,dx, x, \frac{1}{x}\right)}{7b}\\
&= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}\\
&\quad +\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x-\frac{1}{2}\left(3c^2(bc+2ad)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}\\
&\quad -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}\\
&\quad +\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x-\frac{1}{2}\left(3ac^2(bc+2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}$$
$$-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3x-\frac{(3ac^2(bc+2ad))\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{b}$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}$$
$$-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3x+3\sqrt{a}c^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.97

$$\int\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3dx=\frac{\sqrt{a+\frac{b}{x}}(4a^3d^3x^3-2a^2bd^2x^2(d+21cx)+ab^2x(-16d^3-84cd^2x-280c^2dx^2+35c^3x^3)-2b^3(5d^3}{35b^2x^3}$$
$$+3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-16
*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2*x
 + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(35a\,b^2c^3x^4+4x^3a^3d^3-42x^3a^2bc\,d^2-280x^3a\,b^2c^2d-70x^3b^3c^3-2x^2a^2b\,d^3-84x^2a\,b^2c\,d^2-70x^2b^3c^2d-16xa\,b^2d^3-42x\,b^3c\,d^2-10b^3d^3}{35x^3b^2}$
default	$\sqrt{\frac{ax+b}{x}}\,\Big(420a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b\,c^2d\,x^5+210a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b^2c^3x^5+210\ln\Big(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\Big)a^2b^2c^2d\,x^5+105\ln\Big(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax}{2\sqrt{a}}$

```
[In] int((a+b/x)^(3/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(35*a*b^2*c^3*x^4+4*a^3*d^3*x^3-42*a^2*b*c*d^2*x^3-280*a*b^2*c^2*d*x^3
-70*b^3*c^3*x^3-2*a^2*b*d^3*x^2-84*a*b^2*c*d^2*x^2-70*b^3*c^2*d*x^2-16*a*b^
2*d^3*x-42*b^3*c*d^2*x-10*b^3*d^3)/x^3/b^2*((a*x+b)/x)^(1/2)+3/2*(2*a*d+b*c
)*a^(1/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \left(a+\frac{b}{x}\right)^{3/2}\bigg(c+\frac{d}{x}\bigg)^3\,dx=\left[\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3c}{} \right.$$

$$-\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3c^3+140\,ab^2c^2d+21\,a^2}{35\,b^2x^3}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 1
40*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*a*b
^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b
)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c
^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d +
```

```
42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a
*x + b)/x))/(b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 33.36 (sec) , antiderivative size = 1828, normalized size of antiderivative = 11.15

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(21/2
)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*
a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(23/2
)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a
**(7/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**6*d**3*x**(1
1/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(
9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
```

```
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**7*b*d**3*x**(7/2)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**2*c*d
**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a*
*6*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 12*a**5*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) + a*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 3*a*c**2*d*Piecew
ise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0))
, (-sqrt(a)*log(x), True)) + 3*a*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) - b*c**3*Piecewise((2*a*atan(sqrt(a + b/x
)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True))
 + 3*b*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx &= -\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b}\\ &+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax-3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}b\right)c^3\\ &-\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}a\right)c^2d\\ &-\frac{2}{35}\left(\frac{5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^2}\right)d^3\end{aligned}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((
sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2
*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b^2 -
 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.99

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 dx = \left(a+\frac{b}{x}\right)^{5/2} \left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right) + \sqrt{a+\frac{b}{x}} \left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\left(\frac{6\,a\,d^3-6\,b\,c}{b^2}\right.\right. + \left(a+\frac{b}{x}\right)^{3/2} \left(\frac{2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}-\frac{2\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{3\,b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{7/2}}{7\,b^2}+a\,c^3\,x\,\sqrt{a+\frac{b}{x}}-2\,c^2\,a$$

```
[In] int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (
4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d^3
- 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 - 6*b*
c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/(3*b
^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*c^2*at
an((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2*d + 3*
a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

3.232 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx = -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -1/3*c*(4*a*d+3*b*c)*(a+b/x)^(3/2)/a-2/5*d^2*(a+b/x)^(5/2)/b+c^2*(a+b/x)^(5
/2)*x/a+c*(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-c*(4*a*d+3*b
*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 91, 81, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx = \sqrt{a}\text{carctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+3bc) + \frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a} - \frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a} - c\sqrt{a+\frac{b}{x}}(4ad+3bc) - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(c(3bc+4ad))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}\\
&\quad - \frac{1}{2}(c(3bc+4ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}\\
&\quad + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(ac(3bc+4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}\\
&\quad + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(ac(3bc+4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}\\
&\quad + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}c(3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}\left(-6a^2d^2x^2+abx(-12d^2-80cdx+15c^2x^2)-2b^2(3d^2+10cdx+15c^2x^2)\right)}{15bx^2} + \sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-6*a^2*d^2*x^2 + a*b*x*(-12*d^2 - 80*c*d*x + 15*c^2*x^2) -
2*b^2*(3*d^2 + 10*c*d*x + 15*c^2*x^2)))/(15*b*x^2) + Sqrt[a]*c*(3*b*c + 4*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\left(-15ab\,c^2x^3+6a^2d^2x^2+80abcd\,x^2+30b^2c^2x^2+12xab\,d^2+20x\,b^2cd+6b^2d^2\right)\sqrt{\frac{ax+b}{x}}}{15x^2b}+\frac{(4ad+3bc)\sqrt{a}\,c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax}{}}}{2ax+2b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(120a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,cd\,x^4+90a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b\,c^2x^4-120a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}cd\,x^2+60\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bcd\,x^4+45\ln\left(\frac{2\sqrt{}}{}\right.\right.}{30x^3b\sqrt{x(ax}}$

```
[In] int((a+b/x)^(3/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-15*a*b*c^2*x^3+6*a^2*d^2*x^2+80*a*b*c*d*x^2+30*b^2*c^2*x^2+12*a*b*d
^2*x+20*b^2*c*d*x+6*b^2*d^2)/x^2/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+3*b*c)*a^(1
/2)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b
))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.13

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx=\left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+4}{30\,bx^2}\right.$$

$$-\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^2-}{15\,bx^2}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*
```

```
c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a
*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d +
3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.33

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx = \frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \sqrt{a}bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1} - 2acd\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) - bc^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + 2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(9
/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqr
t(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) + a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*c*d*
```

```
Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(
b, 0)), (-sqrt(a)*log(x), True)) + a*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)),
 (-2*(a + b/x)**(3/2)/(3*b), True)) - b*c**2*Piecewise((2*a*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True
)) + 2*b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax-3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}b\right)c^2 - \frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}a\right)cd$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sq
rt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^2
- 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx = \sqrt{a+\frac{b}{x}} \left(2\,a \left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right) - \frac{2\,(a\,d-b\,c)^2}{b}+\frac{2\,a^2\,d^2}{b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{3\,b}-\frac{4\,a\,d^2}{3\,b}\right) \left(a+\frac{b}{x}\right)^{3/2}-\frac{2\,d^2\,\left(a+\frac{b}{x}\right)^{5/2}}{5\,b} + a\,c^2\,x\,\sqrt{a+\frac{b}{x}}-2\,c\,\mathrm{atan}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\,(4\,a\,d+3\,b\,c)\,\sqrt{-\frac{a}{4}}}{4\,d\,a^2\,c+3\,b\,a\,c^2}\right)\,(4\,a\,d+3\,b\,c)\,\sqrt{-\frac{a}{4}}$$

```
[In] int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a +
 b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) - 2*c
*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4*a^2
*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

3.233 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \, dx = -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2} x}{a} + \sqrt{a}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -1/3*(2*a*d+3*b*c)*(a+b/x)^(3/2)/a+c*(a+b/x)^(5/2)*x/a+(2*a*d+3*b*c)*arctan
h((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-(2*a*d+3*b*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) \, dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+3bc) - \frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a} - \sqrt{a+\frac{b}{x}}(2ad+3bc) + \frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

```
[In] Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\left(\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(3bc+2ad)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(a(3bc+2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(a(3bc+2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = \frac{\sqrt{a+\frac{b}{x}}(ax(-8d+3cx)-2b(d+3cx))}{3x} + \sqrt{a}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x),x]
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*b
*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(3ac\,x^2-8adx-6bcx-2bd)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{(2ad+3bc)\sqrt{a}\,\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2ax+2b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(12a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,d\,x^3+18a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,bc\,x^3-12a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}dx+6\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2bd\,x^3+9\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\cdots}{2\sqrt{a}}\right.\cdots}{6x^2\sqrt{x(ax+b)}\,\sqrt{a}\,b}$

[In] int((a+b/x)^(3/2)*(c+d/x),x,method=_RETURNVERBOSE)

[Out] 1/3*(3*a*c*x^2-8*a*d*x-6*b*c*x-2*b*d)/x*((a*x+b)/x)^(1/2)+1/2*(2*a*d+3*b*c)*a^(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.64

$$\int\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx=\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{6\,x}, -\frac{3\,(3\,bc+2\,ad)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")

[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x, -1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]

Sympy [A] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\sqrt{a}bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-ad\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)-bc\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)+bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b
 + 1) - a*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a
 + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) - b*c*Piecewise((2*a*atan(sqrt
(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x)
, True)) + b*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax-3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}b\right)c-\frac{1}{3}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}a\right)d$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a +
b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(a +
 b/x)*a)*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = 2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3} - 2\,a\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

```
[In] int((a + b/x)^(3/2)*(c + d/x),x)
```

```
[Out] 2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a*
d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a
*x)/b))/((a*x)/b + 1)^(3/2)
```

3.234 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2} x+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] (a+b/x)^(3/2)*x+3*b*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*b*(a+b/x)^(1/2
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {248, 43, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = 3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}$$

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - (3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x + 3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}(-2b+ax)+3\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

[In] Integrate[(a + b/x)^(3/2),x]

[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

method	result	size
risch	$(ax-2b)\sqrt{\frac{ax+b}{x}}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	78
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(6a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,x^{2}+3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2x\sqrt{x(ax+b)}\,\sqrt{a}}$	100

[In] int((a+b/x)^(3/2),x,method=_RETURNVERBOSE)

[Out] (a*x-2*b)*((a*x+b)/x)^(1/2)+3/2*a^(1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x
)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}, -3\,\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")

[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 1.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = 3\sqrt{a}b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + \frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}} - \frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}ax - \frac{3}{2}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}b$$

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b
/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -\frac{2x\left(a+\frac{b}{x}\right)^{3/2} {}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{ax}{b}\right)}{\left(\frac{ax}{b}+1\right)^{3/2}}$$

```
[In] int((a + b/x)^(3/2),x)
```

```
[Out] -(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1)
^(3/2)
```

3.235 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\frac{a\sqrt{a+\frac{b}{x}}\,x}{c}-\frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

```
[Out] (-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^2-2*(-a*d+b*c)^(3/2
)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(1/2)+a*x*(a+b/x)^(1
/2)/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 100, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=-\frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-2ad)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

```
[In] Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-2ad)-\frac{1}{2}b(2bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \frac{ac\sqrt{a+\frac{b}{x}}x - \frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} - \sqrt{a}(-3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x),x]
[Out] (a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/Sqrt[d] - Sqrt[a]*(-3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/c^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(88) = 176.

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.33

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c}-\frac{\left(\frac{\sqrt{a}\,(2ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c}-\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2c(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^3-2a^{\frac{3}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d-4a^{\frac{3}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2}{cx+d}\right.\right.}{}$

```
[In] int((a+b/x)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*x*a*((a*x+b)/x)^(1/2)-1/2/c*(a^(1/2)*(2*a*d-3*b*c)/c*ln((1/2*b+a*x)/a^(
1/2)+(a*x^2+b*x)^(1/2))-(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)/c^2/((a*d-b*c)*d/c
^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(
1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*(
(a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.90

$$\int\frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log}{2\,c^2}\right.$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d
*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d
)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-
(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2,
1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-
d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt
```

```
(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqrt((a*x +
 b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/c^2]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

```
[In] integrate((a+b/x)**(3/2)/(c+d/x),x)
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c+\frac{d}{x}}\,dx$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.25

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c}$$

$$-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}+\frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d}+\frac{12\,a^{7/2}\,b^4\,a}{12\,a^4\,b^4\,d^4-46\,a^3\,b^5\,c\,d^3+5}\right.}{c^2}$$

$$+\frac{2\,\mathrm{atanh}\left(\frac{12\,a^2\,b^4\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{12\,a^4\,b^4\,d^4-40\,a^3\,b^5\,c\,d^3+44\,a^2\,b^6\,c^2\,d^2-16\,a\,b^7\,c^3\,d}+\frac{16\,a\,b^5\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{40\,a^3\,b^5\,d^3-44\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+16\,a\,b^7\,c^2\,d}\right)\sqrt{d\,(a\,d-}{c^2\,d}$$

```
[In] int((a + b/x)^(3/2)/(c + d/x),x)
```

```
[Out] (a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/2
))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c
^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*c*d
^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/x)^(
1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b^7*c^
3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d
- (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 + (2*ata
nh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2*b^6*c^2*
d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2*b^6*c*d^2
- (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2))/(c^2*d)
```

3.236 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} dx = -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

```
[Out] (-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^3-(-4*a*d+b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^3/d^(1/2)-(-2
*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 100, 156, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} dx = -\frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-4ad)-\frac{1}{2}b(2bc-3ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc-4ad)(bc-ad)+\frac{1}{2}b(bc-2ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}\\
&= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3}\\
&\quad - \frac{((bc-4ad)(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3}\\
&= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3}\\
&\quad - \frac{((bc-4ad)(bc-ad))\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3}\\
&= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}\\
&\quad + \frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-bc+2ad+acx)}{d+cx} - \frac{(b^2c^2-5abcd+4a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} - \sqrt{a}(-3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right.}{c^3}$$

[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]

[Out] ((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*b *c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d] *Sqrt[b*c - a*d]) - Sqrt[a]*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(136) = 272.

Time = 0.23 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.22

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{\sqrt{a}\,(4ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c} - \frac{\left(-6a^2d^2+8abcd-2b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{}$
default	$-\frac{\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+b}{cx+d}\right.\right.}{}$

[In] int((a+b/x)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)

[Out] 1/c^2*x*a*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(1/2)*(4*a*d-3*b*c)/c*ln((1/2*b+a*x) /a^(1/2)+(a*x^2+b*x)^(1/2))-(-6*a^2*d^2+8*a*b*c*d-2*b^2*c^2)/c^2/((a*d-b*c) *d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^ 2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c

```
)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c
)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*(
(a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^
(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 769, normalized size of antiderivative = 4.93

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \left[-\frac{(3\,bcd-4\,ad^2+(3\,bc^2-4\,acd)x)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(bcd-4\,ad^2+\cdots}{\cdots}\right.$$

$$-\frac{2\,(3\,bcd-4\,ad^2+(3\,bc^2-4\,acd)x)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(bcd-4\,ad^2+(bc^2-4\,acd)x)\sqrt{-\frac{bc-ad}{d}}\log\cdots}{2\,(c^4x+c^3d)}$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a
*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*
sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt
((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d - 4*a*d^2 +
 (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c
^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d - 4*a*d^2 + (b
*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((
a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*
x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(cx+d\right)^2}\,dx$$

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
[Out] Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(136) = 272.

Time = 0.34 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\sqrt{ax^2+bx}a\mathrm{sgn}(x)}{c^2}$$
$$+\frac{\left(b^2c^2\mathrm{sgn}(x)-5\,abcd\mathrm{sgn}(x)+4\,a^2d^2\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}c^3}$$
$$-\frac{\left(3\,abc\mathrm{sgn}(x)-4\,a^2d\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^3}$$
$$+\frac{\left(2\sqrt{a}b^2c^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-10\,a^{\frac{3}{2}}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+8\,a^{\frac{5}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+3\sqrt{bcd-ad^2}abc\,\mathrm{lo}\cdots}{2\sqrt{bcd-ad^2}\sqrt{a}c^3}$$
$$+\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)b^2c^2\mathrm{sgn}(x)-3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)abcd\mathrm{sgn}(x)+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^2d^2\mathrm{sgn}(x)-\cdots}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d+bd\right)c^3}$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^2 + (b^2*c^2*sgn(x) - 5*a*b*c*d*sgn(x) + 4*a^2
*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b
```

```
*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3) - 1/2*(3*a*b*c*sgn(x) - 4*a^2*d*sg
n(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^3
) + 1/2*(2*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 10*a^(3/
2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 8*a^(5/2)*d^2*arctan(sqrt(
a)*d/sqrt(b*c*d - a*d^2)) + 3*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 4*sqr
t(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*b*c - 2*sqrt(b
*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3) + ((sqrt(a)*x
 - sqrt(a*x^2 + b*x))*b^2*c^2*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*
b*c*d*sgn(x) + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*d^2*sgn(x) - sqrt(a)*b
^2*c*d*sgn(x) + a^(3/2)*b*d^2*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c
 + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)*c^3)
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\operatorname{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,(4\,a\,d-b\,c}{c^3\,d}$$
$$-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{6\,\sqrt{a}\,b^7\,d\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,d-\frac{14\,a^2\,b^6\,d^2}{c}+\frac{8\,a^3\,b^5\,d^3}{c^2}}-\frac{14\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,c\,d-14\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}+\frac{8\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{8\,a^3\,b^5\,d^3-14\,a^2\,b^6\,c\,d^2+6\,a\,b^7\,c^2\,d}\right)\,\left(4\,a\,d-3\,b\,c\right)}{c^3}$$
$$-\frac{\frac{2\,\left(a\,b^2\,c-a^2\,b\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2}+\frac{b\,{\left(a+\frac{b}{x}\right)}^{3/2}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,{\left(a+\frac{b}{x}\right)}^2-a^2\,d+a\,b\,c}$$

```
[In] int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

```
[Out] (atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^3
 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2 -
b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*d -
 b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(a +
b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (14*a^
(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3*b^5*d
^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2*b^6*c*
d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d)*(a + b
/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d -
 b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

3.237 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a}(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

```
[Out] 3*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^4-3/4*(8*a^2*d^2-8*
a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(1/2)
/(-a*d+b*c)^(1/2)-1/2*(-3*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)^2-3/4*(-4*a*d+
b*c)*(a+b/x)^(1/2)/c^3/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 100, 156, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\frac{3(8a^2d^2-8abcd+b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{c^4} - \frac{3\sqrt{a+\frac{b}{x}}(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)} - \frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -1/2*((b*c - 3*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqr
t[a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a
*d]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/c^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad)-\frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad)+\frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-ad)^2-\frac{3}{4}b(bc-4ad)(bc-ad)^2x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^4} - \frac{(3(b^2c^2-8abcd+8a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{8c^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^4} - \frac{(3(b^2c^2-8abcd+8a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4bc^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-bc(3d+5cx)+2a(6d^2+9cdx+2c^2x^2))}{(d+cx)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} - 12\sqrt{a}(-bc+2}{4c^4}$$

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x^
2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqr
t[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) - 12*Sqrt[a]*(-(b*c
) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(181) = 362.

Time = 0.27 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.82

method	result	size
risch	Expression too large to display	1008
default	Expression too large to display	1817

[In] int((a+b/x)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^3*x*a*((a*x+b)/x)^(1/2)-1/2/c^3*(3*a^(1/2)*(2*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))-(-12*a^2*d^2+12*a*b*c*d-2*b^2*c^2)/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+4/c^3*d*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))-2*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(181) = 362.

Time = 0.34 (sec) , antiderivative size = 1765, normalized size of antiderivative = 8.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")

[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b

```
^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d -
(b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)) -
2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2
*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x
+ b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2
- a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*
d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2
 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log(
(b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
 d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqr
t((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c
^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^
2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8
*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x +
 b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4
*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)
/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*
c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a
*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)
*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d
*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b
*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^
4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c
^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*
c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a
*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(181) = 362.

Time = 0.38 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\sqrt{ax^2+bx}\,a\operatorname{sgn}(x)}{c^3} + \frac{3\left(b^2c^2\operatorname{sgn}(x)-8\,abcd\operatorname{sgn}(x)+8\,a^2d^2\operatorname{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\sqrt{bcd-ad^2}\,c^4} - \frac{3\left(abc\operatorname{sgn}(x)-2\,a^2d\operatorname{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^4} + \frac{\left(3\sqrt{a}b^2c^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-24\,a^{\frac{3}{2}}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+24\,a^{\frac{5}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+6\sqrt{bcd-ad^2}abc\right.}{4\sqrt{bcd-ad^2}\sqrt{a}c^4} + \frac{5\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3\operatorname{sgn}(x)-24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d\operatorname{sgn}(x)+24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^2cd^2\operatorname{sg}}{}$$

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8
*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sq
rt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*
sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c
^4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(
3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12
*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*s
qrt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*sgn(x) - 24*(sqrt(a)*x - sqrt(a*
x^2 + b*x))^3*a*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c
```

```
*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d*sgn(x) -
24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^2*sgn(x) + 40*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*
x))*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d^2*sgn(x
) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^3*sgn(x) - 5*sqrt(a)*b^3*c*d
^2*sgn(x) + 10*a^(3/2)*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*
c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 1664, normalized size of antiderivative = 7.96

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/x)^(3/2)/(c + d/x)^3,x)
```

```
[Out] - ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
- ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (3*
b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*b*c
*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3*d
^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a + b/x
)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*b^6*d^
2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a*d - b*
c))/c^4 - (atan((((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a
*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(
a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*
c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d
^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2
*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*
d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((((a + b/x)^(1/
2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4
 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d
^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a +
 b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^
6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2
 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8
*(a*c^4*d^2 - b*c^5*d)))/((216*a^5*b^3*d^5 - 378*a^4*b^4*c*d^4 - (189*a^2*b
^6*c^3*d^2)/4 + 216*a^3*b^5*c^2*d^3 + (27*a*b^7*c^4*d)/8)/c^9 - (3*(((a + b
/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^
3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(a*d - b*c))^(1/2)*((9*a*b^
4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^
3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))
/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c
```

```
^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)
)/(8*(a*c^4*d^2 - b*c^5*d)) + (3*(((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*
b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8
*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^
9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))
^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(
8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c)
)^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d))))*(d*(
a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(4*(a*c^4*d^2 - b*c
^5*d))
```

3.238 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx = -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2}+\left(a+\frac{b}{x}\right)^{5/2}\left(c\right.$$

```
[Out] -1/3*c^2*(6*a*d+5*b*c)*(a+b/x)^(3/2)-11/9*d*(a+b/x)^(5/2)*(c+d/x)^2-1/315*d
*(a+b/x)^(5/2)*(-20*a^2*d^2+270*a*b*c*d+938*b^2*c^2+5*b*d*(10*a*d+89*b*c)/x
)/b^2+(a+b/x)^(5/2)*(c+d/x)^3*x+a^(3/2)*c^2*(6*a*d+5*b*c)*arctanh((a+b/x)^(
1/2)/a^(1/2))-a*c^2*(6*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 99, 158, 152, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx = a^{3/2}c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+5bc) -\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(-10a^2d^2+135abcd+469b^2c^2)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2} -\frac{1}{3}c^2\left(a+\frac{b}{x}\right)^{3/2}(6ad+5bc)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)+x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x-\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2\left(\frac{1}{2}(5bc+6ad)+\frac{11bdx}{2}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2+\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x\\
&\quad-\frac{2\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)\left(\frac{9}{4}bc(5bc+6ad)+\frac{1}{4}bd(89bc+10ad)x\right)}{x}\,dx, x, \frac{1}{x}\right)}{9b}\\
&= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2}\\
&\quad+\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x-\frac{1}{2}\left(c^2(5bc+6ad)\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\\
&\quad-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2}\\
&\quad+\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x-\frac{1}{2}\left(ac^2(5bc+6ad)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2} + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \frac{1}{2}\left(a^2c^2(5bc+6ad)\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{x}\right)$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2} + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \frac{\left(a^2c^2(5bc+6ad)\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(469b^2c^2+135abcd-10a^2d^2)+\frac{5bd(89bc+10ad)}{x}\right)}{315b^2} + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x + a^{3/2}c^2(5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \frac{\sqrt{a+\frac{b}{x}}(20a^4d^3x^4-10a^3bd^2x^3(d+27cx)-3a^2b^2x^2(50d^3+270cd^2x+966c^2dx^2-105c^3x^3)-2\cdots}{315b^2x\cdots} + a^{3/2}c^2(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2*
x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3 +
135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d^2*
x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c + 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(315a^2b^2c^3x^5+20a^4d^3x^4-270a^3bc\,d^2x^4-2898a^2b^2c^2d\,x^4-1470a\,b^3c^3x^4-10a^3b\,d^3x^3-810a^2b^2c\,d^2x^3-1386a\,b^3c^2d\,x^3-210b^4c^3x^3-}{315x^4b^2}$
default	$\sqrt{\frac{ax+b}{x}}\left(3780a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,b\,c^2d\,x^6+3150a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b^2c^3x^6+1890\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c^2d\,x^6+1575\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right.\right.$

```
[In] int((a+b/x)^(5/2)*(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*(315*a^2*b^2*c^3*x^5+20*a^4*d^3*x^4-270*a^3*b*c*d^2*x^4-2898*a^2*b^2*
c^2*d*x^4-1470*a*b^3*c^3*x^4-10*a^3*b*d^3*x^3-810*a^2*b^2*c*d^2*x^3-1386*a*
b^3*c^2*d*x^3-210*b^4*c^3*x^3-150*a^2*b^2*d^3*x^2-810*a*b^3*c*d^2*x^2-378*b
^4*c^2*d*x^2-190*a*b^3*d^3*x-270*b^4*c*d^2*x-70*b^4*d^3)/x^4/b^2*((a*x+b)/x
)^(1/2)+1/2*(6*a*d+5*b*c)*a^(3/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1
/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.49

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx = \left[\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{a}x^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(315\,a^2b^2c^3x^5-70\,b^4d^3-2\,(}{}\right.$$

$$-\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(315\,a^2b^2c^3x^5-70\,b^4d^3-2\,(735\,ab^3c^3+1449\,a^2b^2}{}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*
a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105
*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b
^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*
b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*
```

```
b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2
*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c
*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d
^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)
*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

Sympy [A] (verification not implemented)

Time = 39.99 (sec) , antiderivative size = 5523, normalized size of antiderivative = 27.89

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

```
[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(
21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6
300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(1
1/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/
2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2
)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x
**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2)
+ 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*sqrt(a*x
/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2
)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
+ 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(2
1/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1
890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(1
5/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**2
0*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x*
*6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 378*a**(
19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2
*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a*
*(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
**10*x**(7/2)) - 1134*a**(17/2)*b**(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a*
```

```
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120
*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b + 1)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x
**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2)
+ 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**
(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2)*c*d**2*x**4*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3
*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**
(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*d**3*x**2*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2
)) - 120*a**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 1
05*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b
+ 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*
c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) + 2*a**(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*x*sqr
t(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2
) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*
a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b
**21*x**(9/2)) - 300*a**(11/2)*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a
**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*
x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(11/2)*
b**(7/2)*c**2*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) + 12*a**(11/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(11/2)*b**(7/2)*d
**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 70*a**(9/2)*b**(47/2)*d**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21
/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 630
0*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 288*a**(9/2)*b**(23/2)*
```

c*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60
*a**(9/2)*b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 6*a**(9/2)*b**(9/2)*c**2*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*
b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 48*a**(9/2)*b**(9/2)*c*d**2*x*
sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6
*a**(9/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 90*a**(7/2)*b**(25/2)*c*d**2*sqrt(a*x/b + 1)/(105*a*
*(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x
(9/2) + 105*a(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(11/2)*c**2*d*x*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 36*a
(7/2)*b(11/2)*c*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(13/2)*c**2*d*sqrt(a*x/b + 1)/(15*a*
*(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c**3*asinh(s
qrt(a)*sqrt(x)/sqrt(b)) - 32*a**15*b**13*d**3*x**(21/2)/(315*a**(21/2)*b**1
5*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/
2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 192*a**14*b**14
*d**3*x**(19/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(1
9/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2
)*b**21*x**(9/2)) - 480*a**13*b**15*d**3*x**(17/2)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(
11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 640*a**12*b**16*d**3
*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2)
+ 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**
21*x**(9/2)) - 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 32*a**11*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x*
*(11/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 96*a**10*b**6*d**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2)
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)

```
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 144*a**9*b**7*c*d**2*x**(
11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**
(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d**2*
x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 32*a**8*b**8*d**3*
x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(
7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9
*c*d**2*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**7*b*
*2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 24*a**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2
)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**2*sqrt(b)*c**3*sqrt(x)*sq
rt(a*x/b + 1) - 3*a**2*c**2*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/s
qrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 3*a**2*c*d
**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) -
2*a*b*c**3*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a
+ b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 6*a*b*c**2*d*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*Piecewise(
(-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7\,b}$$
$$+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^3$$
$$-\frac{1}{5}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}a^2\right)c^2d$$
$$-\frac{2}{63}\left(\frac{7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{b^2}-\frac{9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{b^2}\right)d^3$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*lo
g((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a
))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a +
30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x)^(7/
2)*a/b^2)*d^3
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 10.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.46

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{7\,b^2}-\frac{4\,a\,d^3}{7\,b^2}\right)$$
$$-\sqrt{a+\frac{b}{x}}\left(a^2\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-2\,a\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\frac{6}{}\right.\right.\right.\right.$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,(a\,d-b\,c)^3}{3\,b^2}+\frac{2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)}{3}-\frac{a^2\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}\right.$$

```
[In] int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*
d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*
((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*
a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a + b/
x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2
- (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (a^2*((6
*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*a*((6*a*
```

```
d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*b^2) + (2
*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*(a + b/x)^
(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + 5*b*c)*1i
```

3.239 $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = &-ac(5bc+4ad)\sqrt{a+\frac{b}{x}}\\ &-\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}\\ &+\frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a}+a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

```
[Out] -1/3*c*(4*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*c*(4*a*d+5*b*c)*(a+b/x)^(5/2)/a-2/7*
d^2*(a+b/x)^(7/2)/b+c^2*(a+b/x)^(7/2)*x/a+a^(3/2)*c*(4*a*d+5*b*c)*arctanh((
a+b/x)^(1/2)/a^(1/2))-a*c*(4*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 91, 81, 52, 65, 214}

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = &\, a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+5bc)\\ &+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}\\ &-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4ad+5bc)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}\end{aligned}$$

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{(c(5bc+4ad))\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(c(5bc+4ad))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(ac(5bc+4ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}\left(a^2c(5bc+4ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\left(a^2c(5bc+4ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} + a^{3/2}c(5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}(-30a^3d^2x^3-2b^3(15d^2+42cdx+35c^2x^2)+a^2bx^2(-90d^2-644cdx+105c^2x^2)-2ab^2\cdots}{105bx^3} + a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
[Out] (Sqrt[a + b/x]*(-30*a^3*d^2*x^3 - 2*b^3*(15*d^2 + 42*c*d*x + 35*c^2*x^2) +
a^2*b*x^2*(-90*d^2 - 644*c*d*x + 105*c^2*x^2) - 2*a*b^2*x*(45*d^2 + 154*c*d
*x + 245*c^2*x^2)))/(105*b*x^3) + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\left(-105a^2b\,c^2x^4+30a^3d^2x^3+644a^2bcd\,x^3+490a\,b^2c^2x^3+90x^2a^2b\,d^2+308x^2a\,b^2cd+70x^2b^3c^2+90a\,b^2d^2x+84b^3cdx+30b^3d^2\right)\sqrt{\frac{ax+\cdots}{x}}}{105x^3b}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(840a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,cd\,x^5+1050a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b\,c^2x^5-840a^{\frac{5}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}cd\,x^3-60a^{\frac{5}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}d^2x^2-840a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}b\,c\cdots}{\cdots}$

```
[In] int((a+b/x)^(5/2)*(c+d/x)^2,x,method=_RETURNVERBOSE)
[Out] -1/105*(-105*a^2*b*c^2*x^4+30*a^3*d^2*x^3+644*a^2*b*c*d*x^3+490*a*b^2*c^2*x
^3+90*a^2*b*d^2*x^2+308*a*b^2*c*d*x^2+70*b^3*c^2*x^2+90*a*b^2*d^2*x+84*b^3*
c*d*x+30*b^3*d^2)/x^3/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+5*b*c)*a^(3/2)*c*ln((1
/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a
*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.30

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \left[\frac{105\,(5\,ab^2c^2+4\,a^2bcd)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(105\,a^2bc^2x^4-30\,b^3d^2-2\,(245\,\ldots}{\ldots}\right.$$

$$-\frac{105\,(5\,ab^2c^2+4\,a^2bcd)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(105\,a^2bc^2x^4-30\,b^3d^2-2\,(245\,ab^2c^2+322\,a^2bcd+1\ldots}{105\,bx^3}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*
c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*
a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 32
2*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^
2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 1853, normalized size of antiderivative = 12.19

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
```

```
3/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(23/2)*d*
*2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**
(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(
9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(
5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(25/2)*d**2
*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 16*a**(7/2
)*b**(11/2)*c*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b*
*3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2
)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**7*d**2*x**(11/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**2*d**2*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*c*d*x**(7
/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d
**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a*
*5*b**4*c*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)
) + a**2*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a**2*c*d*Piecewise((2*a*a
tan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a
)*log(x), True)) + a**2*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x
)**(3/2)/(3*b), True)) - 2*a*b*c**2*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(
-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 4*a*
b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
+ b**2*c**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.19

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^2}{7\,b} + \frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x - 15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 24\sqrt{a+\frac{b}{x}}ab\right)c^2 - \frac{2}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 6\left(a+\frac{b}{x}\right)^{\frac{5}{2}} + 10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a + 30\sqrt{a+\frac{b}{x}}a^2\right)cd$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b
- 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a)
)/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 3
0*sqrt(a + b/x)*a^2)*c*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 dx = \left(a + \frac{b}{x}\right)^{3/2} \left(\frac{2\,a\,\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b} - \frac{4\,a\,d^2}{b}\right)}{3} - \frac{2\,(a\,d-b\,c)^2}{3\,b} + \frac{2\,a^2\,d^2}{3\,b}\right) + \left(\frac{4\,a\,d^2-4\,b\,c\,d}{5\,b} - \frac{4\,a\,d^2}{5\,b}\right)\left(a + \frac{b}{x}\right)^{5/2} - \sqrt{a + \frac{b}{x}}\left(a^2\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b} - \frac{4\,a\,d^2}{b}\right) - 2\,a\left(2\,a\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b} - \frac{4\,a\,d^2}{b}\right) - \frac{2\,(a\,d-b\,c)^2}{b} + \frac{2\,a^2\,d^2}{b}\right)\right) -$$

```
[In] int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^2)
/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b - (4*
a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a + b/
x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*c)*1i
```

3.240 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx = -a(5bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a}+\frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a}+a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -1/3*(2*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*(2*a*d+5*b*c)*(a+b/x)^(5/2)/a+c*(a+b/x
)^(7/2)*x/a+a^(3/2)*(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))-a*(2*a*d+5
*b*c)*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+5bc)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a} -\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

```
[In] Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} - \frac{\left(\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} - \frac{1}{2}(5bc+2ad)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} - \frac{1}{2}(a(5bc+2ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} - \frac{1}{2}\left(a^2(5bc+2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} - \frac{\left(a^2(5bc+2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2} x}{a} + a^{3/2}(5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx = \frac{\sqrt{a+\frac{b}{x}}\left(-2b^2(3d+5cx)+a^2x^2(-46d+15cx)-2abx(11d+35cx)\right)}{15x^2} + a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(
11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
risch	$\frac{\left(15 a^{2} c \,x^{3}-46 a^{2} d \,x^{2}-70 a b c \,x^{2}-22 a x b d -10 b^{2} c x -6 b^{2} d \right) \sqrt{\frac{a x +b}{x}}}{15 x^{2}}+\frac{\left(2 a d +5 b c \right) a^{\frac{3}{2}} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x \left(a x +b \right)}}{2 a x +2 b}$
default	$\frac{\sqrt{\frac{a x +b}{x}}\, \left(60 a^{\frac{7}{2}} \sqrt{a \,x^{2}+b x}\, d \,x^{4}+150 a^{\frac{5}{2}} \sqrt{a \,x^{2}+b x}\, b c \,x^{4}-60 a^{\frac{5}{2}} \left(a \,x^{2}+b x \right)^{\frac{3}{2}} d \,x^{2}-120 a^{\frac{3}{2}} \left(a \,x^{2}+b x \right)^{\frac{3}{2}} b c \,x^{2}+30 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +}{2 \sqrt{a}}\right.\right.}{30 x^{3} b \sqrt{x \left(a x +\right.}}$

```
[In] int((a+b/x)^(5/2)*(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*a^2*c*x^3-46*a^2*d*x^2-70*a*b*c*x^2-22*a*b*d*x-10*b^2*c*x-6*b^2*d)
/x^2*((a*x+b)/x)^(1/2)+1/2*(2*a*d+5*b*c)*a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.78

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx = \left[\frac{15\,(5\,abc+2\,a^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a^2}{30\,x^2}\right.$$

$$-\frac{15\,(5\,abc+2\,a^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a^2d)x^2-2\,(5\,b^2c+11\,a}{15\,x^2}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.27

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^2\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-a^2d\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log\left(x\right) & \text{otherwise}\end{cases}\right)-2abc\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log\left(x\right) & \text{otherwise}\end{cases}\right)+2abd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2)*d
*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**
6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) + a**2*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - a**2*d*Piecewise((2*a*atan(
sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*lo
g(x), True)) - 2*a*b*c*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
 + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 2*a*b*d*Piecewise
((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecew
ise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Maxima [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\text{Timed out}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{5/2}}{5}-2\,a^2\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

```
[In] int((a + b/x)^(5/2)*(c + d/x),x)
[Out] - (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)^(
5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

3.241 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -5/3*b*(a+b/x)^(3/2)+(a+b/x)^(5/2)*x+5*a^(3/2)*b*arctanh((a+b/x)^(1/2)/a^(1
/2))-5*a*b*(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {248, 43, 52, 65, 214}

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = 5a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + x\left(a+\frac{b}{x}\right)^{5/2} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} - 5ab\sqrt{a+\frac{b}{x}}$$

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)
\end{aligned}
$$

$$= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2}x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \frac{\sqrt{a+\frac{b}{x}}(-2b^2-14abx+3a^2x^2)}{3x} + 5a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[In] Integrate[(a + b/x)^(5/2),x]
[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{(3a^2x^2-14abx-2b^2)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	94
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(30a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,x^3-24a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}x+15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3-4b\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^2\sqrt{x(ax+b)}\,\sqrt{a}}$	120

```
[In] int((a+b/x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(3*a^2*x^2-14*a*b*x-2*b^2)/x*((a*x+b)/x)^(1/2)+5/2*a^(3/2)*b*ln((1/2*b+
a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.96

$$\int\left(a+\frac{b}{x}\right)^{5/2}dx=\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{6\,x},\right.$$
$$\left.-\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

```
[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*
a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((
a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int\left(a+\frac{b}{x}\right)^{5/2}dx=a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=\sqrt{a+\frac{b}{x}}a^2x-\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-4\sqrt{a+\frac{b}{x}}ab$$

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=-\frac{2\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

```
[In] int((a + b/x)^(5/2),x)
```

```
[Out] -(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
+ 1)^(5/2))
```

3.242 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}+\frac{a^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

```
[Out] a*(a+b/x)^(3/2)*x/c+2*(-a*d+b*c)^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b
*c)^(1/2))/c^2/d^(3/2)+a^(3/2)*(-2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2)
)/c^2-b*(a*d+2*b*c)*(a+b/x)^(1/2)/c/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 100, 159, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-2ad)}{c^2}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

```
[In] Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b\left(2b^2c^2-6abcd+a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{cd}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2}\\
&\quad + \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2d}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}\\
&\quad - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2}\\
&\quad + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2d}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}\\
&\quad + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\left(-2b^2c+a^2dx\right)}{d}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}-a^{3/2}(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] ((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(Sq
rt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 2*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(114) = 228.

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.08

method	result
risch	$\frac{(a^2dx-2b^2c)\sqrt{\frac{ax+b}{x}}}{dc}-\frac{\left(\frac{a^{\frac{3}{2}}d(2ad-5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c}-\frac{\left(-2a^3d^3+6a^2bc\,d^2-6a\,b^2c^2d+2b^3c^3\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\cdots}{\cdots}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}\cdots}\cdots\right)}{2cd(ax+b)}\cdots$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^4x^2-2a^{\frac{5}{2}}\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d^2x^2-6a^{\frac{5}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\cdots}{cx+d}\cdots\right.\cdots\right.}{\cdots}$

```
[In] int((a+b/x)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] (a^2*d*x-2*b^2*c)/d/c*((a*x+b)/x)^(1/2)-1/2/c/d*(a^(3/2)*d*(2*a*d-5*b*c)/c*
ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))-(-2*a^3*d^3+6*a^2*b*c*d^2-6*a*b^2
*c^2*d+2*b^3*c^3)/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-
b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)
+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x
+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 659, normalized size of antiderivative = 4.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \left[-\frac{(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-}{a}}}{2\,c^2d}\right.$$

$$-\frac{(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc}{cx+d}\right.}{c^2d}$$

$$-\frac{4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}\right.}{2\,c^2d}$$

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-}{c^2d}$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*
d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x +
d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*d -
2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((
a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2*c^2)*s
qrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*
c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) +
(5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
 b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt
((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d
)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

```
[In] integrate((a+b/x)**(5/2)/(c+d/x),x)
[Out] Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{c+\frac{d}{x}}\,dx$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 1427, normalized size of antiderivative = 10.65

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\frac{a^2\,b\,d\,\sqrt{a+\frac{b}{x}}}{c\,\left(d\,\left(a+\frac{b}{x}\right)-a\,d\right)}-\frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{d}$$

$$+\frac{\mathrm{atan}\Bigg(\frac{a^3\,b^5\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^5\,d^8-5\,a^4\,b\,c\,d^7+10\,a^3\,b^2\,c^2\,d^6-10\,a^2\,b^3\,c^3\,d^5+5\,a\,b^4\,c^4\,d^4-b^5\,c^5\,d^3}\,160\mathrm{i}}{448\,a^3\,b^8\,c^3\,d-340\,a^6\,b^5\,d^4-128\,a^2\,b^9\,c^4+740\,a^5\,b^6\,c\,d^3+\frac{16\,a\,b^{10}\,c^5}{d}-796\,a^4\,b^7\,c^2\,d^2+\frac{60\,a^7\,b^4\,d^5}{c}}-\frac{a^2\,b^6\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^5\,d^8-5\,a^4\,b\,c\,d^7+10}{16\,a\,b^{10}\,c^4+740\,a^5\,b^6\,d^4-128\,a^2\,b^9\,c^3\,d-}}{}$$

$$+\frac{\mathrm{atan}\Bigg(\frac{b^9\,c^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}\,40\mathrm{i}}{40\,a^2\,b^9\,c^3-790\,a^5\,b^6\,d^3-256\,a^3\,b^8\,c^2\,d+696\,a^4\,b^7\,c\,d^2+\frac{370\,a^6\,b^5\,d^4}{c}-\frac{60\,a^7\,b^4\,d^5}{c^2}}+\frac{a\,b^8\,c^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}\,256\mathrm{i}}{256\,a^3\,b^8\,c^2+790\,a^5\,b^6\,d^2-\frac{40\,a^2\,b^9\,c^3}{d}-\frac{370\,a^6\,b^5\,d^3}{c}+60}}{}$$

```
[In] int((a + b/x)^(5/2)/(c + d/x),x)

[Out] (atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 1
0*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*a^
3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (16*a
*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a + b/
x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10
*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5*b^6*d
^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (340*a^6
*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5*d^8 - b
^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*
a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a^4*b^7*c^3
*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d^2 - (128*a
^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*
c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*16
i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2 + 448*a^3*b^8
*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a*b^10*c^4*d))*(
d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(1/2))/d + (atan(
(b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3 - 790*a^5*b^6*d^3
 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d^4)/c - (60*a^7*b^
4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256i)/(256*a^3*b^8*c^2
 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4
*d^4)/c^2 - 696*a^4*b^7*c*d) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*790
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d
^3)/c + (60*a^7*b^4*d^4)/c^2 - 696*a^4*b^7*c*d) - (a^4*b^5*d^3*(a + b/x)^(1
/2)*(a^3)^(1/2)*370i)/(256*a^3*b^8*c^3 - 370*a^6*b^5*d^3 - 696*a^4*b^7*c^2*
d + 790*a^5*b^6*c*d^2 - (40*a^2*b^9*c^4)/d + (60*a^7*b^4*d^4)/c) + (a^5*b^4
*d^4*(a + b/x)^(1/2)*(a^3)^(1/2)*60i)/(256*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 6
96*a^4*b^7*c^3*d - 370*a^6*b^5*c*d^3 + 790*a^5*b^6*c^2*d^2 - (40*a^2*b^9*c^
5)/d) - (a^2*b^7*c*d*(a + b/x)^(1/2)*(a^3)^(1/2)*696i)/(256*a^3*b^8*c^2 + 7
90*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4*d^4
)/c^2 - 696*a^4*b^7*c*d))*(2*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^2 + (a^2*b*d*(a
 + b/x)^(1/2))/(c*(d*(a + b/x) - a*d))
```

3.243 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)-(-a*d+b*c)^(3/2)*(4*a*d+b*c)*arctan(d^(1/2)*(a+
b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)+a^(3/2)*(-4*a*d+5*b*c)*arctanh((a+
b/x)^(1/2)/a^(1/2))/c^3+(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 100, 154, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-4ad)}{c^3} - \frac{(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)} + \frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b\left(b^2c^2+2abcd-2a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((bc-ad)^2(bc+4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} - \frac{((bc-ad)^2(bc+4ad))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(b^2c^2-2abcd+a^2d(2d+cx))}{d(d+cx)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - a^{3/2}(-5bc+4ad)\text{arctan}}{c^3}$$

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x)
) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(146) = 292.

Time = 0.28 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.11

method	result
risch	$\frac{a^{2} x \sqrt{\frac{a x+b}{x}}}{c^{2}}-\frac{\left(\frac{a^{\frac{3}{2}}(4ad-5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{c}+\frac{6a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^{2}}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\sqrt{a\left(x+\frac{d}{c}\right)^{2}-}}{x+\frac{d}{c}}\right.}{c^{2}\sqrt{\frac{(ad-bc)d}{c^{2}}}}\right.}{}$
default	Expression too large to display

```
[In] int((a+b/x)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*a^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(3/2)*(4*a*d-5*b*c)/c*ln((1/2*b+a*
x)/a^(1/2)+(a*x^2+b*x)^(1/2))+6*a/c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c
)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c
^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c
))+1/c^3*(2*a^3*d^3-6*a^2*b*c*d^2+6*a*b^2*c^2*d-2*b^3*c^3)*(-1/(a*d-b*c)/d*
c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(
2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a
*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d
/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/
(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 1001, normalized size of antiderivative = 6.03

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \Bigg[-\frac{(5\,abcd^2 - 4\,a^2d^3 + (5\,abc^2d - 4\,a^2cd^2)x)\sqrt{a}\log\left(2\,ax - 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + (b^2c^2d}{}$$

$$-\frac{2\,(5\,abcd^2 - 4\,a^2d^3 + (5\,abc^2d - 4\,a^2cd^2)x)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right) + (b^2c^2d + 3\,abcd^2 - 4\,a^2d^3 + (b^2c^3 +}{2}$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 - 4
*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c
*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt
((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a
*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x)
 + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*
c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), 1/2*(2*(b^2
*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*
log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), (
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b
*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a
*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{(cx+d)^2}\,dx$$

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")

[Out] integrate((a + b/x)^(5/2)/(c + d/x)^2, x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(146) = 292$.

Time = 0.34 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\sqrt{ax^2+bx}\,a^2\mathrm{sgn}(x)}{c^2} - \frac{\left(5\,a^2bc\mathrm{sgn}(x)-4\,a^3d\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^3} + \frac{\left(b^3c^3\mathrm{sgn}(x)+2\,ab^2c^2d\mathrm{sgn}(x)-7\,a^2bcd^2\mathrm{sgn}(x)+4\,a^3d^3\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3d} + \frac{\left(2\sqrt{a}b^3c^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+4\,a^{\frac{3}{2}}b^2c^2d\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-14\,a^{\frac{5}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+8\,a^{\frac{7}{2}}d^3\arctan\cdots\right.}{\cdots} - \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c^3\mathrm{sgn}(x)-4\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{3}{2}}b^2c^2d\mathrm{sgn}(x)+5\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^{\frac{5}{2}}bcd^2\mathrm{s}\cdots}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{a\cdots}\right.\right.}$$

[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")

[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^2 - 1/2*(5*a^2*b*c*sgn(x) - 4*a^3*d*sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^3) + (b^3*c^3*sgn(x) + 2*a*b^2*c^2*d*sgn(x) - 7*a^2*b*c*d^2*sgn(x) + 4*a^3*d^3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3*d) + 1/2*(2*sqrt(a)*b^3*c^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 4*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 14*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 8*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 5*sqrt(b*c*d - a*d^2)*a^2*b*c*d*log(abs(b)) - 4*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - 2*sqrt(b*c*d - a*d^2)*a*b^2*c^2 + 4*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 2*sqrt(b*c*d - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3*d) - ((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^3*sgn(x) - 4*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a

```
^(3/2)*b^2*c^2*d*sgn(x) + 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*c*d^2
*sgn(x) - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3*sgn(x) - a*b^3*c^2*
d*sgn(x) + 2*a^2*b^2*c*d^2*sgn(x) - a^3*b*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a
*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)*sqrt(
a)*c^3*d)
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.95

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{\sqrt{a+\frac{b}{x}}\,(2\,a^3\,b\,d^2-3\,a^2\,b^2\,c\,d+a\,b^3\,c^2)}{c^2\,d}-\frac{b\,\left(a+\frac{b}{x}\right)^{3/2}\,(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{c^2\,d}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,\left(a+\frac{b}{x}\right)^2-a^2\,d+a\,b\,c}$$
$$-\frac{\mathrm{atanh}\left(\frac{10\,b^9\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{10\,a^2\,b^9+\frac{32\,a^3\,b^8\,d}{c}-\frac{132\,a^4\,b^7\,d^2}{c^2}+\frac{130\,a^5\,b^6\,d^3}{c^3}-\frac{40\,a^6\,b^5\,d^4}{c^4}}+\frac{32\,a\,b^8\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{32\,a^3\,b^8+\frac{10\,a^2\,b^9\,c}{d}-\frac{132\,a^4\,b^7\,d}{c}+\frac{130\,a^5\,b^6\,d^2}{c^2}-\frac{40\,a^6\,b^5\,d^3}{c^3}}-\frac{}{32\,a^3\,b^8\,c-1}\right.}{}$$
$$+\frac{\mathrm{atanh}\left(\frac{30\,a^3\,b^6\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^6-3\,a^2\,b\,c\,d^5+3\,a\,b^2\,c^2\,d^4-b^3\,c^3\,d^3}}{14\,a^2\,b^9\,c^3+110\,a^5\,b^6\,d^3-4\,a^3\,b^8\,c^2\,d-82\,a^4\,b^7\,c\,d^2+\frac{2\,a\,b^{10}\,c^4}{d}-\frac{40\,a^6\,b^5\,d^4}{c}}+\frac{18\,a^2\,b^7\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^6-3\,a^2\,b\,c\,d^5+3\,a\,b^2\,c^2\,d^4-}}{2\,a\,b^{10}\,c^3-82\,a^4\,b^7\,d^3+14\,a^2\,b^9\,c^2\,d-4\,a^3\,b^8\,c\,d^2+\frac{110\,a^5\,b^6}{c}}\right.}{}$$

```
[In] int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

```
[Out] (((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*(
a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*a*
d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/2)*
(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (130*
a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(a^3)^
(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*b^6*d^
2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/2)
)/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6*d^2)/c
- (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2))/(32
*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5*d^3)/c -
132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/(32*a^3*b^8
*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 + (10*a^2*b^9
*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^6*(a + b/x)^(
1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(14*a
^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*c*d^2 + (2*a*b^
10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/2)*(a^3*d^6 - b^
3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b^10*c^3 - 82*a^4*
b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b^6*d^4)/c - (40*a^
6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*
b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 + 40*a^6*b^5*d^3 + 82*a^
4*b^7*c^2*d - 110*a^5*b^6*c*d^2 - (2*a*b^10*c^5)/d^2 - (14*a^2*b^9*c^4)/d)
- (2*a*b^8*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2
*b*c*d^5)^(1/2))/(4*a^3*b^8*d^3 - 14*a^2*b^9*c*d^2 + (82*a^4*b^7*d^4)/c - (
```

```
110*a^5*b^6*d^5)/c^2 + (40*a^6*b^5*d^6)/c^3 - 2*a*b^10*c^2*d))*(d^3*(a*d -
b*c)^3)^(1/2)*(4*a*d + b*c))/(c^3*d^3)
```

3.244 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx = \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}(b^2c^2+8abcd-24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}} + \frac{a^{3/2}(5bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)^2+a^(3/2)*(-6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/c^4-1/4*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/
2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^4/d^(3/2)+1/2*(-3*a*d+b*c)*(-a*d+b*
c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2+7*a*b*c*d+b^2*c^2)*(a+b/x
)^(1/2)/c^3/d/(c+d/x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 100, 154, 156, 162, 65, 214, 211}

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} \, dx=\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-6ad)}{c^4}-\frac{\sqrt{bc-ad}(-24a^2d^2+8abcd+b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}(-12a^2d^2+7abcd+b^2c^2)}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)(bc-ad)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b(b^2c^2+6abcd-9a^2d^2)x}{x\sqrt{a+bx}(c+dx)^2}\,dx,x,\frac{1}{x}\right)}{2c^2d}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-a^2d(5bc-6ad)(bc-ad)-\frac{1}{4}b(bc-ad)(b^2c^2+7abcd-12a^2d^2)x}{x\sqrt{a+bx}(c+dx)}\,dx,x,\frac{1}{x}\right)}{2c^3d(bc-ad)}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2}-\frac{(a^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{x}\right)}{2c^4} - \frac{((bc-ad)\left(b^2c^2+8abcd-24a^2d^2\right))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx,x,\frac{1}{x}\right)}{8c^4d}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2}-\frac{(a^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{bc^4} - \frac{((bc-ad)\left(b^2c^2+8abcd-24a^2d^2\right))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{4bc^4d}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2}-\frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}(b^2c^2+8abcd-24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}+\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(b^2c^2(-d+cx)-abcd(7d+11cx)+2a^2d(6d^2+9cdx+2c^2x^2))}{d(d+cx)^2}-\frac{(b^3c^3+7ab^2c^2d-32a^2bcd^2+24a^3d^3)\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.}{d^{3/2}\sqrt{bc-ad}}}{4c^4}$$

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d*
(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - ((b^3*c^3 + 7*a*b^2*c^2*d
 - 32*a^2*b*c*d^2 + 24*a^3*d^3)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a
*d]])/(d^(3/2)*Sqrt[b*c - a*d]) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(4*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(209) = 418$.

Time = 0.34 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.37

method	result	size
risch	Expression too large to display	1035
default	Expression too large to display	1638

```
[In] int((a+b/x)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^3*x*((a*x+b)/x)^(1/2)-1/2/c^3*(a^(3/2)*(6*a*d-5*b*c)/c*ln((1/2*b+a*x)
/a^(1/2)+(a*x^2+b*x)^(1/2))+6/c^2*a*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)/((a*d-b*c
)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c
^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c
))+(8*a^3*d^3-18*a^2*b*c*d^2+12*a*b^2*c^2*d-2*b^3*c^3)/c^3*(-1/(a*d-b*c)/d*
c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(
2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a
*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d
/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))-2*d*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c
*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)
/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/
2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(
2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(
x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d
/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)
```

```
^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))
)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 1445, normalized size of antiderivative = 6.10

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(
5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^
3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)
*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*
sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 +
 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(
-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*
a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^
3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*a^2*c^3*d
*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c
^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4
*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2
 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d
 - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*
d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*
a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(-d*
sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2
*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*
x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*a^2*c^3*d*x^3 + (b^2*
c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*
a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(209) = 418$.

Time = 0.35 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))*
log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^4) + 1/
4*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^3*d
^3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c
*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan(sqrt
(a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*
d - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24
*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d - a*d^2)
*a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - sqrt(
b*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*sqrt(b*c*d
 - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) - 1/4*((sqrt(
a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a)*x - sqrt(a
*x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x)
```

```
)^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(7/2)
*c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^3*d*sgn(x) - 3*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) + 48*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x)
)^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*c^3*d*sgn(
x) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*d^2*sgn(x) + 52*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqr
t(a*x^2 + b*x))*a^(7/2)*b*d^4*sgn(x) - a*b^4*c^2*d^2*sgn(x) + 11*a^2*b^3*c*
d^3*sgn(x) - 10*a^3*b^2*d^4*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c +
 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)*c^4*d)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 1476, normalized size of antiderivative = 6.23

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

```
[Out] (atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*d
)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a +
b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b^7*
d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/
2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) + (45*
a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/(4*(8*
a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*b^7*c*d
)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4*c^3 - 2
4*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a + b/x)^(
5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(1/2)*(12*a
^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*d))/((a + b
/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d
^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (5*a^2*b^9*c^6
 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^
4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d) - ((((a + b
/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4
*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6
*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16
*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8*c^10*d^4))*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*
c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*c))
^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3) - (log(((((a + b/x)^(
1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4*d^2
```

```
- 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6*d) +
 (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16*c^9*
d) + ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(64*c^10*d^4))*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3))*(d^3*(a*d - b*c))^
(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3) - (5*a^2*b^9*c^6 + 17
28*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^4*d^2
 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d))*(d^3*(a*d - b*c
))^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3)
```

3.245 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -c^2*(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-1/3*d*(-4*a^2*d^2+
18*a*b*c*d+6*b^2*c^2+b*d*(2*a*d+3*b*c)/x)*(a+b/x)^(1/2)/a/b^2+c*(c+d/x)^2*x
*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 100, 152, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)}{a^{3/2}} - \frac{d\sqrt{a+\frac{b}{x}}\left(2(-2a^2d^2+9abcd+3b^2c^2)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2} + \frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

```
[In] Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -1/3*(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c
+ 2*a*d))/x))/(a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad)-\frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} \\
&\quad + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc-6ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} \\
&\quad + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc-6ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab} \\
&= -\frac{d\sqrt{a+\frac{b}{x}}\left(2(3b^2c^2+9abcd-2a^2d^2)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} \\
&\quad + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx &= \frac{\sqrt{a+\frac{b}{x}}\,(4a^2d^3x+3b^2c^3x^2-2abd^2(d+9cx))}{3ab^2x} \\
&\quad + \frac{c^2(-bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

```
[In] Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a*
b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(ax+b)\left(3c^3b^2x^2+4x\,a^2d^3-18xabc\,d^2-2ab\,d^3\right)}{3b^2x^2a\sqrt{\frac{ax+b}{x}}}+\frac{(6ad-bc)c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(6a^{\frac{7}{2}}\sqrt{x(ax+b)}\,d^3x^3-18a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bc\,d^2x^3+18a^{\frac{3}{2}}\sqrt{x(ax+b)}\,b^2c^2d\,x^3-6\sqrt{a}\,\sqrt{x(ax+b)}\,b^3c^3x^3+6a^{\frac{7}{2}}\sqrt{a\,x^2+bx}\,d^3x^3\right.}{}$

[In] int((c+d/x)^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(a*x+b)*(3*b^2*c^3*x^2+4*a^2*d^3*x-18*a*b*c*d^2*x-2*a*b*d^3)/b^2/x^2/a/((a*x+b)/x)^(1/2)+1/2*(6*a*d-b*c)*c^2/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.85

$$\int\frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$=\left[-\frac{3\left(b^3c^3-6\,ab^2c^2d\right)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,ab^2c^3x^2-2\,a^2bd^3-2\left(9\,a^2bcd^2-2\,a^3d^3\right)x\right.}{6\,a^2b^2x}\right.$$

[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x)]

Sympy [A] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.10

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} + \frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - 3c^2d\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + 3cd^2\left(\begin{cases}-\frac{1}{\sqrt{a}x} & \text{for } b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right) + \frac{\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

```
[In] integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3
*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*a*
*(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d**3
*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*
a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**
(3/2)) - 3*c**2*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b,
 0)), (-log(x)/sqrt(a), True)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b,
0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a -
 b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.32

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}c^3\left(\frac{2\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2}{3}d^3\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^2} - \frac{3\sqrt{a+\frac{b}{x}}a}{b^2}\right) - \frac{3c^2d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{6\sqrt{a+\frac{b}{x}}cd^2}{b}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2 -
3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b^2}+\frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1\mathrm{i}}{a^{3/2}}$$

```
[In] int((c + d/x)^3/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a + b
/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/2)
*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

3.246 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}+\frac{c^2\sqrt{a+\frac{b}{x}}x}{a}-\frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -c*(-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-2*d^2*(a+b/x)^(1/2)/
b+c^2*x*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 91, 81, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

```
[In] Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{c^2\sqrt{a+\frac{b}{x}}x}{a} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(bc-4ad)+ad^2x}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\ &= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}x}{a} + \frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\end{aligned}$$

$$= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}x}{a} + \frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}$$

$$= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}x}{a} - \frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}(-2ad^2+bc^2x)}{ab} + \frac{c(-bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

method	result
risch	$-\frac{(ax+b)\left(-bx\,c^2+2a\,d^2\right)}{bax\sqrt{\frac{ax+b}{x}}}+\frac{(4ad-bc)c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,d^2x^2-4a^{\frac{3}{2}}\sqrt{x(ax+b)}\,bcd\,x^2+2\sqrt{a}\,\sqrt{x(ax+b)}\,b^2c^2x^2+2a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,d^2x^2+4a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,bcd\,x^2+\ln\left(\frac{2\sqrt{a\,x}\right.\right.$

```
[In] int((c+d/x)^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a*x+b)*(-b*c^2*x+2*a*d^2)/b/a/x/((a*x+b)/x)^(1/2)+1/2*(4*a*d-b*c)*c/a^(3/
2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b)
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \left[-\frac{(b^2c^2-4\,abcd)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(abc^2x-2\,a^2d^2)\sqrt{\frac{ax+b}{x}}}{2\,a^2b}, \frac{(b^2c^2-4\,abcd)\sqrt{-a}\,\mathrm{arc}\right.$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = -2cd\left(\begin{cases} \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0 \\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise} \end{cases}\right) + d^2\left(\begin{cases} -\frac{1}{\sqrt{a}x} & \text{for } b=0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

```
[In] integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] -2*c*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log
(x)/sqrt(a), True)) + d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a
 + b/x)/b, True)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - b*c**2*asinh(s
qrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}c^2\left(\frac{2\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+\frac{b}{x}}d^2}{b}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{c^2\,x\,\sqrt{a+\frac{b}{x}}}{a} - \frac{2\,d^2\,\sqrt{a+\frac{b}{x}}}{b} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-b\,c)}{a^{3/2}}$$

```
[In] int((c + d/x)^2/(a + b/x)^(1/2),x)
```

```
[Out] (c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)^
(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

3.247 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+c*x*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {382, 79, 65, 214}

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{cx\sqrt{a+\frac{b}{x}}}{a} - \frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{3/2}}$$

```
[In] Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{c+dx}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(-\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(2\left(-\frac{bc}{2}+ad\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\
&= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} + \frac{(-bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[(c + d/x)/Sqrt[a + b/x],x]
[Out] (c*Sqrt[a + b/x]*x)/a + ((-(b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a
^(3/2)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.65

method	result
risch	$\frac{c(ax+b)}{a\sqrt{\frac{ax+b}{x}}} + \frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}\,x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{3}{2}}\sqrt{x(ax+b)}\,d-2\sqrt{a}\,\sqrt{x(ax+b)}\,bc-2a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,d-\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd-\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\right.}{2\sqrt{x(ax+b)}\,b\,a^{\frac{3}{2}}}$

```
[In] int((c+d/x)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2*(2*a*d-b*c)/a^(3/2)*ln((1/2*b+a*x)/a^(1
/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{acx\sqrt{\frac{ax+b}{x}} + (bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\right.}{}\right.}{a^2}\right.$$

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

Sympy [A] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -d\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

```
[In] integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] -d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(x)/
sqrt(a), True)) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - b*c*asinh(sqrt(a)*s
qrt(x)/sqrt(b))/a**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}c\left(\frac{2\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt(
a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{(bc\log(|b|) - 2\,ad\log(|b|))\mathrm{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{\sqrt{ax^2+bx}c}{a\mathrm{sgn}\,(x)} + \frac{(bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c*log(abs(b)) - 2*a*d*log(abs(b)))*sgn(x)/a^(3/2) + sqrt(a*x^2 + b*
x)*c/(a*sgn(x)) + 1/2*(b*c - 2*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x
))*sqrt(a) + b))/(a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\,c\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x} + \frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

```
[In] int((c + d/x)/(a + b/x)^(1/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x)
^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2
)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

3.248 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+x*(a+b/x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {248, 44, 65, 214}

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{a+\frac{b}{x}}x}{a} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{\sqrt{a+\frac{b}{x}}x}{a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a} \\
&= \frac{\sqrt{a+\frac{b}{x}}x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x}{a} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[1/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2 \sqrt{x (ax+b)}\, \sqrt{a}-b \ln \left(\frac{2 \sqrt{x (ax+b)}\, \sqrt{a}+2 a x+b}{2 \sqrt{a}}\right)\right)}{2 \sqrt{x (ax+b)}\, a^{\frac{3}{2}}}$	71
risch	$\frac{ax+b}{a \sqrt{\frac{ax+b}{x}}}-\frac{b \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a\, x^{2}+b x}\right) \sqrt{x (ax+b)}}{2 a^{\frac{3}{2}} x \sqrt{\frac{ax+b}{x}}}$	75

```
[In] int(1/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a+b/x)**(1/2),x)
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2
 + b*x))*sqrt(a) + b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

```
[In] int(1/(a + b/x)^(1/2),x)
```

```
[Out] (2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*
1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/
2))
```

3.249 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{ac}-\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

```
[Out] -(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^2-2*d^(3/2)*arctan(d^
(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(1/2)+x*(a+b/x)^(1/2)/
a/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 105, 162, 65, 214, 211}

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc)}{a^{3/2}c^2}-\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{ac} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+2ad)+\frac{bdx}{2}}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2} + \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{abc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^2}$$

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2))/c^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(90) = 180$.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.11

method	result
default	$-\frac{\left(2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,acd+\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,b\,c^2-2\sqrt{x(ax+b)}\,c^2\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^2}}+2a^{\frac{3}{2}}\ln\cdots\right.}{2a^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^3\sqrt{x(ax+b)}}$
risch	$\frac{ax+b}{ca\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2a\,d^2\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+(a\cdots}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\cdots\right.}{2cax\sqrt{\frac{ax+b}{x}}}$

```
[In] int(1/(c+d/x)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*a*c*d+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*
d-b*c)*d/c^2)^(1/2)*b*c^2-2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*((a*d-b*c)*d/c^2)
^(1/2)+2*a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+
b*c*x-b*d)/(c*x+d))*d^2)*x*((a*x+b)/x)^(1/2)/a^(3/2)/((a*d-b*c)*d/c^2)^(1/2
)/c^3/(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

$$= \left[\frac{2\,a^2d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,acx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\cdots\right.}{2\,a^2c^2}\right.$$

$$-\frac{4\,a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-2\,acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}\cdots\right.}{2\,a^2c^2}$$

$$\left.-\frac{2\,a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2c^2}\right]$$

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d
))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((a*
x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + a*c*x
*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/
x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
)/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*
c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

```
[In] integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 1183, normalized size of antiderivative = 10.95

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx=\frac{x\sqrt{a+\frac{b}{x}}}{a\,c}$$

$$-\frac{\operatorname{atanh}\left(\frac{12\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{12\,b^4\,d^4}{a}+\frac{10\,b^5\,c\,d^3}{a^2}+\frac{2\,b^6\,c^2\,d^2}{a^3}\right)}+\frac{10\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{10\,b^5\,d^3}{a}+\frac{12\,b^4\,d^4}{c}+\frac{2\,b^6\,c\,d^2}{a^2}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{2\,b^6\,d^2}{a}+\frac{10\,b^5\,d^3}{c}+\frac{12\,a\,b^4\,d^4}{c^2}\right)}\right)\,(2\,a\,d+}{c^2\,\sqrt{a^3}}$$

$$-\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(\frac{2\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4}{a^2\,c^2}\right)}{b\,c^3-a\,c^2\,d}}{\frac{\left(\frac{\left(\frac{2\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3}{a^2\,c^2}\right)}{b\,c^3-a\,c^2\,d}}\right)}{}$$

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)),x)
```

```
[Out] (x*(a + b/x)^(1/2))/(a*c) - (atan(((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*
d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2
)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(
1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 +
 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d) -
 (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^
5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^
2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a +
b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4
 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b
^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/
x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c
*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^
2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*
d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b^4*c^5*d^2 + 2*
a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a
 + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4
- b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b
^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2*d) - (atanh((12*b^4*d^
4*(a + b/x)^(1/2))/((a^3)^(1/2)*((12*b^4*d^4)/a + (10*b^5*c*d^3)/a^2 + (2*b
^6*c^2*d^2)/a^3)) + (10*b^5*d^3*(a + b/x)^(1/2))/((a^3)^(1/2)*((10*b^5*d^3)
```

```
/a + (12*b^4*d^4)/c + (2*b^6*c*d^2)/a^2)) + (2*b^6*d^2*(a + b/x)^(1/2))/((a
^3)^(1/2)*((2*b^6*d^2)/a + (10*b^5*d^3)/c + (12*a*b^4*d^4)/c^2)))*(2*a*d +
b*c))/(c^2*(a^3)^(1/2))
```

3.250 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}$$

```
[Out] -d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/
(-a*d+b*c)^(3/2)-(4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^3+d*(
-2*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d
/x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{382, 105, 156, 162, 65, 214, 211\}$

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+bc)}{a^{3/2}c^3} - \frac{d^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+4ad)+\frac{3bdx}{2}}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(bc+4ad)-\frac{1}{2}bd(bc-2ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac^2(bc-ad)}\\
&= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{(d^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)}\\
&\quad + \frac{(bc+4ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} \\
&\quad - \frac{(d^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)} \\
&\quad + \frac{(bc+4ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{abc^3} \\
&= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} \\
&\quad - \frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$
\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(-bc(d+cx)+ad(2d+cx))}{a(-bc+ad)(d+cx)} + \frac{d^{3/2}(-5bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^3}
$$

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(d + c*x)) + a*d*(2*d + c*x)))/(a*(-(b*c) + a*d)
*(d + c*x)) + (d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/a^(3/2))/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(152) = 304$.

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.73

method	result
risch	$\frac{ax+b}{a\,c^2\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(4ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2a\,d^3\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)}-\frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad}{}}{}\right.}{}\right.}{c^3}\right.}{}$
default	Expression too large to display

```
[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a*((4*a*d+b*c)/c*ln((1/2*b+a*x)/a
^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+2*a*d^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(
a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/
(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+
d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)
*d/c^2)^(1/2))/(x+d/c)))+6*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c
)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d
-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^(1/2)*(x*(a
*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 1163, normalized size of antiderivative = 6.76

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*
b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))
*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*
c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*
c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^
```

```
2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a
*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3
 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*
sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x),
 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(
2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*
x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*
x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/
(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*
d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c
^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*
b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\,(cx+d)^2}\,dx$$

```
[In] integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
[Out] Integral(x**2/(sqrt(a + b/x)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(152) = 304$.

Time = 0.32 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\left(10\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-8\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-\sqrt{bcd-ad^2}b^2c^2\log\left(|b|\right)-3\sqrt{bcd-ad^2}abcd\log\right.}{2\left(\sqrt{bcd-ad^2}a^{\frac{3}{2}}bc^4-\sqrt{bcd-ad^2}a^{\frac{5}{2}}c^3d\right)}$$
$$+\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\sqrt{bcd-ad^2}}$$
$$+\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd^2-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^3-\sqrt{a}bd^3}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c+2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d+bd\right)}$$
$$+\frac{\sqrt{ax^2+bx}}{ac^2\mathrm{sgn}\left(x\right)}+\frac{\left(bc+4\,ad\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^3\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(10*a^(3/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(5/2)*d
^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b^2*c^2*log(
abs(b)) - 3*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) + 4*sqrt(b*c*d - a*d^2)
*a^2*d^2*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a^2*d^2)*sgn(x)/(sqrt(b*c*d -
a*d^2)*a^(3/2)*b*c^4 - sqrt(b*c*d - a*d^2)*a^(5/2)*c^3*d) + (5*b*c*d^2 - 4*
a*d^3)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d -
 a*d^2))/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*sqrt(b*c*d - a*d^2)) + ((sqrt(a)*
x - sqrt(a*x^2 + b*x))*b*c*d^2 - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*d^3 -
sqrt(a)*b*d^3)/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 +
b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)) + sqrt(a*x^
2 + b*x)/(a*c^2*sgn(x)) + 1/2*(b*c + 4*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x
^2 + b*x))*sqrt(a) + b))/(a^(3/2)*c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 3813, normalized size of antiderivative = 22.17

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)
```

```
[Out] (((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b*
c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a + b/
x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x)^(
1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 2
6*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^
6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^
2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(
4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6
*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*
d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/2)) + (((2
*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b
^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)
 - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*
c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*
a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8
*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^
5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(
1/2)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*d^5 - 48*a^2*b^4*c
*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((2*(a + b/x)^(1/2)*(
32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*
b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^6*c^9*
d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*
c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*
b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))
/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*
c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)) + (((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - (((4*a
*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/
(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*a*d + b*c
)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*
c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4
*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2))))*(4*a
*d + b*c)*1i)/(c^3*(a^3)^(1/2)) - (atan((((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d
 - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^
4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
```

a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))*(d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*1i)/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)

3.251 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3} dx = \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} - \frac{(bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}$$

```
[Out] -1/4*d^(3/2)*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2
)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(5/2)-(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(3/2)/c^4+1/2*d*(-3*a*d+2*b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c
+d/x)^2+1/4*d*(-4*a*d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^(1/2)/a/c^3/(-a*d+b*c)^2/
(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d/x)^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 156, 162, 65, 214, 211}

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)}{a^{3/2}c^4}-\frac{d^{3/2}(24a^2d^2-56abcd+35b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+6ad)+\frac{5bdx}{2}}{x\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac} \\
&= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-((bc-ad)(bc+6ad))-\frac{3}{2}bd(2bc-3ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}
\end{aligned}
$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{(bc-ad)^2(bc+6ad)+\frac{1}{4}bd(bc-4ad)(4bc-3ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2ac^3(bc-ad)^2}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^4} - \frac{(d^2(35b^2c^2-56abcd+24a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{8c^4(bc-ad)^2}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{abc^4} - \frac{(d^2(35b^2c^2-56abcd+24a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4bc^4(bc-ad)^2}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} - \frac{(bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

$$= \frac{\frac{c\sqrt{a+\frac{b}{x}}x(4b^2c^2(d+cx)^2+2a^2d^2(6d^2+9cdx+2c^2x^2)-abcd(19d^2+29cdx+8c^2x^2))}{a(bc-ad)^2(d+cx)^2} - \frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \ldots}{4c^4}$$

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

[Out] ((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x + 2
*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/(a*(b*c - a*d)^2*(d +
 c*x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(4*c^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(222) = 444$.

Time = 0.33 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.81

method	result
risch	$\frac{ax+b}{a\,c^3\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(6ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{8a\,d^3\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\frac{(ad-bc)d}{c^2}}}{(ad-bc)d\left(x+\frac{d}{c}\right)} - \frac{(2ad-bc)c\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-}{}}{}\right.}{}\right.}{c^3}\right.}{}$
default	Expression too large to display

[In] int(1/(c+d/x)^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/a/c^3*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^3/a*((6*a*d+b*c)/c*ln((1/2*b+a*x)/a
^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+8*a*d^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(
a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/
(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+
d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)
*d/c^2)^(1/2))/(x+d/c)))-2*a*d^4/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+d/c)^2*(a*(x+
d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-

```
b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-
b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln(
(2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/
c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))+1/2*a/(a*d-b*c
)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)
+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c
^2)^(1/2))/(x+d/c)))+12*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d
/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*
c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+
b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(222) = 444$.

Time = 0.60 (sec) , antiderivative size = 2307, normalized size of antiderivative = 9.23

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*
c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d
+ 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 +
 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 +
2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2
)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*
c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c
^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(
8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*
b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (
35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c
^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^
2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^
2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*
b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 +
```

```
 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^
2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2
+ 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c
*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*
c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2
*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4
)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5
 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 1
8*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)
*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2
*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^
6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^
4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*
a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*a
rctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) -
 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a
*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*
sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (
8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 -
 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 -
2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(222) = 444$.

Time = 0.38 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\left(35\,a^{\frac{3}{2}}b^2c^2d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-56\,a^{\frac{5}{2}}bcd^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)+24\,a^{\frac{7}{2}}d^4\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)-2\sqrt{bcd-a\cdots}\right.}{4\left(\sqrt{b\cdots}\right.}$$
$$+\frac{(35\,b^2c^2d^2-56\,abcd^3+24\,a^2d^4)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{4\,(b^2c^6\mathrm{sgn}(x)-2\,abc^5d\mathrm{sgn}(x)+a^2c^4d^2\mathrm{sgn}(x))\sqrt{bcd-ad^2}}$$
$$+\frac{13\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3b^2c^3d^2-40\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3abc^2d^3+24\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^2cd^4+7\left(\cdots\right.}{\cdots}$$
$$+\frac{\sqrt{ax^2+bx}}{ac^3\mathrm{sgn}(x)}+\frac{(bc+6\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^4\mathrm{sgn}(x)}$$

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(35*a^(3/2)*b^2*c^2*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 56*a^(5
/2)*b*c*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(7/2)*d^4*arctan(s
qrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^3*c^3*log(abs(b)) -
 8*sqrt(b*c*d - a*d^2)*a*b^2*c^2*d*log(abs(b)) + 22*sqrt(b*c*d - a*d^2)*a^2
*b*c*d^2*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^3*log(abs(b)) + 13*sqrt
(b*c*d - a*d^2)*a^2*b*c*d^2 - 10*sqrt(b*c*d - a*d^2)*a^3*d^3)*sgn(x)/(sqrt(
b*c*d - a*d^2)*a^(3/2)*b^2*c^6 - 2*sqrt(b*c*d - a*d^2)*a^(5/2)*b*c^5*d + sq
rt(b*c*d - a*d^2)*a^(7/2)*c^4*d^2) + 1/4*(35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 2
4*a^2*d^4)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c
*d - a*d^2))/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*sgn(x))*sq
rt(b*c*d - a*d^2)) + 1/4*(13*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d^2
- 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^3 + 24*(sqrt(a)*x - sqrt(a
*x^2 + b*x))^3*a^2*c*d^4 + 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*
c^2*d^3 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^4 + 40*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^5 + 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))
*b^3*c^2*d^3 - 60*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d^4 + 40*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*a^2*b*d^5 - 13*sqrt(a)*b^3*c*d^4 + 10*a^(3/2)*b^2*d
^5)/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*sgn(x))*((sqrt(a)*x
 - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b
*d)^2) + sqrt(a*x^2 + b*x)/(a*c^3*sgn(x)) + 1/2*(b*c + 6*a*d)*log(abs(2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(3/2)*c^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 2890, normalized size of antiderivative = 11.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
[Out] (log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a*
b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b^2
*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3
*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(12*a^
2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a + b/x)
^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(4*a
*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 56*a^2*b
^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((a + b/x)
^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*
(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d - b*c)^5)
^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*b*c*d))/(8*(
b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 - 10*a^3*b^2*c
^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16
*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^
5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^
6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^
8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4
*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)
*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^
11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7)
)/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5
*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*
d + b*c)*1i)/(2*c^4*(a^3)^(1/2)) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 1
6*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d
^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c
^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^
5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 -
4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2
)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c
^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7
))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^
5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a
*d + b*c)*1i)/(2*c^4*(a^3)^(1/2)))/((216*a^5*b^3*d^9 + (35*b^8*c^5*d^4)/2 -
```

(49*a*b^7*c^4*d^5)/8 - 810*a^4*b^4*c*d^8 - (1877*a^2*b^6*c^3*d^6)/4 + 1044*a^3*b^5*c^2*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)) - ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2))))*(6*a*d + b*c)*1i)/(c^4*(a^3)^(1/2))

3.252 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*c^2*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+((-2*a*d+b*c)*(2
*a^2*d^2-2*a*b*c*d+3*b^2*c^2)-a*b*d^2*(2*a*d+b*c)/x)/a^2/b^2/(a+b/x)^(1/2)+
c*(c+d/x)^2*x/a/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 100, 151, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{3c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{5/2}} + \frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

```
[In] Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}\\
&\quad + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}}\\
&\quad + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}\\
&= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}\\
&\quad + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\left(3b^3c^3x-4a^3d^3x-2a^2bd^2(d-3cx)+ab^2c^2x(-6d+cx)\right)}{a^2b^2(b+ax)} + \frac{3c^2(-bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*b^3*c^3*x - 4*a^3*d^3*x - 2*a^2*b*d^2*(d - 3*c*x) + a*b^2
*c^2*x*(-6*d + c*x)))/(a^2*b^2*(b + a*x)) + (3*c^2*(-(b*c) + 2*a*d)*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{(ax+b)(-b^2x\,c^3+2a^2d^3)}{b^2a^2x\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b^2c^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+6\sqrt{a}\,b\,c^2d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{2\left(-2a^3d^3+6a^2bc\,d^2-6a\,b^2c^2d+2\right.}{ab\left(x+\right.}\right.}{2a^2bx\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^4c^3x^4+6a^{\frac{9}{2}}\sqrt{x(ax+b)}\,bc\,d^2x^4-12a^{\frac{7}{2}}\sqrt{x(ax+b)}\,b^2c^2d\,x^4-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right.}{}$

```
[In] int((c+d/x)^3/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x+b)*(-b^2*c^3*x+2*a^2*d^3)/b^2/a^2/x/((a*x+b)/x)^(1/2)+1/2/a^2/b*(-3*b
^2*c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+6*a^(1/2)*b*c^2*d*
ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+2*(-2*a^3*d^3+6*a^2*b*c*d^2-6*a*b
^2*c^2*d+2*b^3*c^3)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x
)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.55

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\left[-\frac{3\left(b^4c^3-2\,ab^3c^2d+\left(ab^3c^3-2\,a^2b^2c^2d\right)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2b^2\right.}{2\left(a^4b^2x+a^3b^3\right)}\right.$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(a
)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a
^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*s
qrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b
^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a)
+ (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d/x)**3/(a+b/x)**(3/2),x)
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.52

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}c^3\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - 3c^2d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right) - 2d^3\left(\frac{\sqrt{a+\frac{b}{x}}}{b^2}+\frac{a}{\sqrt{a+\frac{b}{x}}b^2}\right)+\frac{6cd^2}{\sqrt{a+\frac{b}{x}}b}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
[Out] 1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^2))
+ 6*c*d^2/(sqrt(a + b/x)*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}-a\,b^2\,\sqrt{a+\frac{b}{x}}} - \frac{2\,d^3\,\sqrt{a+\frac{b}{x}}}{b^2}+\frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(2\,a\,d-b\,c\right)}{a^{5/2}}$$

```
[In] int((c + d/x)^3/(a + b/x)^(3/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2*
a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)^(
3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*atanh(
(a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

3.253 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -c*(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(2*a^2*d^2+b*c*(-4
*a*d+3*b*c))/a^2/b/(a+b/x)^(1/2)+c^2*x/a/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 91, 79, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)}{a^{5/2}}+\frac{\frac{3bc^2}{a}+\frac{2ad^2}{b}-4cd}{a\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

```
[In] Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((3*b*c^2)/a - 4*c*d + (2*a*d^2)/b)/(a*Sqrt[a + b/x]) + (c^2*x)/(a*Sqrt[a +
 b/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(3bc-4ad)+ad^2x}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{3bc^2}{a} - 4cd + \frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{3bc^2}{a} - 4cd + \frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{\frac{3bc^2}{a} - 4cd + \frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(3b^2c^2+2a^2d^2+abc(-4d+cx))}{a^2b(b+ax)} + \frac{c(-3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x)))/(a^2*b*(b +
a*x)) + (c*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(84) = 168.

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.94

method	result
risch	$\frac{c^2(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}} + \frac{\left(-\frac{3bc^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)}{\sqrt{a}} + 4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right) + \frac{2\left(2a^2d^2-4abcd+2b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-4\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab^4cd-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b^2c^2x^2+16\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b^2cdx-8(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}bcd-8\ln\left(\frac{2\sqrt{x(a}}{}\right.\right.}{}$

```
[In] int((c+d/x)^2/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] c^2/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(-3*b*c^2*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)
^(1/2))+2*(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a
))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.89

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^3c^2-4\,ab^2cd+(3\,ab^2c^2-4\,a^2bcd)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2bc^2}{2\,(a^4bx+a^3b^2)}\right.$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*lo
g(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^2*
c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2), (
(3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d
+ 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}\,c^2\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - 2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)+\frac{2\,d^2}{\sqrt{a+\frac{b}{x}}b}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(84) = 168.

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^2\mathrm{sgn}\,(x)} + \frac{(3\,bc^2-4\,acd)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\,(x)} - \frac{(3\,b^2c^2\log\left(|b|\right)-4\,abcd\log\left(|b|\right)+4\,b^2c^2-8\,abcd+4\,a^2d^2)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}b} + \frac{2\left(\sqrt{a}b^2c^2-2\,a^{\frac{3}{2}}bcd+a^{\frac{5}{2}}d^2\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}\,(x)}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^2/(a^2*sgn(x)) + 1/2*(3*b*c^2 - 4*a*c*d)*log(abs(2*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) - 1/2*(3*b^2*c^2
*log(abs(b)) - 4*a*b*c*d*log(abs(b)) + 4*b^2*c^2 - 8*a*b*c*d + 4*a^2*d^2)*s
gn(x)/(a^(5/2)*b) + 2*(sqrt(a)*b^2*c^2 - 2*a^(3/2)*b*c*d + a^(5/2)*d^2)/(((
sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-3\,b\,c)}{a^{5/2}} - \frac{\frac{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{a}-\frac{\left(a+\frac{b}{x}\right)\,(2\,a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2)}{a^2}}{b\,\left(a+\frac{b}{x}\right)^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

```
[In] int((c + d/x)^2/(a + b/x)^(3/2),x)
```

```
[Out] (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d))/
a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

3.254 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}} dx = \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{(3bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(-2*a*d+3*b*c)/a^2/(
a+b/x)^(1/2)+c*x/a/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 79, 53, 65, 214}

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-2ad)}{a^{5/2}} + \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

```
[In] Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{\left(-\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}+\frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(3bc-2ad+acx)}{a^2(b+ax)}+\frac{(-3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(66) = 132.

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.09

method	result
risch	$\frac{c(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{3bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}-\frac{4(ad-bc)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4a^{\frac{7}{2}}\sqrt{x(ax+b)}\,d\,x^2-6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bc\,x^2-4a^{\frac{5}{2}}(x(ax+b))^{\frac{3}{2}}d+8a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bdx+4a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}bc-12a^{\frac{3}{2}}\sqrt{x(ax+b}\right.}{}$

```
[In] int((c+d/x)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] c/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))-3*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)
-4*(a*d-b*c)/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.76

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^2c-2\,abd+(3\,abc-2\,a^2d)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2cx^2+(3\,a}{2\,(a^4x+a^3b)}\right.$$

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sqrt
((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c - 2
*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(65) = 130$.

Time = 14.55 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right) + d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^2b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

```
[In] integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/
b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqrt(
1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x +
a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
 - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/
(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(66) = 132.

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}\,c\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)$$

[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")

[Out] 1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) - d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(sqrt(a + b/x)*a))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(66) = 132.

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.96

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,bc\log(|b|)-2\,ad\log(|b|)+4\,bc-4\,ad)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{\sqrt{ax^2+bx}c}{a^2\mathrm{sgn}(x)} + \frac{(3\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)} + \frac{2\left(\sqrt{a}b^2c-a^{\frac{3}{2}}bd\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}(x)}$$

[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")

[Out] -1/2*(3*b*c*log(abs(b)) - 2*a*d*log(abs(b)) + 4*b*c - 4*a*d)*sgn(x)/a^(5/2) + sqrt(a*x^2 + b*x)*c/(a^2*sgn(x)) + 1/2*(3*b*c - 2*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + 2*(sqrt(a)*b^2*c - a^(3/2)*b*d)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x))

Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}}+\frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] int((c + d/x)/(a + b/x)^(3/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
(2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/
x)^(3/2))
```

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx = \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}} - \frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/
x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {248, 44, 53, 65, 214}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}} dx = -\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}}$$

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (3*b)/(a^2*Sqrt[a + b/x]) + x/(a*Sqrt[a + b/x]) - (3*b*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x}{a\sqrt{a+\frac{b}{x}}} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}
\end{aligned}
$$

$$
= \frac{3b}{a^2\sqrt{a+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{b}{x}}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$
\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(3b+ax)}{a^2(b+ax)} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
$$

```
[In] Integrate[(a + b/x)^(-3/2),x]
[Out] (Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/a^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

method	result
risch	$\frac{ax+b}{a^2\sqrt{\frac{ax+b}{x}}} + \frac{\left(-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{2a^{\frac{5}{2}}} + \frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,x^2-4a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}+12a^{\frac{3}{2}}\sqrt{x(ax+b)}\,bx-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^2-6\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right.}{2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,(ax+b)^2}$

```
[In] int(1/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/2*b/a^(5/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x
^2+b*x)^(1/2))+2*b/a^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)
^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{3\,(abx+b^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(a^2x^2+3\,abx)\sqrt{\frac{ax+b}{x}}}{2\,(a^4x+a^3b)}, \frac{3\,(abx+b^2)\sqrt{}}{}\right.$$

[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")

[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

[In] integrate(1/(a+b/x)**(3/2),x)

[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")

[Out] (3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,b\log\left(|b|\right)+4\,b)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a^2\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*log(abs(b)) + 4*b)*sgn(x)/a^(5/2) + 3/2*b*log(abs(2*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a^2*
sgn(x)) + 2*b^2/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^(5/2)*sgn(
x))
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,x\left(\frac{a\,x}{b}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] int(1/(a + b/x)^(3/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)
^(3/2))
```

3.256 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=\frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}+\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}-\frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

```
[Out] 2*d^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(3/
2)-(2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^2+b*(-a*d+3*b*c)/
a^2/c/(-a*d+b*c)/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+3bc)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc-ad)}+\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad)+\frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad)+\frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{a^2c(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)} + \frac{(3bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2c^2}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2(bc-ad)} + \frac{(3bc+2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2bc^2}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(-3b^2c+a^2dx+ab(d-cx))}{a^2(-bc+ad)(b+ax)}+\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{c^2}$$

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/(a^2*(-(b*c) + a*
d)*(b + a*x)) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/(b*c - a*d)^(3/2) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/
2))/c^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(127) = 254.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.02

method	result
risch	$\frac{ax+b}{a^2c\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{4c\,b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{(ad-bc)a\left(x+\frac{b}{a}\right)}+\frac{2a^2d^3\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(\right.}}{x+\frac{d}{c}}\right.}{c^2(ad-bc)\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2a^2cx\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\left(2a^{\frac{9}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+bcx-bd}{cx+d}\right)d^3x^2-2a^{\frac{7}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c^2d\,x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{x(ax+b)}\sqrt{\frac{(ad-bc)d}{c^2}}\,c-2adx+\right.}{cx+d}\right.\right.}{}$

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^2/c*((2*a*d+3*b*c)/c*ln((1/2*b+a*x)
/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+4*c*b^2/(a*d-b*c)/a/(x+b/a)*(a*(x+b/a)^
2-b*(x+b/a))^(1/2)+2/c^2*a^2*d^3/(a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a
*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-
(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^(1/2)
*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(127) = 254$.

Time = 0.35 (sec) , antiderivative size = 1075, normalized size of antiderivative = 7.31

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \left[\frac{(3\,b^3c^2 - ab^2cd - 2\,a^2bd^2 + (3\,ab^2c^2 - a^2bcd - 2\,a^3d^2)x)\sqrt{a}\log\left(2\,ax - 2\,\sqrt{a}x\right.}{}\right.$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a
^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^4*
d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b*c
^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^
2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d -
 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*l
og(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*
d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*
d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*
d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*
b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2
 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2
*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b*c^2 - a
^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3
- a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)}\,dx$$

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 3000, normalized size of antiderivative = 20.41

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)),x)
[Out] (atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 6
6*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c
^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) +
 ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 13
2*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b
^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^
2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 +
200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*
d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b
*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 6
8*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c
^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)
^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 -
```

128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a
*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*
d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6
- 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a
*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^5 - 96*a^7*b^7*c^6
*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10*b^4*c^3*d^9 + 8*a^
11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c
^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62
*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*
c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^
12*d^2 - 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6
+ 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*
b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*
c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8
))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3) + ((d^5*
(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9
*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^
11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d -
b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^
10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 +
((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b
^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*
c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))
/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))*(d^5*(a*d - b*c)^3)^(1/2)*2i)/
(c^2*(a*d - b*c)^3) - (atanh((54*a^5*b^11*c^10*d^2*(a + b/x)^(1/2))/((a^5)^
(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 +
124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b
^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (216*a^6*b^10*c^9*d^3*(a + b/x)^(1/2))
/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^
8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 1
10*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (234*a^7*b^9*c^8*d^4*(a + b/x)
^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5
*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*
d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (124*a^8*b^8*c^7*d^5*(a
+ b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 +
234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b
^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (366*a^9*b^7*c^6
d^6(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9
*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 12
0*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (120*a^10
*b^6*c^5*d^7*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*
b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*
d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (
110*a^11*b^5*c^4*d^8*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 -
216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*

```
b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d
^9)) - (60*a^12*b^4*c^3*d^9*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10
*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 3
66*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^
4*c^3*d^9)))*(2*a*d + 3*b*c))/(c^2*(a^5)^(1/2)) - ((2*b^2)/(a^2*d - a*b*c)
+ (b*(a + b/x)*(a*d - 3*b*c))/(a^2*c*(a*d - b*c)))/(a*(a + b/x)^(1/2) - (a
+ b/x)^(3/2))
```

3.257 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}-\frac{(3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}$$

```
[Out] d^(5/2)*(-4*a*d+7*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/(
-a*d+b*c)^(5/2)-(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^3+b*
(2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(1/2)+d*(-2*a*
d+b*c)/a/c^2/(-a*d+b*c)/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+3bc)}{a^{5/2}c^3}+\frac{b(2a^2d^2-2abcd+3b^2c^2)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{\text{Subst}\left(\int\frac{\frac{1}{2}(3bc+4ad)+\frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2}\,dx,x,\frac{1}{x}\right)}{ac}$$

$$=\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}-\frac{\text{Subst}\left(\int\frac{-\frac{1}{2}(bc-ad)(3bc+4ad)-\frac{3}{2}bd(bc-2ad)x}{x(a+bx)^{3/2}(c+dx)}\,dx,x,\frac{1}{x}\right)}{ac^2(bc-ad)}$$

$$=\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}-\frac{2\text{Subst}\left(\int\frac{-\frac{1}{4}(bc-ad)^2(3bc+4ad)-\frac{1}{4}bd(3b^2c^2-2abcd+2a^2d^2)x}{x\sqrt{a+bx}(c+dx)}\,dx,x,\frac{1}{x}\right)}{a^2c^2(bc-ad)^2}$$

$$=\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{(d^3(7bc-4ad))\,\text{Subst}\left(\int\frac{1}{\sqrt{a+bx}(c+dx)}\,dx,x,\frac{1}{x}\right)}{2c^3(bc-ad)^2}+\frac{(3bc+4ad)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{x}\right)}{2a^2c^3}$$

$$=\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{(d^3(7bc-4ad))\,\text{Subst}\left(\int\frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)^2}+\frac{(3bc+4ad)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{a^2bc^3}$$

$$=\frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}-\frac{(3bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(3b^3c^2(d+cx)+a^3d^2x(2d+cx)+a^2bd\left(2d^2-cdx-2c^2x^2\right)+ab^2c\left(-2d^2-cdx+c^2x^2\right)\right)}{a^2(bc-ad)^2(b+ax)(d+cx)}+\frac{d^{5/2}(7bc-4a\cdots}{\cdots}}{c^3}$$

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(3*b^3*c^2*(d + c*x) + a^3*d^2*x*(2*d + c*x) + a^2*b*d*
(2*d^2 - c*d*x - 2*c^2*x^2) + a*b^2*c*(-2*d^2 - c*d*x + c^2*x^2)))/(a^2*(b*
c - a*d)^2*(b + a*x)*(d + c*x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(202) = 404.

Time = 0.38 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.38

method	result
risch	$\frac{ax+b}{a^2c^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^3}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^2}+\frac{2b^3\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3(ad-bc)^2\left(x+\frac{b}{a}\right)}+\frac{d^3\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\cdots}}{c^3(ad-bc)^2\left(x+\frac{d}{c}\right)}\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/c^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-2/a^(3/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/
2))*b+2/a^3*b^3/(a*d-b*c)^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+1/c^3*d^3
/(a*d-b*c)^2/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1
/2)-2*a/c^4*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(
2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(
x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+7/2/c^3*d^3/(a*d-b*c)^2/((a*d-b*c)*
d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2
)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))
*b)/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(202) = 404.

Time = 0.62 (sec) , antiderivative size = 2321, normalized size of antiderivative = 10.36

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a
*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^
4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*
d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*
d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
 - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^
2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a
^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^
3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 -
 a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 -
 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3
*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt
(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c
^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5
*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^
2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*
d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d
 - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 +
(3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^
4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(
-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 +
 (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 -
 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*
c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c
*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^
```

```
4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^
3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a
*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*
a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*
d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3
*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((
a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3
*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)^2}\,dx$$

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
[Out] Integral(x**2/((a + b/x)**(3/2)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 4274, normalized size of antiderivative = 19.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
[Out] ((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b*
c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3*b
^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/x)^
(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^11*c
^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i - a^1
0*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*1694i
- a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x)^(1/2)
*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*(a + b/x
)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*c^4*d^10*
(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i + a^11*b^13*
c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^7*c^7*d^7 -
 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d^10 + 140*a^
4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 1694*a^2*b^10*c^
10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^3*b^14*c^14 +
189*a^4*b^13*c^13*d)))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2)) - (atan((((d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^
3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 -
 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 -
3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12
+ 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) -
((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*
b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^1
2*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15
*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b
^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^
(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a
^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376
*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280
*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*
a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b
^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d
^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^
7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2
 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d
 - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 +
362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 10
```

04*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 59
60*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 5
76*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d - b*c)^5)^(1/2)*(
4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^1
2*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*
b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6
*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^
10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^1
0*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^1
3*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^
16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^
19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5
*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*
d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^
2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))*1i)/(2*(b^5*c^8 - a^5
*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*
b^4*c^7*d)))/(((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(
18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*
a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a
^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768
*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^
18*b^2*c^6*d^15) - ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*
c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*
c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*
c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^
12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^
5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^
12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*
b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*
b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*
b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10
*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) - ((d^5*(a*d - b*c)
^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*
b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^
10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^
7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*
b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d -
b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^
3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*
d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^
9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12
+ 8*a^19*b^3*c^10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d

 - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^2
1*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^
18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^
15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^1
2*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 -
 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d)) - 126*a^6*b^13*c^14*d^5 + 744*a^7*b^12*c^13*d^6 -
1742*a^8*b^11*c^12*d^7 + 1756*a^9*b^10*c^11*d^8 + 322*a^10*b^9*c^10*d^9 - 3
248*a^11*b^8*c^9*d^10 + 4606*a^12*b^7*c^8*d^11 - 3668*a^13*b^6*c^7*d^12 + 1
804*a^14*b^5*c^6*d^13 - 512*a^15*b^4*c^5*d^14 + 64*a^16*b^3*c^4*d^15))*(d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*1i)/(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*
c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)

3.258 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{3d^{5/2}(21b^2c^2-24abcd+8a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} - \frac{3(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}$$

```
[Out] 3/4*d^(5/2)*(8*a^2*d^2-24*a*b*c*d+21*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/
(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(7/2)-3*(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(5/2)/c^4+3/4*b*(-a*d+2*b*c)*(4*a^2*d^2-a*b*c*d+2*b^2*c^2)/a^2/c
^3/(-a*d+b*c)^3/(a+b/x)^(1/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(c+d/x)
^2/(a+b/x)^(1/2)+1/4*d*(12*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2
/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)^2/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {382, 105, 156, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc)}{a^{5/2}c^4} + \frac{3d^{5/2}(8a^2d^2-24abcd+21b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} + \frac{d(12a^2d^2-21abcd+4b^2c^2)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3} + \frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+2ad)+\frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-3(bc-ad)(bc+2ad)-\frac{5}{2}bd(2bc-3ad)x}{x(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{3(bc-ad)^2(bc+2ad)+\frac{3}{4}bd(4b^2c^2-21abcd+12a^2d^2)x}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2ac^3(bc-ad)^2}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc-ad)^3(bc+2ad)+\frac{3}{8}bd(2bc-ad)(2b^2c^2-abcd+4a^2d^2)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{a^2c^3(bc-ad)^3}$$

$$= \frac{3b(2bc-ad)(2b^2c^2-abcd+4a^2d^2)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{(3(bc+2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2c^4} + \frac{(3d^3(21b^2c^2-24abcd+8a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{8c^4(bc-ad)^3}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{(3(bc+2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2bc^4} + \frac{(3d^3(21b^2c^2-24abcd+8a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4bc^4(bc-ad)^3}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-21abcd+12a^2d^2)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{3d^{5/2}(21b^2c^2-24abcd+8a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} - \frac{3(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{c\sqrt{a+\frac{b}{x}}x(-12b^4c^3(d+cx)^2-4ab^3c^2(-3d+cx)(d+cx)^2+2a^4d^3x(6d^2+9cdx+2c^2x^2)+a^3bd^2(12d^3-9cd^2x-37}{a^2(-bc+ad)^3(b+ax)(d+cx)^2}$$

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-12*b^4*c^3*(d + c*x)^2 - 4*a*b^3*c^2*(-3*d + c*x)*(d
+ c*x)^2 + 2*a^4*d^3*x*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + a^3*b*d^2*(12*d^3 -
9*c*d^2*x - 37*c^2*d*x^2 - 12*c^3*x^3) + a^2*b^2*c*d*(-27*d^3 - 29*c*d^2*x
+ 12*c^2*d*x^2 + 12*c^3*x^3)))/(a^2*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x)^2)
 + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a
+ b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (12*(b*c + 2*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/a^(5/2))/(4*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. 2(288) = 576.

Time = 0.35 (sec) , antiderivative size = 983, normalized size of antiderivative = 3.07

method	result
risch	$\frac{ax+b}{a^{2}c^{3}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)d}{a^{\frac{3}{2}}c^{4}}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)b}{2a^{\frac{5}{2}}c^{3}}-\frac{2b^{4}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{a^{3}(ad-bc)^{3}\left(x+\frac{b}{a}\right)}-\frac{d^{4}\sqrt{a\left(x+\frac{d}{c}\right)^{2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+(\cdots}}{2c^{5}(ad-bc)^{2}\left(x+\frac{d}{c}\right)^{2}}\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(3/2)/c^4*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/
2))*b-2/a^3*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^5*d
^4/(a*d-b*c)^2/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2
)^(1/2)+5/2*a/c^4*d^4/(a*d-b*c)^3/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c
)+(a*d-b*c)*d/c^2)^(1/2)-17/4/c^3*d^3/(a*d-b*c)^3/(x+d/c)*(a*(x+d/c)^2-(2*a
*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2/c^5*d^5/(a*d-b*c)^3/((a*
d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c
)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(
x+d/c))+19/2*a/c^4*d^4/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*
d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b
*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))*b-63/8/c^3*d^3/(a*d-b*c)^3/(
(a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-
b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)
)/(x+d/c))*b^2+1/2*a/c^5*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d
-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2
*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(288) = 576.

Time = 1.45 (sec) , antiderivative size = 4093, normalized size of antiderivative = 12.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5
 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c
^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 -
a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4
*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)
*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^3*b^3*
c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c
^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40
*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d
^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)
*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d
)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^
2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*
b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d
 - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2
 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a
^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) -
3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^
2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^
2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27
*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-
(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*
d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 -
a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 3
7*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^
2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c
^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*
x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b
*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*
x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 -
2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d
^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b
^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2
*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 1
6*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5
 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*
```

```
d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a
^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*
b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c
*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^
2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5
*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a
^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 +
 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*
d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x +
 b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^
4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3
 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a
^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3
+ a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*
c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^
4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a
^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 +
8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))
*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2
*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d -
 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^
4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d
^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(288) = 576$.

Time = 0.40 (sec) , antiderivative size = 1089, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
[Out] 2*sqrt(a)*b^5/((a^3*b^3*c^3*sgn(x) - 3*a^4*b^2*c^2*d*sgn(x) + 3*a^5*b*c*d^2
*sgn(x) - a^6*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)) -
1/4*(63*a^(5/2)*b^2*c^2*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 72*a^(7
/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(9/2)*d^5*arctan(s
qrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*b^4*c^4*log(abs(b)) -
 6*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 18*sqrt(b*c*d - a*d^2)*a^2
*b^2*c^2*d^2*log(abs(b)) + 30*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3*log(abs(b)) -
 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) + 8*sqrt(b*c*d - a*d^2)*b^4*c^4
 + 17*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3 - 10*sqrt(b*c*d - a*d^2)*a^4*d^4)*sgn
(x)/(sqrt(b*c*d - a*d^2)*a^(5/2)*b^3*c^7 - 3*sqrt(b*c*d - a*d^2)*a^(7/2)*b^
2*c^6*d + 3*sqrt(b*c*d - a*d^2)*a^(9/2)*b*c^5*d^2 - sqrt(b*c*d - a*d^2)*a^(
11/2)*c^4*d^3) - 3/4*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5)*arctan(-((
sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^3*c^
7*sgn(x) - 3*a*b^2*c^6*d*sgn(x) + 3*a^2*b*c^5*d^2*sgn(x) - a^3*c^4*d^3*sgn(
```

x))*sqrt(b*c*d - a*d^2)) - 1/4*(17*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d^3 - 48*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^4 + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^5 + 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d^4 - 72*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^5 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^6 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^4 - 76*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d^5 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^6 - 17*sqrt(a)*b^3*c*d^5 + 10*a^(3/2)*b^2*d^6)/((b^3*c^7*sgn(x) - 3*a*b^2*c^6*d*sgn(x) + 3*a^2*b*c^5*d^2*sgn(x) - a^3*c^4*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) + sqrt(a*x^2 + b*x)/(a^2*c^3*sgn(x)) + 3/2*(b*c + 2*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*c^4*sgn(x))

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 8936, normalized size of antiderivative = 27.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

[In] int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)

[Out] ((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)^2*(24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72*a^3*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a + b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d

- b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192
*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^
4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^1
5*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^
9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 10660249
6*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21
*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^
25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 +
7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*
d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 +
7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*
d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a
^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*
c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21
*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (((a + b/x)^(1/2)*(18432*a^6*b^19*c^2
6*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b
^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 127
6416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^1
1*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13
- 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a
^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^
18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d
- b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*
d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*
b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 +
10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^
11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 +
30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b
^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 122
88*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2
*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17
*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636
800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c
^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 14
6432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b
^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1
843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17
d^18))/(8(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 -
35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^1
0*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 -
35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^1
0*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/
(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*
b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))/(
290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^16*c^

19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 24731136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^14 - 343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216*a^18*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19 + 221184*a^21*b^3*c^6*d^20 - (3*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (3*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 1411

2768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(4*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (atan((((2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)) + ((2*a*d + b*c)*((a + b/x

)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^
17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 60076
80*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^1
9*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 52
1961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*
b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17
- 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^
2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c
^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472
*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24
*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616
*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19
*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*
a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 +
(3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^
17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 186
36800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12
*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 -
146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21
*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 +
1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^
17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)
))/(290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^1
6*c^19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 247
31136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*
b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^
14 - 343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216
*a^18*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19
+ 221184*a^21*b^3*c^6*d^20 - (3*(2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*
b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 17510
40*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d
^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*
a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^
16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164
257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4
*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2
*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a
^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6
- 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15
*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^1
1 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^
20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16
+ 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)
*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 11059
20*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^2

9*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 1347174
40*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*
c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26
091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c
^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a
^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)) + (3*(2*a*d + b*c)*(
(a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90316
8*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7
 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*
b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d
^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288096
00*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^
12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456
*a^23*b^2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^
9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 +
 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b
^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 +
45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*
b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 -
1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^1
4*d^18 + (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264
*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d
^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a
^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25
*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383
232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^2
0*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^2
6*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^
(1/2))))*(2*a*d + b*c)*3i)/(c^4*(a^5)^(1/2))

3.259 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-2a^2bd(3d+5cx)+ab^2c(-3d+20cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] c*(c+d/x)^2*x/a/(a+b/x)^(3/2)+1/3*(-a*d+b*c)*(15*b^3*c^2-4*a^3*d^2*x-2*a^2*
b*d*(5*c*x+3*d)+a*b^2*c*(20*c*x-3*d))/a^3/b^2/(a+b/x)^(3/2)/x-c^2*(-6*a*d+5
*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 100, 150, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-6ad)}{a^{7/2}} + \frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}} + \frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(5bc-6ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-6ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-6ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,x(15b^4c^3+4a^4d^3x+3a^2b^2c^2x(-8d+cx)+6a^3bd^2(d+cx)+2ab^3c^2(-9d+10cx}{3a^3b^2(b+ax)^2} + \frac{c^2(-5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^4*c^3 + 4*a^4*d^3*x + 3*a^2*b^2*c^2*x*(-8*d + c*x) +
 6*a^3*b*d^2*(d + c*x) + 2*a*b^3*c^2*(-9*d + 10*c*x)))/(3*a^3*b^2*(b + a*x)
^2) + (c^2*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(131) = 262$.

Time = 0.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.11

method	result
risch	$\frac{c^3(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5c^3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+6\sqrt{a}\,c^2d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{12c\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}+\frac{\left(2a^3d\right.}{}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	Expression too large to display

```
[In] int((c+d/x)^3/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(-5*c^3*b*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))/a^(1/2)+6*a^(1/2)*c^2*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))+12*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/
a))^(1/2)+(2*a^3*d^3-6*a^2*b*c*d^2+6*a*b^2*c^2*d-2*b^3*c^3)/a^2*(2/3/b/(x+b
/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a
))^(1/2)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.38

$$\int\frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\left[-\frac{3\left(5\,b^5c^3-6\,ab^4c^2d+\left(5\,a^2b^3c^3-6\,a^3b^2c^2d\right)x^2+2\left(5\,ab^4c^3-6\,a^2b^3c^2d\right)x\right)\sqrt{a}\log\left(2\,a\right.}{}\right.$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d
+ 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4),
 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
+ 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
 b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2*d + 3*a
^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2*a^4*b*d^
3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.59

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{1}{6}\,c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-c^2d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2}{3}\,d^3\left(\frac{3}{\sqrt{a+\frac{b}{x}}b^2}-\frac{a}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}\right)+\frac{2\,cd^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
[Out] 1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3*d
^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b/x)^(
3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(132) = 264.

Time = 0.32 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.00

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\sqrt{ax^2+bx}c^3}{a^3\mathrm{sgn}\left(x\right)}+\frac{\left(5\,bc^3-6\,ac^2d\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}-\frac{\left(15\,b^3c^3\log\left(|b|\right)-18\,ab^2c^2d\log\left(|b|\right)+28\,b^3c^3-48\,ab^2c^2d+12\,a^2bcd^2+8\,a^3d^3\right)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b^2}+\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^3-18\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bc^2d+9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3cd^2+15\left(\sqrt{a}x\right.\right.}{}$$

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^3/(a^3*sgn(x)) + 1/2*(5*b*c^3 - 6*a*c^2*d)*log(abs(2*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^3*
c^3*log(abs(b)) - 18*a*b^2*c^2*d*log(abs(b)) + 28*b^3*c^3 - 48*a*b^2*c^2*d
+ 12*a^2*b*c*d^2 + 8*a^3*d^3)*sgn(x)/(a^(7/2)*b^2) + 2/3*(9*(sqrt(a)*x - sq
rt(a*x^2 + b*x))^2*a*b^2*c^3 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c
^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*c*d^2 + 15*(sqrt(a)*x - sqrt
(a*x^2 + b*x))*sqrt(a)*b^3*c^3 - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)
*b^2*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*c*d^2 + 3*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3 + 7*b^4*c^3 - 12*a*b^3*c^2*d + 3*a^2*b^
2*c*d^2 + 2*a^3*b*d^3)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(
7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.36

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,a}+\frac{{\left(a+\frac{b}{x}\right)}^2\,\left(2\,a^3\,d^3-6\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{a^3}-\frac{2\,\left(a+\frac{b}{x}\right)\,\left(4\,a^3\,d^3-3\,a^2\,b\,c\,d^2-6\,a\,b^2\cdots\right)}{3\,a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{5/2}-a\,b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}}+\frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(6\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

```
[In] int((c + d/x)^3/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x)
^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3 +
 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/2)
- a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d - 5*b
*c))/a^(7/2)
```

3.260 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 1/3*(2*a^2*d^2+b*c*(-4*a*d+5*b*c))/a^2/b/(a+b/x)^(3/2)+c^2*x/a/(a+b/x)^(3/2
)-c*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+c*(-4*a*d+5*b*c)/
a^3/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {382, 91, 79, 53, 65, 214}

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} dx = -\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-4ad)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{\frac{5bc^2}{a}+\frac{2ad^2}{b}-4cd}{3a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((5*b*c^2)/a - 4*c*d + (2*a*d^2)/b)/(3*a*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a
*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(5bc-4ad)+ad^2x}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^3c^2+2a^3d^2x+a^2bcx(-16d+3cx)+4ab^2c(-3d+5cx))}{3a^3b(b+ax)^2} + \frac{c(-5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^3*c^2 + 2*a^3*d^2*x + a^2*b*c*x*(-16*d + 3*c*x) + 4*
a*b^2*c*(-3*d + 5*c*x)))/(3*a^3*b*(b + a*x)^2) + (c*(-5*b*c + 4*a*d)*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(108) = 216$.

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.35

method	result
risch	$\frac{c^2(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5bc^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)+\frac{2\left(2a^2d^2-8abcd+6b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}-\frac{2\left(a^2d^2\right.}{}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(24a^{\frac{9}{2}}\sqrt{x(ax+b)}\,cd\,x^3-30a^{\frac{7}{2}}\sqrt{x(ax+b)}\,b\,c^2x^3-24a^{\frac{7}{2}}(x(ax+b))^{\frac{3}{2}}cdx+72\sqrt{x(ax+b)}\,a^{\frac{7}{2}}bcd\,x^2-4(x(ax+b))^{\frac{3}{2}}a^{\frac{7}{2}}d^2+2\right.}{}$

```
[In] int((c+d/x)^2/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] c^2/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(-5*b*c^2*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)
^(1/2))+2*(2*a^2*d^2-8*a*b*c*d+6*b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a
))^(1/2)-2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)^2-
b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)))/x/((a*x+
b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.34

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^4c^2-4\,ab^3cd+(5\,a^2b^2c^2-4\,a^3bcd)x^2+2\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{a}\log\left(2\,ax+\right.}{6\,(a^6}\right.$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*(
5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
```

```
 b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4*d^
2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x^2 +
 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 -
 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*
c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/
(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c+d/x)**2/(a+b/x)**(5/2),x)
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.56

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{1}{6}\,c^2\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{2}{3}\,cd\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2\,d^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
[Out] 1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3
*d^2/((a + b/x)^(3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(108) = 216.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.98

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^3\mathrm{sgn}\left(x\right)} + \frac{\left(5\,bc^2-4\,acd\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} - \frac{\left(15\,b^2c^2\log\left(|b|\right)-12\,abcd\log\left(|b|\right)+28\,b^2c^2-32\,abcd+4\,a^2d^2\right)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^2-12\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bcd+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3d^2+15\left(\sqrt{a}x\right.\right.}{3\left(\left(\sqrt{a}x-\sqrt{ax^2\right.\right.}$$

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*c^2/(a^3*sgn(x)) + 1/2*(5*b*c^2 - 4*a*c*d)*log(abs(2*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^2*c^
2*log(abs(b)) - 12*a*b*c*d*log(abs(b)) + 28*b^2*c^2 - 32*a*b*c*d + 4*a^2*d^
2)*sgn(x)/(a^(7/2)*b) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2
- 12*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d + 3*(sqrt(a)*x - sqrt(a*x^
2 + b*x))^2*a^3*d^2 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2 -
18*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*c*d + 3*(sqrt(a)*x - sqrt(a*
x^2 + b*x))*a^(5/2)*b*d^2 + 7*b^4*c^2 - 8*a*b^3*c*d + a^2*b^2*d^2)/(((sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\left(a+\frac{b}{x}\right)\left(a^2\,d^2+4\,a\,b\,c\,d-5\,b^2\,c^2\right)}{3\,a^2}-\frac{2\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a}+\frac{b\left(a+\frac{b}{x}\right)^2\left(5\,b\,c^2-4\,a\,c\,d\right)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-a\,b\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\left(4\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

```
[In] int((c + d/x)^2/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(a
+ b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*
a*d - 5*b*c))/a^(7/2)
```

3.261 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 1/3*(-2*a*d+5*b*c)/a^2/(a+b/x)^(3/2)+c*x/a/(a+b/x)^(3/2)-(-2*a*d+5*b*c)*arc
tanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+(-2*a*d+5*b*c)/a^3/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {382, 79, 53, 65, 214}

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-2ad)}{a^{7/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{(5bc-2ad)\text{Subst}\left(\int\frac{1}{x(a+bx)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a^2}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{(5bc-2ad)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,\frac{1}{x}\right)}{2a^3}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}+\frac{(5bc-2ad)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int\frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(15b^2c+a^2x(-8d+3cx)+ab(-6d+20cx))}{3a^3(b+ax)^2}+\frac{(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[In] Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2*c + a^2*x*(-8*d + 3*c*x) + a*b*(-6*d + 20*c*x)))/(
3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7
/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(89) = 178.

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.47

method	result
risch	$\frac{c(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{5bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+\frac{2(ad-bc)b^2\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\cdots\right.}}{3b^2\left(x+\frac{b}{a}\right)}\cdots\right.}{a^2}\cdots\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(12a^{\frac{9}{2}}\sqrt{x(ax+b)}\,d\,x^3-30a^{\frac{7}{2}}\sqrt{x(ax+b)}\,bc\,x^3-12a^{\frac{7}{2}}(x(ax+b))^{\frac{3}{2}}dx+36a^{\frac{7}{2}}\sqrt{x(ax+b)}\,bd\,x^2+24a^{\frac{5}{2}}(x(ax+b))^{\frac{3}{2}}bcx-90a\cdots\right.}{\cdots}$

```
[In] int((c+d/x)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1/
2)+(a*x^2+b*x)^(1/2))-5*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/
2)+2*(a*d-b*c)*b^2/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a
/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))-4*(2*a*d-3*b*c)/a/(x+b/a)*(a*(x
+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.21

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\left[-\frac{3\left(5\,b^3c-2\,ab^2d+(5\,a^2bc-2\,a^3d)x^2+2\,(5\,ab^2c-2\,a^2bd)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\cdots}\right.}{6\left(a^6x^2+2\,a^5bx+\cdots\right.}\cdots\right.$$

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3
*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqr
t((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*
d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^
2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x +
a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(90) = 180.

Time = 33.93 (sec) , antiderivative size = 1479, normalized size of antiderivative = 14.36

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*
x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1
 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*
a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) +
 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) +
1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17
/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/
2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**
(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqr
t(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
+ 3*a**(13/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)
*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*
x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5
*b**2*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b
**2*x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
- 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
5/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
```

```
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**
3))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.65

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2\,a^2 b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) -\frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)$$

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a
^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(89) = 178.

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.51

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,bc\log\left(|b|\right)-6\,ad\log\left(|b|\right)+28\,bc-16\,ad)\mathrm{sgn}\left(x\right)}{6\,a^{\frac{7}{2}}} +\frac{\sqrt{ax^2+bx}c}{a^3\mathrm{sgn}\left(x\right)}+\frac{(5\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} +\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c-6\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bd+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c-9\left(\sqrt{a}x-\right.}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b*c*log(abs(b)) - 6*a*d*log(abs(b)) + 28*b*c - 16*a*d)*sgn(x)/a^(7
/2) + sqrt(a*x^2 + b*x)*c/(a^3*sgn(x)) + 1/2*(5*b*c - 2*a*d)*log(abs(2*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + 2/3*(9*(sqrt(a
```

```
)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c - 6*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^
2*b*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c - 9*(sqrt(a)*x - s
qrt(a*x^2 + b*x))*a^(3/2)*b^2*d + 7*b^4*c - 4*a*b^3*d)/(((sqrt(a)*x - sqrt(
a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{2\,d\,\left(a+\frac{b}{x}\right)}{a^2}}{\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,\left(a+\frac{b}{x}\right)^{5/2}}$$

```
[In] int((c + d/x)/(a + b/x)^(5/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/x
))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2],
9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

3.262 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{5b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 5/3*b/a^2/(a+b/x)^(3/2)+x/a/(a+b/x)^(3/2)-5*b*arctanh((a+b/x)^(1/2)/a^(1/2)
)/a^(7/2)+5*b/a^3/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {248, 44, 53, 65, 214}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=-\frac{5b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (5*b)/(3*a^2*(a + b/x)^(3/2)) + (5*b)/(a^3*Sqrt[a + b/x]) + x/(a*(a + b/x)^
(3/2)) - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5b}{a^3\sqrt{a+\frac{b}{x}}} + \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}
\end{aligned}$$

$$= \frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5b}{a^3\sqrt{a+\frac{b}{x}}} + \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}$$

$$= \frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5b}{a^3\sqrt{a+\frac{b}{x}}} + \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^2+20abx+3a^2x^2)}{3a^3(b+ax)^2} - \frac{5b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[In] Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2) - (5*
b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(65) = 130.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result
risch	$\frac{ax+b}{a^3\sqrt{\frac{ax+b}{x}}} + \frac{\left(-\frac{5b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{2a^{\frac{7}{2}}} - \frac{2b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^5\left(x+\frac{b}{a}\right)^2} + \frac{14b\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^4\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}x^3-24(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}x+90\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b\,x^2-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b\,x^3-20b\,a^{\frac{3}{2}}(x(ax+b))^{\frac{3}{2}}\right.}{6}$

```
[In] int(1/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-5/2/a^(7/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x
^2+b*x)^(1/2))-2/3/a^5*b^2/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+14/3/a^4
*b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+15\,a\right.}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)}\right.$$

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(66) = 132.

Time = 2.71 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{6a^{17}x^4\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{46a^{16}bx^3\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{15a^{16}bx^3\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{30a^{16}bx^3\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{70a^{15}b^2x^2\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{45a^{15}b^2x^2\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{30a^{14}b^3x\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{45a^{14}b^3x\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{90a^{14}b^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{15a^{13}b^4\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} - \frac{30a^{13}b^4\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}$$

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
```

```
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{15\left(a+\frac{b}{x}\right)^{2}b-10\left(a+\frac{b}{x}\right)ab-2\,a^{2}b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
(a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
 sqrt(a)))/a^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.16

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\frac{(15\,b\log\left(|b|\right)+28\,b)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} \\ &\quad+\frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}+\frac{\sqrt{ax^{2}+bx}}{a^{3}\mathrm{sgn}\left(x\right)} \\ &\quad+\frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)^{2}ab^{2}+15\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}b^{3}+7\,b^{4}\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right)^{3}a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}\end{aligned}$$

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b*log(abs(b)) + 28*b)*sgn(x)/a^(7/2) + 5/2*b*log(abs(2*(sqrt(a)*x
- sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a^
3*sgn(x)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2 + 15*(sqrt(a)*x
- sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))
*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{2\,x\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,{\left(a+\frac{b}{x}\right)}^{5/2}}$$

```
[In] int(1/(a + b/x)^(5/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)
^(5/2))
```

3.263 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}-\frac{(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}$$

```
[Out] 1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(3/2)+x/a/c/(a+b/x)^(3/2)-2*d
^(7/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(5/2)-
(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^2+b*(a^2*d^2-8*a*b*c
*d+5*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {382, 105, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+5bc)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b(a^2d^2-8abcd+5b^2c^2)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 -
 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/
x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^
2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^
(7/2)*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
```

```
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+2ad)+\frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac} \\
&= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)(5bc+2ad)+\frac{3}{4}bd(5bc-3ad)x}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^2c(bc-ad)} \\
&= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{\frac{3}{8}(bc-ad)^2(5bc+2ad)+\frac{3}{8}bd(5b^2c^2-8abcd+a^2d^2)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^3c(bc-ad)^2} \\
&= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} \\
&\quad - \frac{d^4\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)^2} + \frac{(5bc+2ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3c^2} \\
&= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} \\
&\quad + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(2d^4)\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2(bc-ad)^2} \\
&\quad + \frac{(5bc+2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3bc^2}
\end{aligned}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}} - \frac{(5bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(15b^4c^2+3a^4d^2x^2+6a^3bdx(d-cx)+4ab^3c(-6d+5cx)+a^2b^2(3d^2-32cdx+3c^2x^2))}{a^3(bc-ad)^2(b+ax)^2} - \frac{6d^{7/2}\arctan\left(\ldots\right.}{(bc-ad\ldots}}{3c^2}$$

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(15*b^4*c^2 + 3*a^4*d^2*x^2 + 6*a^3*b*d*x*(d - c*x) + 4
*a*b^3*c*(-6*d + 5*c*x) + a^2*b^2*(3*d^2 - 32*c*d*x + 3*c^2*x^2)))/(a^3*(b*
c - a*d)^2*(b + a*x)^2) - (6*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(b*c - a*d)^(5/2) - (3*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2))/(3*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.00

method	result
risch	$\frac{ax+b}{a^3c\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(2ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} - \frac{2c\,b^4\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)a^2} + \frac{2a^3d^4\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\ldots}{\ldots}\right.}{\ldots}\right.}{2a^3cx\ldots}$
default	Expression too large to display

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^3/c*((2*a*d+5*b*c)/c*ln((1/2*b+a*x)
/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)-2*c*b^4/(a*d-b*c)/a^2*(2/3/b/(x+b/a)^2*
(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/
2))+2/c^2*a^3*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2
```

```
-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c
*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+4*c*b^2*(4*a*d-3*b*c)/(a*d-b*c)^2
/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(
1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(177) = 354$.

Time = 1.52 (sec) , antiderivative size = 1990, normalized size of antiderivative = 9.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2
*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3
- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*
d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((
a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4
*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*
c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 -
 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*
b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^
2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2
*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4
*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*
a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((
a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^
4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a
^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt
(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*
d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(
5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c
^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b
)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a
```

```
^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^
2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c
- a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x
+ b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5
*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*
c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x
^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4
*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4
- 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^
2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 5387, normalized size of antiderivative = 26.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

```
[Out] - ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*(a
^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (atan((
(((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^1
1*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*
a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^1
7*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^
3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^
2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*
d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11
*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d
^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1/2)*(2*a*d +
 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*
d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13
*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10
*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^1
3))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c)*1i)/(2*c^2*(
a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*
d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^
11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*
d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^1
3 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^1
2*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a
^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768
*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*
a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^
(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a
^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280
*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^
26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c
)*1i)/(2*c^2*(a^7)^(1/2)))/(100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^10*d^7
+ 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*d^10 -
 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^14 + 8*
a^18*b^3*c^2*d^15 + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13
*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b
```

^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2)) - (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2))))*(2*a*d + 5*b*c)*1i)/(c^2*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5))*1i)/(c^2*(a*d - b*c)^5) + ((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10

- 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 8
8*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(2
0*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2
860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 +
5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 -
228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*
(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c
^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*
c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*
c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c
^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5))
1i)/(c^2(a*d - b*c)^5))/((((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*
a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280
*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 21
08*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a
^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b
^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13
*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a
^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*
a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22
*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^
(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^
4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d
^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d
^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13)
)/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5) - ((d^7*(
a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*
c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^
10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7
*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^
5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((d^7*(a*d - b*c)^5
)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^
15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*
c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*
c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^1
3 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a
^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600
*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320
*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*
a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d
- b*c)^5)))/(c^2*(a*d - b*c)^5) + 100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^1
0*d^7 + 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*
d^10 - 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^1
4 + 8*a^18*b^3*c^2*d^15))*(d^7*(a*d - b*c)^5)^(1/2)*2i)/(c^2*(a*d - b*c)^5)

3.264 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 287

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx &= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} \\ &+ \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} \\ &+ \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} \\ &- \frac{d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}} - \frac{(5bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}\end{aligned}$$

```
[Out] 1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+d*
(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+
d/x)-d^(7/2)*(-4*a*d+9*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/
c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c
^3+b*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+b/x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad+5bc)}{a^{7/2}c^3} + \frac{b(6a^2d^2-6abcd+5b^2c^2)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2} + \frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(bc-ad)^3} - \frac{d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}} + \frac{d(bc-2ad)}{ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+4ad)+\frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(5bc+4ad)-\frac{5}{2}bd(bc-2ad)x}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac^2(bc-ad)}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{2\text{Subst}\left(\int \frac{-\frac{3}{4}(bc-ad)^2(5bc+4ad)-\frac{3}{4}bd(5b^2c^2-6abcd+6a^2d^2)x}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^2c^2(bc-ad)^2}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{4\text{Subst}\left(\int \frac{-\frac{3}{8}(bc-ad)^3(5bc+4ad)-\frac{3}{8}bd(bc-2ad)(5b^2c^2-abcd+a^2d^2)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^3c^2(bc-ad)^3}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2-abcd+a^2d^2)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{(d^4(9bc-4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)^3} + \frac{(5bc+4ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3c^3}$$

$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}}$$
$$+ \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$
$$- \frac{(d^4(9bc-4ad))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)^3}$$
$$+ \frac{(5bc+4ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3bc^3}$$
$$= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}}$$
$$+ \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$
$$- \frac{d^{7/2}(9bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}} - \frac{(5bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{c\sqrt{a+\frac{b}{x}}\,x\left(-15b^5c^3(d+cx)+3a^5d^3x^2(2d+cx)+ab^4c^2\left(33d^2+13cdx-20c^2x^2\right)-3a^4bd^2x\left(-4d^2+cdx+3c^2x^2\right)+\right.}{a^3(-bc+ad)^3(b+ax)^2(d+cx)}$$

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-15*b^5*c^3*(d + c*x) + 3*a^5*d^3*x^2*(2*d + c*x) + a*
b^4*c^2*(33*d^2 + 13*c*d*x - 20*c^2*x^2) - 3*a^4*b*d^2*x*(-4*d^2 + c*d*x +
3*c^2*x^2) + a^2*b^3*c*(-9*d^3 + 35*c*d^2*x + 41*c^2*d*x^2 - 3*c^3*x^3) + 3
*a^3*b^2*d*(2*d^3 - 5*c*d^2*x - 3*c^2*d*x^2 + 3*c^3*x^3)))/(a^3*(-(b*c) + a
*d)^3*(b + a*x)^2*(d + c*x)) + (3*d^(7/2)*(-9*b*c + 4*a*d)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (3*(5*b*c + 4*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(3*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(261) = 522$.

Time = 0.37 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.32

method	result
risch	$\frac{ax+b}{a^3c^2\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(4ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2c^2b^5\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)^2a^2} + \frac{2a^3d^5\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right)^2-\ldots}}{(\ldots}\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a^3*((4*a*d+5*b*c)/c*ln((1/2*b+
a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+2*c^2*b^5/(a*d-b*c)^2/a^2*(2/3/b/(x
+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b
/a))^(1/2))+2/c^3*a^3*d^5/(a*d-b*c)^2*(-1/(a*d-b*c)/d*c^2/(x+d/c)*(a*(x+d/c
)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c
)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*(
(a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^
(1/2))/(x+d/c)))-4*c^2*b^3*(5*a*d-3*b*c)/(a*d-b*c)^3/a/(x+b/a)*(a*(x+b/a)^2
-b*(x+b/a))^(1/2)+2/c^2*a^3*d^4*(3*a*d-5*b*c)/(a*d-b*c)^3/((a*d-b*c)*d/c^2)
^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2
)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)))/x/((
a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(261) = 522$.

Time = 1.66 (sec) , antiderivative size = 3887, normalized size of antiderivative = 13.54

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d
^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2
+ 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5
*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b
^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2
*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)
*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*
d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c
- a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c
*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b
*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3
*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11
*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x
))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4
 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2
*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^
3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c
^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9
*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a
^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(
d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/
(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*
a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b
^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^
4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)
*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 +
 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a
^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 +
3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*
a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4
*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x
 + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*
c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x
^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 -
 a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^
7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2
 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 1
1*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 +
(10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 -
 a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d
^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a
```

```
^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7
*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d
/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)
- b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d +
 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d +
9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13
*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*
x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2
*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^
3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*
d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d
^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^
6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/3*(3*(9*a^4*b^3
*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*
c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*
d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c
 - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d
^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 -
 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3
+ (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3
 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3
*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arc
tan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a
^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^
4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2
*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2
+ 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d
^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^
4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2
- a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 +
 a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^
3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)^2}\,dx$$

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
[Out] Integral(x**2/((a + b/x)**(5/2)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(261) = 522$.

Time = 0.40 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.22

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\left(54\,a^{\frac{7}{2}}bcd^4\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 24\,a^{\frac{9}{2}}d^5\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 15\,\sqrt{bcd-ad^2}b^4c^4 \cdots}{\cdots}$$

$$+ \frac{(9\,bcd^4 - 4\,ad^5)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^6\mathrm{sgn}(x) - 3\,ab^2c^5d\mathrm{sgn}(x) + 3\,a^2bc^4d^2\mathrm{sgn}(x) - a^3c^3d^3\mathrm{sgn}(x)\right)\sqrt{bcd-ad^2}}$$

Numerator (as visible): $\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd^4 - 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^5 - \sqrt{a}bd^5$

Denominator (as visible): $+\left(b^3c^6\mathrm{sgn}(x) - 3\,ab^2c^5d\mathrm{sgn}(x) + 3\,a^2bc^4d^2\mathrm{sgn}(x) - a^3c^3d^3\mathrm{sgn}(x)\right)\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x - \cdots\right.\right.$

Numerator (as visible): $2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^5c - 15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2b^4d + 15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^6c - 27\left(\sqrt{a}x \cdots\right.\right.$

Denominator (as visible): $+3\left(a^{\frac{7}{2}}b^3c^3\mathrm{sgn}(x) - 3\,a^{\frac{9}{2}}b^2c^2d\mathrm{sgn}(x) + 3\,a^{\frac{11}{2}}bcd^2\mathrm{sgn}(x) - a^{\frac{13}{2}}d^3\mathrm{sgn}(x)\right)\left(\left(\sqrt{a}x - \cdots\right.\right.$

$$+ \frac{\sqrt{ax^2+bx}}{a^3c^2\mathrm{sgn}(x)} + \frac{(5\,bc+4\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}c^3\mathrm{sgn}(x)}$$

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")

[Out] 1/6*(54*a^(7/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(9/2)*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 15*sqrt(b*c*d - a*d^2)*b^4*c^4*log(abs(b)) + 33*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 9*sqrt(b*c*d - a*d^2)*a^2*b^2*c^2*d^2*log(abs(b)) - 21*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3*log(abs(b)) + 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) - 28*sqrt(b*c*d - a*d^2)*b^4*c^4 + 52*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d + 6*sqrt(b*c*d - a*d^2)*a^4*d^4)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^3*c^6 - 3*sqrt(b*c*d - a*d^2)*a^(9/2)*b^2*c^5*d + 3*sqrt(b*c*d - a*d^2)*a^(11/2)*b*c^4*d^2 - sqrt(b*c*d - a*d^2)*a^(13/2)*c^3*d^3) + (9*b*c*d^4 - 4*a*d^5)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^3*c^6*sgn(x) - 3*a*b^2*c^5*d*sgn(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x))*sqrt(b*c

```
*d - a*d^2)) + ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d^4 - 2*(sqrt(a)*x - sq
rt(a*x^2 + b*x))*a*d^5 - sqrt(a)*b*d^5)/((b^3*c^6*sgn(x) - 3*a*b^2*c^5*d*sg
n(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^
2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)) + 2/3*(
9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*c - 15*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^2*a^2*b^4*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^6*c - 27*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^5*d + 7*b^7*c - 13*a*b^6*d)/((a^(7
/2)*b^3*c^3*sgn(x) - 3*a^(9/2)*b^2*c^2*d*sgn(x) + 3*a^(11/2)*b*c*d^2*sgn(x)
 - a^(13/2)*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3) +
sqrt(a*x^2 + b*x)/(a^3*c^2*sgn(x)) + 1/2*(5*b*c + 4*a*d)*log(abs(2*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 5789, normalized size of antiderivative = 20.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d -
 a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2 -
58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/x)^
3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*c)^3
))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5/2)*(
2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^17*c^17
*d^2*(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*37776i + a
^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a + b/x)^(1/
2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12*c^12*d^7
*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*184563i + a^24
*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a + b/x)^(1/2)*
107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c^7*d^12*(a +
 b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i + a^29*b^5*c^
5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(1/2)*4515i + a
^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a + b/x)^(1/2)*17
00i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107740*a*b^9*c^9*d^
10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 52836*a^4*b^6*c^6*d^
13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420*a^7*b^3*c^3*d^16)
 + 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a^2*b^15*c^15*d^4 -
126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 + 40965*a^5*b^12*c^12*d^
7 - 184563*a^6*b^11*c^11*d^8) + 125*a^5*b^19*c^19 - 1700*a^6*b^18*c^18*d)))
*(4*a*d + 5*b*c)*1i)/(c^3*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^7)^(1/2)*
((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11
*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 5551
```

0*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)*1i)/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)) + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^13*b^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476*a^16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 - 109648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^17*d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^7*c^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b^4*c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^

5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)*1i)/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/(4880*a^10*b^16*c^17*d^7 - 450*a^9*b^17*c^18*d^6 - 23428*a^11*b^15*c^16*d^8 + 65234*a^12*b^14*c^15*d^9 - 115136*a^13*b^13*c^14*d^10 + 129800*a^14*b^12*c^13*d^11 - 83040*a^15*b^11*c^12*d^12 + 5916*a^16*b^10*c^11*d^13 + 45702*a^17*b^9*c^10*d^14 - 51528*a^18*b^8*c^9*d^15 + 32500*a^19*b^7*c^8*d^16 - 13790*a^20*b^6*c^7*d^17 + 4012*a^21*b^5*c^6*d^18 - 736*a^22*b^4*c^5*d^19 + 64*a^23*b^3*c^4*d^20 + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)) - ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^1

```
3*b^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476
*a^16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 -
 109648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^
17*d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^
7*c^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b
^4*c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^
(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080
*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 -
45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21
*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25
*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120
*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16
*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2
*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*
b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
- 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^
4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a
^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))*(d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c
)*1i)/(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a
^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
```

3.265 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3} dx = \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{d^{7/2}(99b^2c^2-88abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}} - \frac{(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}$$

```
[Out] 1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*d
+b*c)^3/(a+b/x)^(3/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(
c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)
^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+d/x)^2-1/4*d^(7/2)*(24*a^2*d^2-88*a*b
*c*d+99*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b
*c)^(9/2)-(6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^4+1/4*b*(1
2*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2-56*a*b^3*c^3*d+20*b^4*c^4)/a^3/
c^3/(-a*d+b*c)^4/(a+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 105, 156, 157, 162, 65, 214, 211}

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+5bc)}{a^{7/2}c^4} - \frac{d^{7/2}(24a^2d^2-88abcd+99b^2c^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}} + \frac{d(12a^2d^2-23abcd+4b^2c^2)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{b(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3)}{12a^2c^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^3} + \frac{b(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4} + \frac{d(2bc-3ad)}{2ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+6ad)+\frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-((bc-ad)(5bc+6ad))-\frac{7}{2}bd(2bc-3ad)x}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{(bc-ad)^2(5bc+6ad)+\frac{5}{4}bd(4b^2c^2-23abcd+12a^2d^2)x}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2ac^3(bc-ad)^2}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc-ad)^3(5bc+6ad)+\frac{3}{8}bd(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)x}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^2c^3(bc-ad)^3}$$

$$= \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{2\text{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)^4(5bc+6ad)+\frac{3}{16}bd(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{3a^3c^3(bc-ad)^4}$$

$$= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2 - 35a^3bcd^3 + 12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 23abcd + 12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{(5bc+6ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3c^4} - \frac{(d^4(99b^2c^2 - 88abcd + 24a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{8c^4(bc-ad)^4}$$

$$= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2 - 35a^3bcd^3 + 12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 23abcd + 12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{(5bc+6ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3bc^4} - \frac{(d^4(99b^2c^2 - 88abcd + 24a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4bc^4(bc-ad)^4}$$

$$= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2 - 35a^3bcd^3 + 12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 23abcd + 12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{d^{7/2}(99b^2c^2 - 88abcd + 24a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}} - \frac{(5bc+6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}$$

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{c\sqrt{a+\frac{b}{x}}\,x\left(60b^6c^4(d+cx)^2+8ab^5c^3(d+cx)^2(-21d+10cx)+6a^6d^4x^2\left(6d^2+9cdx+2c^2x^2\right)+4a^2b^4c^2(d+cx)^2(18\ldots\right.}{\ldots}$$

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(60*b^6*c^4*(d + c*x)^2 + 8*a*b^5*c^3*(d + c*x)^2*(-21*
d + 10*c*x) + 6*a^6*d^4*x^2*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + 4*a^2*b^4*c^2*(
d + c*x)^2*(18*d^2 - 56*c*d*x + 3*c^2*x^2) + 3*a^5*b*d^3*x*(24*d^3 + c*d^2*
x - 45*c^2*d*x^2 - 16*c^3*x^3) + 6*a^4*b^2*d^2*(6*d^4 - 26*c*d^3*x - 39*c^2
*d^2*x^2 + 8*c^3*d*x^3 + 12*c^4*x^4) - 3*a^3*b^3*c*d*(35*d^4 + 5*c*d^3*x -
64*c^2*d^2*x^2 - 16*c^3*d*x^3 + 16*c^4*x^4)))/(a^3*(b*c - a*d)^4*(b + a*x)^
2*(d + c*x)^2) - (3*d^(7/2)*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(
Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(9/2) - (12*(5*b*c + 6
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(12*c^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. 2(373) = 746.

Time = 0.46 (sec) , antiderivative size = 1138, normalized size of antiderivative = 2.78

method	result	size
risch	Expression too large to display	1138
default	Expression too large to display	7300

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(5/2)/c^4*ln((1/2*b+a*x)/a^(1/2)+
(a*x^2+b*x)^(1/2))*d-5/2/a^(7/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/
2))*b+2/3/a^5*b^5/(a*d-b*c)^3/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3/a
^4*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^5*d^5/(a*d-b
*c)^3/(x+d/c)^2*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)+5
/2*a/c^4*d^5/(a*d-b*c)^4/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*
c)*d/c^2)^(1/2)-21/4/c^3*d^4/(a*d-b*c)^4/(x+d/c)*(a*(x+d/c)^2-(2*a*d-b*c)/c
*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2/c^5*d^6/(a*d-b*c)^4/((a*d-b*c)*d/
c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^
(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))+2
3/2*a/c^4*d^5/(a*d-b*c)^4/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*
a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+
d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))*b-99/8/c^3*d^4/(a*d-b*c)^4/((a*d-b*c)
```

```
*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^
2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c)
)*b^2+1/2*a/c^5*d^5/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c
^2-(2*a*d-b*c)/c*(x+d/c)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+d/c)^2-(2*a*d-b*c)
/c*(x+d/c)+(a*d-b*c)*d/c^2)^(1/2))/(x+d/c))-12/a^3*b^4/(a*d-b*c)^4/(x+b/a)*
(a*(x+b/a)^2-b*(x+b/a))^(1/2)*d+6/a^4*c*b^5/(a*d-b*c)^4/(x+b/a)*(a*(x+b/a)^
2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(373) = 746$.

Time = 4.57 (sec) , antiderivative size = 6171, normalized size of antiderivative = 15.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. 2(373) = 746.

Time = 0.49 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] 1/12*(297*a^(7/2)*b^2*c^2*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 264*a
^(9/2)*b*c*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 72*a^(11/2)*d^6*arct
an(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 30*sqrt(b*c*d - a*d^2)*b^5*c^5*log(abs(
b)) + 84*sqrt(b*c*d - a*d^2)*a*b^4*c^4*d*log(abs(b)) - 36*sqrt(b*c*d - a*d^
2)*a^2*b^3*c^3*d^2*log(abs(b)) - 96*sqrt(b*c*d - a*d^2)*a^3*b^2*c^2*d^3*log
(abs(b)) + 114*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4*log(abs(b)) - 36*sqrt(b*c*d
- a*d^2)*a^5*d^5*log(abs(b)) - 56*sqrt(b*c*d - a*d^2)*b^5*c^5 + 128*sqrt(b*
c*d - a*d^2)*a*b^4*c^4*d + 63*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4 - 30*sqrt(b*c
*d - a*d^2)*a^5*d^5)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^4*c^8 - 4*sqrt(b
*c*d - a*d^2)*a^(9/2)*b^3*c^7*d + 6*sqrt(b*c*d - a*d^2)*a^(11/2)*b^2*c^6*d^
2 - 4*sqrt(b*c*d - a*d^2)*a^(13/2)*b*c^5*d^3 + sqrt(b*c*d - a*d^2)*a^(15/2)
*c^4*d^4) + 1/4*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(-((sqrt
(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^4*c^8*sg
n(x) - 4*a*b^3*c^7*d*sgn(x) + 6*a^2*b^2*c^6*d^2*sgn(x) - 4*a^3*b*c^5*d^3*sg
n(x) + a^4*c^4*d^4*sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(21*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^3*b^2*c^3*d^4 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2
*d^5 + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^6 + 15*(sqrt(a)*x - sqr
t(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d^5 - 88*(sqrt(a)*x - sqrt(a*x^2 + b*x))^
2*a^(3/2)*b*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^7 + 19*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^5 - 92*(sqrt(a)*x - sqrt(a*x^2 + b
*x))*a*b^2*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^7 - 21*sqrt(a
)*b^3*c*d^6 + 10*a^(3/2)*b^2*d^7)/((b^4*c^8*sgn(x) - 4*a*b^3*c^7*d*sgn(x) +
 6*a^2*b^2*c^6*d^2*sgn(x) - 4*a^3*b*c^5*d^3*sgn(x) + a^4*c^4*d^4*sgn(x))*((
sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt
(a)*d + b*d)^2) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^6*c - 18*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^5*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x
))*sqrt(a)*b^7*c - 33*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^6*d + 7*b^8
*c - 16*a*b^7*d)/((a^(7/2)*b^4*c^4*sgn(x) - 4*a^(9/2)*b^3*c^3*d*sgn(x) + 6*
a^(11/2)*b^2*c^2*d^2*sgn(x) - 4*a^(13/2)*b*c*d^3*sgn(x) + a^(15/2)*d^4*sgn(
x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3) + sqrt(a*x^2 + b*x)/(a
^3*c^3*sgn(x)) + 1/2*(5*b*c + 6*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*
x))*sqrt(a) + b))/(a^(7/2)*c^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 4284, normalized size of antiderivative = 10.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)
[Out] ((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*d
 - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*d^
2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^3*(
a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4*d^2
 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3*(a^2
*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5*d + 4
96*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a^4*b*c*
d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*(3*a^2*d
^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan((
a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b/x)^(1/2)*2
77440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^20*c^20*d^4*(
a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8371440i + a^2
1*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7*(a + b/x)^(1/
2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i + a^24*b^15*c^15
*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b/x)^(1/2)*666652
80i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27*b^12*c^12*d^12*(
a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^(1/2)*85652532i +
a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9*c^9*d^15*(a + b/x)
^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)*29178270i - a^32*b^7*c
^7*d^17*(a + b/x)^(1/2)*9940590i + a^33*b^6*c^6*d^18*(a + b/x)^(1/2)*228782
4i - a^34*b^5*c^5*d^19*(a + b/x)^(1/2)*320859i + a^35*b^4*c^4*d^20*(a + b/x
)^(1/2)*20790i - a^16*b^23*c^23*d*(a + b/x)^(1/2)*34800i)/(a^7*(a^7)^(1/2)*
(a^7*(a^7*(a^7*(29178270*b^8*c^8*d^16 - 9940590*a*b^7*c^7*d^17 + 2287824*a^
2*b^6*c^6*d^18 - 320859*a^3*b^5*c^5*d^19 + 20790*a^4*b^4*c^4*d^20) + 636772
18*b^15*c^15*d^9 - 66665280*a*b^14*c^14*d^10 + 24871035*a^2*b^13*c^13*d^11
+ 40203170*a^3*b^12*c^12*d^12 - 85652532*a^4*b^11*c^11*d^13 + 88170192*a^5*
b^10*c^10*d^14 - 60362445*a^6*b^9*c^9*d^15) + 277440*b^22*c^22*d^2 - 132598
4*a*b^21*c^21*d^3 + 4135824*a^2*b^20*c^20*d^4 - 8371440*a^3*b^19*c^19*d^5 +
 9129120*a^4*b^18*c^18*d^6 + 3058605*a^5*b^17*c^17*d^7 - 32337558*a^6*b^16*
c^16*d^8) + 2000*a^5*b^24*c^24 - 34800*a^6*b^23*c^23*d)))*(6*a*d + 5*b*c)*1
i)/(c^4*(a^7)^(1/2)) + (log(400*b^25*c^25*d^4 - 8240*a*b^24*c^24*d^5 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 79696*a^2*b^23*c^23*d^6 - 478768*a^3*
b^22*c^22*d^7 + 1987568*a^4*b^21*c^21*d^8 - 5978896*a^5*b^20*c^20*d^9 + 131
76240*a^6*b^19*c^19*d^10 - 20525703*a^7*b^18*c^18*d^11 + 18765714*a^8*b^17*
```

c^17*d^12 + 3763331*a^9*b^16*c^16*d^13 - 49787452*a^10*b^15*c^15*d^14 + 104
120705*a^11*b^14*c^14*d^15 - 140185682*a^12*b^13*c^13*d^16 + 139985251*a^13
*b^12*c^12*d^17 - 108046616*a^14*b^11*c^11*d^18 + 65184867*a^15*b^10*c^10*d
^19 - 30607170*a^16*b^9*c^9*d^20 + 10996689*a^17*b^8*c^8*d^21 - 2926572*a^1
8*b^7*c^7*d^22 + 544467*a^19*b^6*c^6*d^23 - 63294*a^20*b^5*c^5*d^24 + 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
(d^7(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
c)^9)^(1/2)(3*a^2*d^2 + (99*b^2*c^2)/8 - 11*a*b*c*d))/(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d) - (log(8240*a*b^24*c^24*d^5 - 400*b^25*c^25*d^4 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 79696*a^2*b^23*c^23*d^6 + 478768*a^3*
b^22*c^22*d^7 - 1987568*a^4*b^21*c^21*d^8 + 5978896*a^5*b^20*c^20*d^9 - 131
76240*a^6*b^19*c^19*d^10 + 20525703*a^7*b^18*c^18*d^11 - 18765714*a^8*b^17*
c^17*d^12 - 3763331*a^9*b^16*c^16*d^13 + 49787452*a^10*b^15*c^15*d^14 - 104
120705*a^11*b^14*c^14*d^15 + 140185682*a^12*b^13*c^13*d^16 - 139985251*a^13
*b^12*c^12*d^17 + 108046616*a^14*b^11*c^11*d^18 - 65184867*a^15*b^10*c^10*d
^19 + 30607170*a^16*b^9*c^9*d^20 - 10996689*a^17*b^8*c^8*d^21 + 2926572*a^1
8*b^7*c^7*d^22 - 544467*a^19*b^6*c^6*d^23 + 63294*a^20*b^5*c^5*d^24 - 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
(d^7(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*

```
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
 b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(24*a^2*d^2 + 99*b^2*c^2 - 88*a*b*c*d))/(8*(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d))
```

3.266 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x + \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

```
[Out] (a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(1
/2)-2*arctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*b^(1/2)*d^(1/2)+
x*(a+b/x)^(1/2)*(c+d/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {382, 99, 163, 65, 223, 212, 95, 214}

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right) + x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

```
[In] Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-\text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-(bd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\frac{1}{x}\right)\\
&\quad -\frac{1}{2}(bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)\\
&\quad -(bc+ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-(2d)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left((bc+ad)\sqrt{b+ax}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)+\sqrt{a}\sqrt{c}\left((b+ax)\sqrt{d+cx}-2\sqrt{b}\sqrt{d}\sqrt{b+ax}\,\text{arcta}\right.\right.}{\sqrt{a}\sqrt{c}(b+ax)\sqrt{d+cx}}$$

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*((b*c + a*d)*Sqrt[b + a*x]*ArcTanh[(Sqrt[a]*
Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])] + Sqrt[a]*Sqrt[c]*((b + a*x)*Sqrt[d
 + c*x] - 2*Sqrt[b]*Sqrt[d]*Sqrt[b + a*x]*ArcTanh[(Sqrt[b]*Sqrt[d + c*x])/(
Sqrt[d]*Sqrt[b + a*x])])))/(Sqrt[a]*Sqrt[c]*(b + a*x)*Sqrt[d + c*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(95) = 190.

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\sqrt{\frac{cx+d}{x}}\left(\sqrt{bd}\,\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ad+\sqrt{bd}\,\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)bc-2bd\ln\left(\frac{adx+bcx+2\sqrt{}}{}\right.\right.}{2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\,\sqrt{bd}}$

```
[In] int((c+d/x)^(1/2)*(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*a*c*x+2*((
a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+(b*d)^(1/2)*ln(
1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*
c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((a*x+b)*(c*x+d))^(1/2)+2*b*d)/x)*(a*
c)^(1/2)+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((a*x+b)*(c*x+d
))^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 890, normalized size of antiderivative = 7.24

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+2\,\sqrt{bd}ac\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,bdx+(bc+ad)x^2)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd+abd^2)\right.}{x^2}}{}\right.$$

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(8
*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x
^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x
)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a
^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*
x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2
)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (
b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2
 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x
+ b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*
c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2
*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (b^2*c*d + a*b*d^
2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

```
[In] integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 4674, normalized size of antiderivative = 38.00

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \text{Too large to display}$$

```
[In] int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
[Out] atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/
2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3
 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2)
)*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*
c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^
6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5
))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) -
 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b
^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)
*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c
^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/
2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2
)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^
(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a +
b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6
```

*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i - (b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i)/((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16* a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (7*a^(7/2)*b^12*c^(21/2)*d - 7*a^(9/2)*b^11*c^(19/2)*d^2 - 21*a^(11/2)*b^10*c^(17/2)*d^3 + 21*a^(13/2)*b^9*c^(15/2)*d^4 + 21*a^(15/2)*b^8*c^(13/2)*d^5 - 21*a^(17/2)*b^7*c^(11/2)*d^6 - 7*a^(19/2)*b^6*c^(9/2)*d^7 + 7*a^(21/2)*b^5*c^(7/2)*d^8)/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(112*a^5*b^10*c^9*d^3 - 56*a^4*b^11*c^10*d^2 + 56*a^6*b^9*c^8*d^4 - 224*a^7*b^8*c^7*d^5 + 56*a^8*b^7*c^6*d^6 + 112*a^9*b^6*c^5*d^7 - 56*a^10*b^5*c^4*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))))*(b*d)^(1/2)*4i - ((((a + b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(

```
1/2) - c^(1/2))) - b^2/(4*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d^2)/4
+ (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c*d*((c + d/x)^(1/2) - c^(1/2))^2))/(((a
 + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a + b/x)^(1
/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2) - a^(1/
2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2)) + (d*
((a + b/x)^(1/2) - a^(1/2)))/(4*((c + d/x)^(1/2) - c^(1/2))) + (log(((a + b
/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a*d + b*c))/(2*a^(1/2)*c
^(1/2)) - (log(((c^(1/2)*(a + b/x)^(1/2) - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1
/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2))))
/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(2*a
*c)
```

3.267 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} + \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}$$

```
[Out] (-a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(3/2)/a^(
1/2)+x*(a+b/x)^(1/2)*(c+d/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {382, 96, 95, 214}

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx = \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

```
[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c}\\
&= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c}\\
&= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{d+cx}\left(\frac{\sqrt{b+ax}\sqrt{d+cx}}{c}+\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)}{\sqrt{a}c^{3/2}}\right)}{\sqrt{c+\frac{d}{x}}\sqrt{b+ax}}$$

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[d + c*x]*((Sqrt[b + a*x]*Sqrt[d + c*x])/c + ((b*c - a*d
)*ArcTanh[(Sqrt[a]*Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])])/(Sqrt[a]*c^(3/2
))))/(Sqrt[c + d/x]*Sqrt[b + a*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(65) = 130.

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.91

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\sqrt{\frac{cx+d}{x}}\left(-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ad+\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\right)}{2\sqrt{(ax+b)(cx+d)}\,c\sqrt{ac}}$

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x
+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+ln(1/2*(2*a*c*x+2*((a*x+b)
*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*((a*x+b)*(c*x+d))^(
1/2)*(a*c)^(1/2))/((a*x+b)*(c*x+d))^(1/2)/c/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

$$=\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\sqrt{ac}(bc-ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx^2+(bc+ad)x)\sqrt{ac}\right.}{4\,ac^2}\right.$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*l
og(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a
*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d
)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(-a*c
)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*
a*c*x + b*c + a*d)))/(a*c^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx = \int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx = \int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Giac [F]

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx = \int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 478, normalized size of antiderivative = 5.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4\,c\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{b^2}{4\,c\,d}+\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^3}{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{a+\frac{b}{x}}-\sqrt{a}\,\sqrt{c+\frac{d}{x}}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}$$

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

```
[Out] (d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - ((((a +
 b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c + d/
x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d
^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/2))^
2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a +
 b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2)
 - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2
)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)
*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b/x)^(1/2)
 - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1
/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b
*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

3.268 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}$$

```
[Out] (-3*a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(5/2)/a
^(1/2)+(a+b/x)^(3/2)*x/a/c/(c+d/x)^(1/2)-(-3*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(
c+d/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {382, 98, 96, 95, 214}

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\frac{(bc-3ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{\left(-\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left(\sqrt{a}\sqrt{c}\sqrt{b+ax}(3d+cx)+(bc-3ad)\sqrt{d+cx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{b+ax}}{\sqrt{a}\sqrt{d+cx}}\right)\right)}{\sqrt{a}c^{5/2}\sqrt{b+ax}(d+cx)}$$

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*(Sqrt[a]*Sqrt[c]*Sqrt[b + a*x]*(3*d + c*x) +
 (b*c - 3*a*d)*Sqrt[d + c*x]*ArcTanh[(Sqrt[c]*Sqrt[b + a*x])/(Sqrt[a]*Sqrt[
d + c*x])]))/(Sqrt[a]*c^(5/2)*Sqrt[b + a*x]*(d + c*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(102) = 204.

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\sqrt{\frac{cx+d}{x}}\left(-3\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)acdx+\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)b\,c^2x+2cx\sqrt{(ax+b)(cx+}\right.}{2\sqrt{ac}\,(cx+d)\sqrt{(ax+}}$

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(c
*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*c*d*x+ln(1/2*(2*a*c*x+2*((
a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c^2*x+2*c*x*((a*x
+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1/2)*
(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))
```

```
^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^(1/2)*
(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \left[-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{ac}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx^2+\right.}{4\,(ac^4x}\right.$$

$$\left.-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{-ac}\arctan\left(\frac{2\sqrt{-ac}x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)-2\,(ac^2x^2+3\,acdx)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(ac^4x+ac^3d)}\right]$$

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2
 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x
^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), -
1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)
*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^
2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$$

[In] int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)

[Out] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)

3.269 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+p} \left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}$$

```
[Out] -b*(a+b/x)^(p+1)*(c+d/x)^q*AppellF1(p+1,2,-q,2+p,(a+b/x)/a,-d*(a+b/x)/(-a*d
+b*c))/a^2/(p+1)/((b*(c+d/x)/(-a*d+b*c))^q)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {382, 142, 141}

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{p+1} \left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

```
[In] Int[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^
q))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= -\left(\left(\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 206 vs. $2(96) = 192$.

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.15

$$\int \left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q\,dx$$
$$= \frac{bd(-2+p+q)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q x\,\text{AppellF1}}{(-1+p+q)\left(-bd(-2+p+q)\,\text{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax}{b},-\frac{cx}{d}\right)+x\left(adp\,\text{AppellF1}\left(2\right.\right.\right.}$$

```
[In] Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2 -
 p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*AppellF1
[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*AppellF1
[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[
2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q}dx$$

```
[In] int((a+b/x)^p*(c+d/x)^q,x)
```

```
[Out] int((a+b/x)^p*(c+d/x)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q}dx=\int \left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q}dx$$

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q}dx=\int \left(a+\frac{b}{x}\right)^{p}\left(c+\frac{d}{x}\right)^{q}dx$$

```
[In] integrate((a+b/x)**p*(c+d/x)**q,x)
```

```
[Out] Integral((a + b/x)**p*(c + d/x)**q, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

```
[In] int((a + b/x)^p*(c + d/x)^q,x)
[Out] int((a + b/x)^p*(c + d/x)^q, x)
```

3.270 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

```
[Out] a*x/c+(-a*d+b*c)*arctan(x*c^(1/2)/d^(1/2))/c^(3/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {381, 396, 211}

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}} + \frac{ax}{c}$$

```
[In] Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
```

```
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+ax^2}{d+cx^2}\,dx \\ &= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^2}\,dx}{c} \\ &= \frac{ax}{c} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} - \frac{(-bc+ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

```
[In] Integrate[(a + b/x^2)/(c + d/x^2),x]
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{ax}{c}+\frac{(-ad+bc)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{c\sqrt{cd}}$	34
risch	$\frac{ax}{c}-\frac{\ln\left(cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}+\frac{\ln\left(cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}+\frac{\ln\left(-cx-\sqrt{-cd}\right)ad}{2c\sqrt{-cd}}-\frac{\ln\left(-cx-\sqrt{-cd}\right)b}{2\sqrt{-cd}}$	106

```
[In] int((a+b/x^2)/(c+d/x^2),x,method=_RETURNVERBOSE)
[Out] a*x/c+(-a*d+b*c)/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \left[\frac{2\,acdx+(bc-ad)\sqrt{-cd}\log\left(\frac{cx^2+2\,\sqrt{-cd}x-d}{cx^2+d}\right)}{2\,c^2d}, \frac{acdx+(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{c^2d}\right]$$

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/(
c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/d
))/(c^2*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{a\,x}{c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

```
[In] int((a + b/x^2)/(c + d/x^2),x)
[Out] (a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

3.271 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x}+\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{\sqrt{c}(bc+ad)\sqrt{a+\frac{b}{x^2}}\,\mathrm{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/x/(c+d/x^2)^(1/2)-(a*d+b*c)*(x^2*c/d/(1+x^2*c/d))^(1/2
)/x*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b
/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1
+x^2*c/d))^(1/2)/x*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c
/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2
)+x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {382, 484, 545, 429, 506, 422}

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}} - \frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\operatorname{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[In] Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2
]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[
d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^
2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqr
t[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - 2\text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - (2bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&\quad - (bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - \frac{\sqrt{c}(bc+ad)\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}\\
&\quad + (2cd)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$
$$- \frac{\sqrt{c}(bc+ad)\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx =$$
$$-\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{\frac{a}{b}}(b+ax^2)(d+cx^2)+2iadx\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+i(bc-\right.}{\sqrt{\frac{a}{b}}(b+ax^2)(d+cx^2)}$$

```
[In] Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) + (
2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt
[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^
2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b + a*x^
2)*(d + c*x^2)))
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,x\sqrt{\frac{c\,x^2+d}{x^2}}\left(-\sqrt{-\frac{c}{d}}\,ac\,x^4+\sqrt{\frac{c\,x^2+d}{d}}\,\sqrt{\frac{a\,x^2+b}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)adx-cb\sqrt{\frac{c\,x^2+d}{d}}\,\sqrt{\frac{a\,x^2+b}{b}}\,xF\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)+2cb\sqrt{\frac{c\,x}{}}\right.}{(a\,x^4c+ad\,x^2+cb\,x^2+bd)\sqrt{-\frac{c}{d}}}$
risch	$-x\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{\frac{c\,x^2+d}{x^2}}+\frac{\left(\frac{ad\sqrt{1+\frac{c\,x^2}{d}}\,\sqrt{1+\frac{a\,x^2}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^4c+ad\,x^2+cb\,x^2+bd}}+\frac{bc\sqrt{1+\frac{c\,x^2}{d}}\,\sqrt{1+\frac{a\,x^2}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^4c+ad\,x^2+cb\,x^2+bd}}-\frac{2cb\sqrt{1}}{}\right.}{(a\,x}$

```
[In] int((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*a*c*x^4+((c*x^
2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))
*a*d*x-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticF(x*(-c/d)^(1/
2),(a*d/b/c)^(1/2))+2*c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ellipti
cE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-(-c/d)^(1/2)*a*d*x^2-(-c/d)^(1/2)*b*c*x^
2-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

```
[In] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2),x)
[Out] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,\mathrm{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -d*(a+b/x^2)^(1/2)/c/x/(c+d/x^2)^(1/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x*(1+x
^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1
/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+(x^2*c/d/(1+x^2*c/d))
^(1/2)/x/c*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1
/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+x*(a+b
/x^2)^(1/2)*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {382, 486, 21, 433, 429, 506, 422}

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*Sqrt[c + d/
x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c
+ d/x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c} - \frac{\text{Subst}\left(\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c} - \frac{b\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{c}
\end{aligned}
$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c} - b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - \frac{(bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + d\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{d+cx^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{c}{d}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{b+ax^2}{b}}}$$

```
[In] Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
[Out] (Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*d
)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,E\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right)\sqrt{\frac{a\,x^{2}+b}{b}}\,\sqrt{\frac{c\,x^{2}+d}{d}}\,b}{\left(a\,x^{2}+b\right)\sqrt{-\frac{c}{d}}\,\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	94

```
[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*(
(a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/((c*x^2+d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}} - \sqrt{ac}b\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}) + \sqrt{ac}(a+b)\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc})}{ac}$$

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2) - sqrt(a*c)*b*sqrt(-b/a)*e
lliptic_e(arcsin(sqrt(-b/a)/x), a*d/(b*c)) + sqrt(a*c)*(a + b)*sqrt(-b/a)*e
lliptic_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)))/(a*c)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2),x)
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx = -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}x} - \frac{\sqrt{a+\frac{b}{x^2}}x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c^2} + \frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/c^2/x/(c+d/x^2)^(1/2)-x*(a+b/x^2)^(1/2)/c/(c+d/x^2)^(1
/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*
c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(
1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c^2*d*(1+x^2*c/d)^(1/2
)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/
x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
/c^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {382, 480, 597, 545, 429, 506, 422}

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[In] Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]*x)/(c*Sqr
t[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c^2 + (2*Sqrt[d]*Sqrt
[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(c^(3/
2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x
^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{\text{Subst}\left(\int \frac{-2a-bx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2}-\frac{\text{Subst}\left(\int \frac{abc+2abdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{ac^2}
\end{aligned}$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c^2} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(2bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}x} - \frac{\sqrt{a+\frac{b}{x^2}}x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c^2} - \frac{b\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + \frac{(2d)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}x} - \frac{\sqrt{a+\frac{b}{x^2}}x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x}{c^2} + \frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{\frac{a}{b}}cx(b+ax^2) + 2iad\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right) + i(bc-2ad)\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\right.}{\sqrt{\frac{a}{b}}c^2\sqrt{c+\frac{d}{x^2}}(b+ax^2)}$$

```
[In] Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] -((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)/
b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(
b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

method	result	si
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(\sqrt{-\frac{c}{d}}\,a\,x^{3}+b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,F\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right)-2b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,E\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right)+\sqrt{-\frac{c}{d}}\,b x\right)\left(c\,x^{2}+d\right)}{x^{2}\left(a\,x^{2}+b\right)\sqrt{-\frac{c}{d}}\,c\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}}$	18

[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*((-c/d)^(1/2)*a*x^3+b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-2*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))+(-c/d)^(1/2)*b*x)*(c*x^2+d)/(-c/d)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{2\,(bcx^2+bd)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}) - ((a+2\,b)cx^2+(a+2\,b)d)\sqrt{ac}\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}}{ac^3x^2+ac^2d}$$

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] -(2*(b*c*x^2 + b*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(sqrt(-b/a)/x), a*d/(b*c)) - ((a + 2*b)*c*x^2 + (a + 2*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)) - (a*c*x^3 + 2*a*d*x)*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2))/(a*c^3*x^2 + a*c^2*d)

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)

[Out] Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)
```

3.274 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p(c+dx^2)^q}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{x^2}\,dx, x,\right.\right.$$

$$= \left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-1+2p+2q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.275 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx=\frac{ax}{c}-\frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}$$

```
[Out] a*x/c+1/3*(-a*d+b*c)*ln(d^(1/3)+c^(1/3)*x)/c^(4/3)/d^(2/3)-1/6*(-a*d+b*c)*l
n(d^(2/3)-c^(1/3)*d^(1/3)*x+c^(2/3)*x^2)/c^(4/3)/d^(2/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(d^(1/3)-2*c^(1/3)*x)/d^(1/3)*3^(1/2))/c^(4/3)/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {381, 396, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx=-\frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}+\frac{ax}{c}$$

```
[In] Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{b+ax^3}{d+cx^3}\,dx\\
&= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^3}\,dx}{c}\\
&= \frac{ax}{c} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{c}x}\,dx}{3cd^{2/3}} + \frac{(bc-ad)\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{3cd^{2/3}}\\
&= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2c^{2/3}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{6c^{4/3}d^{2/3}}\\
&\quad + \frac{(bc-ad)\int \frac{1}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{2c\sqrt[3]{d}}\\
&= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}\\
&\quad + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}\right)}{c^{4/3}d^{2/3}}\\
&= \frac{ax}{c} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}\\
&\quad - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$
$$= \frac{6a\sqrt[3]{c}d^{2/3}x - 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right) - (bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\right.}{6c^{4/3}d^{2/3}}$$

```
[In] Integrate[(a + b/x^3)/(c + d/x^3),x]
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Log[
d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{ax}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3+d\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{3c^2}$	42
default	$\frac{ax}{c}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{c}$	110

```
[In] int((a+b/x^3)/(c+d/x^3),x,method=_RETURNVERBOSE)
[Out] a*x/c+1/3/c^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*c+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \left[\frac{6\,acd^2x - 3\,\sqrt{\frac{1}{3}}(bc^2d - acd^2)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^3+3\,(-cd^2)^{\frac{1}{3}}dx-d^2-3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(-cd^2)^{\frac{2}{3}}x+(-cd^2)^{\frac{1}{3}}d\right)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}}}{cx^3+d}\right)}{6\,c^2d} \right.$$

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)*
log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-c*
d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) - (-c
*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)*d)
+ 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2), 1/6*
(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c)*arct
an(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^(1/3)/c
)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^
2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^
2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

```
[In] integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2
*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2-x\left(\frac{d}{c}\right)^{\frac{1}{3}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/c
)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d/c
)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(d/c
)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,(-c^2d)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,(-c^2d)^{\frac{2}{3}}} + \frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1/
3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2/3
))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d/c)
^(1/3)))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{a\,x}{c} - \frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} + \frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} - \frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}$$

```
[In] int((a + b/x^3)/(c + d/x^3),x)
```

```
[Out] (a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (log
(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

3.276 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx = -\frac{2(bc-ad)\sqrt{x}}{d^2} + \frac{bx}{d} + \frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

```
[Out] b*x/d+2*c*(-a*d+b*c)*ln(c+d*x^(1/2))/d^3-2*(-a*d+b*c)*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {383, 78}

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx = \frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3} - \frac{2\sqrt{x}(bc-ad)}{d^2} + \frac{bx}{d}$$

```
[In] Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)\,dx, x, \sqrt{x}\right) \\
&= -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$
\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{\left(-2bc+2ad+bd\sqrt{x}\right)\sqrt{x}}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}
$$

```
[In] Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
[Out] ((-2*b*c + 2*a*d + b*d*Sqrt[x])*Sqrt[x])/d^2 + (2*c*(b*c - a*d)*Log[c + d*S
qrt[x]])/d^3
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln\left(c+d\sqrt{x}\right)}{d^3}$	48
default	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln\left(c+d\sqrt{x}\right)}{d^3}$	48

```
[In] int((a+b*x^(1/2))/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/2*b*d*x+a*d*x^(1/2)-b*c*x^(1/2))-2*(a*d-b*c)*c/d^3*ln(c+d*x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\frac{bd^2x+2\,(bc^2-acd)\log\left(d\sqrt{x}+c\right)-2\,(bcd-ad^2)\sqrt{x}}{d^3}$$

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x)
)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2}+\frac{2a\sqrt{x}}{d}+\frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3}-\frac{2bc\sqrt{x}}{d^2}+\frac{bx}{d} & \text{for } d\neq 0\\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/
d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(
3/2)/3)/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2}+\frac{2\,(bc^2-acd)\log\left(d\sqrt{x}+c\right)}{d^3}$$

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)/
d^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{bdx-2\,bc\sqrt{x}+2\,ad\sqrt{x}}{d^2}+\frac{2\,(bc^2-acd)\log\left(\left|d\sqrt{x}+c\right|\right)}{d^3}$$

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*s
qrt(x) + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \sqrt{x}\left(\frac{2\,a}{d}-\frac{2\,b\,c}{d^2}\right)+\frac{\ln\left(c+d\,\sqrt{x}\right)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{d^3}+\frac{b\,x}{d}$$

```
[In] int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

```
[Out] x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))/
d^3 + (b*x)/d
```

3.277 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {383, 78}

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx = -3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

```
[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 3\text{Subst}\left(\int \frac{(-1+x)x^2}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\text{Subst}\left(\int \left(2-2x+x^2-\frac{2}{1+x}\right)dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
default	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(1+x^{\frac{1}{3}}\right)$	21
trager	$-1+x+6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	32
meijerg	$\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}-6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1+x^{\frac{1}{3}}\right)+\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}$	39

```
[In] int((x^(1/3)-1)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = -3x^{\frac{2}{3}} + 6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right)$$

```
[In] integrate((-1+x**(1/3))/(1+x**(1/3)),x)
[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 6\,\ln\left(x^{1/3}+1\right) + 6\,x^{1/3} - 3\,x^{2/3}$$

```
[In] int((x^(1/3) - 1)/(x^(1/3) + 1),x)
[Out] x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)
```

3.278 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx = 6\sqrt[3]{x}+x-6\text{arctanh}\left(\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/3)+x-6*arctanh(x^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {383, 470, 327, 213}

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx = -6\text{arctanh}\left(\sqrt[3]{x}\right)+x+6\sqrt[3]{x}$$

```
[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^2(1+x^2)}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= x + 6\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= 6\sqrt[3]{x} + x + 6\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= 6\sqrt[3]{x} + x - 6\tanh^{-1}\left(\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x} + x - 6\text{arctanh}\left(\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
default	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	24
trager	$-2+x+6x^{\frac{1}{3}}+3\ln\left(-\frac{2x^{\frac{2}{3}}-2x^{\frac{1}{3}}-x+1}{1+x}\right)$	34
meijerg	$-\frac{3i\left(2ix^{\frac{1}{3}}-2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3i\left(-\frac{2ix^{\frac{1}{3}}\left(5x^{\frac{2}{3}}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}$	43

```
[In] int((1+x^(2/3))/(-1+x^(2/3)),x,method=_RETURNVERBOSE)
[Out] x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(1+x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

```
[In] integrate((1+x**(2/3))/(-1+x**(2/3)),x)
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x + 6\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right) + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x + 6\,x^{\frac{1}{3}} - 3\,\log\left(x^{\frac{1}{3}} + 1\right) + 3\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x - 6\,\mathrm{atanh}\left(x^{1/3}\right) + 6\,x^{1/3}$$

```
[In] int((x^(2/3) + 1)/(x^(2/3) - 1),x)
[Out] x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

3.279 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)$$

```
[Out] -128*x^(1/4)+x+256/3*2^(1/3)*ln(2*2^(1/3)+x^(1/4))-128/3*2^(1/3)*ln(4*2^(2/
3)-2*2^(1/3)*x^(1/4)+x^(1/2))-256/3*2^(1/3)*arctan(1/6*(2^(1/3)-x^(1/4))*2^
(2/3)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {383, 470, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -\frac{256\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}+x-128\sqrt[4]{x} + \frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)$$

```
[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int \frac{x^3(-16+x^3)}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= x - 128\text{Subst}\left(\int \frac{x^3}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x} + x + 2048\text{Subst}\left(\int \frac{1}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x} + x + \frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+x}\,dx, x, \sqrt[4]{x}\right)\\
&\quad + \frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{4\sqrt[3]{2}-x}{4\ 2^{2/3}-2\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x} + x + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)\\
&\quad - \frac{1}{3}\left(128\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{-2\sqrt[3]{2}+2x}{4\ 2^{2/3}-2\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&\quad + \left(256\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{4\ 2^{2/3}-2\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x} + x + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}\right.\\
&\qquad \left.+\sqrt{x}\right) + \left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}\right)\\
&= -128\sqrt[4]{x} + x - \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2-2^{2/3}\sqrt[4]{x}}{2\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(4+2^{2/3}\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(-8+2\ 2^{2/3}\sqrt[4]{x}-\sqrt[3]{2}\sqrt{x}\right)$$

```
[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[1/Sqrt[3] - x^(1/4)/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (128*2^(1/3)*Log[-
8 + 2*2^(2/3)*x^(1/4) - 2^(1/3)*Sqrt[x]])/3
```

Maple [A] (verified)

Time = 56.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
derivativedivides	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right)}{3}$
default	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right)}{3}$
meijerg	$-\frac{128\,2^{\frac{1}{3}}\left(\frac{3\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}\left(\frac{2\,2^{\frac{1}{3}}\ln\left(1+\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}\right)}{x^{\frac{1}{4}}}-\frac{2^{\frac{1}{3}}\ln\left(1-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}+\frac{2^{\frac{1}{3}}\sqrt{x}}{8}\right)}{x^{\frac{1}{4}}}+\frac{2\,2^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{8-2^{\frac{2}{3}}x^{\frac{1}{4}}}\right)}{x^{\frac{1}{4}}}\right)}{4}\right)}{3}+\frac{128\,2^{\frac{1}{3}}}{}$
trager	Expression too large to display

```
[In] int((-16+x^(3/4))/(16+x^(3/4)),x,method=_RETURNVERBOSE)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)-
16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8*1
6^(2/3)*x^(1/4)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}x^{\frac{1}{4}} - \frac{1}{3}\sqrt{3}\right) - \frac{128}{3} \cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}} - 2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}} + \sqrt{x}\right) + \frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}} + x^{\frac{1}{4}}\right) + x - 128\,x^{\frac{1}{4}}$$

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*l
og(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x} + x + \frac{256\cdot\sqrt[3]{2}\log\left(\sqrt[4]{x} + 2\cdot\sqrt[3]{2}\right)}{3} - \frac{128\cdot\sqrt[3]{2}\log\left(-2\cdot\sqrt[3]{2}\sqrt[4]{x} + \sqrt{x} + 4\cdot 2^{\frac{2}{3}}\right)}{3} + \frac{256\cdot\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/3
)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(3)
*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}} - x^{\frac{1}{4}}\right)\right) - \frac{128}{3} \cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}} - 2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}} + \sqrt{x}\right) + \frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}} + x^{\frac{1}{4}}\right) + x - 128\,x^{\frac{1}{4}}$$

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = x+\frac{256\,2^{1/3}\,\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}-6144\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

```
[In] int((x^(3/4) - 16)/(x^(3/4) + 16),x)
```

```
[Out] x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128*
2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))
/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2
)*1i + 1))/3
```

3.280 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {381, 383, 78}

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx = -3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx \\
&= 3\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)dx, x, \sqrt[3]{x}\right) \\
&= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(-1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[-1 + x^(1/3)]
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
default	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
trager	$2-x-6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x+1\right)$	34
meijerg	$-\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}+6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1-x^{\frac{1}{3}}\right)-\frac{x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}+6\right)}{2}$	41

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -3x^{\frac{2}{3}}-6\sqrt[3]{x}-x-6\log\left(\sqrt[3]{x}-1\right)$$

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-6\,\ln\left(x^{1/3}-1\right)-6\,x^{1/3}-3\,x^{2/3}$$

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

3.281 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx = \frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] a^2*x*hypergeom([-3/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a
+b*x^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {259, 252, 251}

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx = \frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[In] Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int\left(a^2-b^2x^{2n}\right)^{3/2}\,dx}{\sqrt{a^2-b^2x^{2n}}}\\ &= \frac{\left(a^2\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int\left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\\ &= \frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int(a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[In] Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1 +
 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F]

$$\int (a - b\,x^{n})^{\frac{3}{2}} (a + b\,x^{n})^{\frac{3}{2}}\,dx$$

```
[In] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
[Out] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a - bx^{n})^{3/2} (a + bx^{n})^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (a - bx^{n})^{3/2} (a + bx^{n})^{3/2}\,dx = \int (a - bx^{n})^{\frac{3}{2}} (a + bx^{n})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
[Out] Integral((a - b*x**n)**(3/2)*(a + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (a - bx^{n})^{3/2} (a + bx^{n})^{3/2}\,dx = \int (bx^{n} + a)^{\frac{3}{2}}(-bx^{n} + a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \int (a+b\,x^n)^{3/2}\,(a-b\,x^n)^{3/2}\,dx$$

```
[In] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2),x)
[Out] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2), x)
```

3.282 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] x*hypergeom([-1/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a+b*x
^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {259, 252, 251}

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[In] Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int\sqrt{a^2-b^2x^{2n}}\,dx}{\sqrt{a^2-b^2x^{2n}}}$$

$$= \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int\sqrt{1-\frac{b^2x^{2n}}{a^2}}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

$$= \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int\sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[In] Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/(
2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F]

$$\int \sqrt{a - b\,x^n}\,\sqrt{a + b\,x^n}dx$$

```
[In] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
[Out] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a - bx^n}\sqrt{a + bx^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{a - bx^n}\sqrt{a + bx^n}\,dx = \int \sqrt{a - bx^n}\sqrt{a + bx^n}\,dx$$

```
[In] integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
[Out] Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{a - bx^n}\sqrt{a + bx^n}\,dx = \int \sqrt{bx^n + a}\sqrt{-bx^n + a}\,dx$$

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{a+b\,x^n}\,\sqrt{a-b\,x^n}\,dx$$

```
[In] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2),x)
[Out] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2), x)
```

3.283 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), \frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)/
((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {259, 252, 251}

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), \frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/
2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a-bx^n)^p(a+bx^n)^p\left(a^2-b^2x^{2n}\right)^{-p}\right)\int\left(a^2-b^2x^{2n}\right)^p\,dx\\ &= \left((a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\ &= x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int(a-bx^n)^p(a+bx^n)^p\,dx=x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
(b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F]

$$\int (a - b\,x^n)^p\,(a + b\,x^n)^p\,dx$$

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p,x)
[Out] int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a - bx^n)^p\,(a + bx^n)^p\;dx = \int (bx^n + a)^p(-bx^n + a)^p\,dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F]

$$\int (a - bx^n)^p\,(a + bx^n)^p\;dx = \int (a - bx^n)^p\,(a + bx^n)^p\;dx$$

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int (a - bx^n)^p\,(a + bx^n)^p\;dx = \int (bx^n + a)^p(-bx^n + a)^p\,dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Giac [F]

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (bx^n + a)^p (-bx^n + a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (a + b\,x^n)^p \, (a - b\,x^n)^p \, dx$$

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p,x)
[Out] int((a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.284 $\int (a+bx^n)(c+dx^n)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int (a+bx^n)(c+dx^n)^4 \, dx = ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d^3(4bc+ad)x^{1+4n}}{1+4n}+\frac{bd^4x^{1+5n}}{1+5n}$$

```
[Out] a*c^4*x+c^3*(4*a*d+b*c)*x^(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^(1+2*n)/(1+2*
n)+2*c*d^2*(2*a*d+3*b*c)*x^(1+3*n)/(1+3*n)+d^3*(a*d+4*b*c)*x^(1+4*n)/(1+4*n
)+b*d^4*x^(1+5*n)/(1+5*n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^4 + c^3(bc+4ad)x^n + 2c^2d(2bc+3ad)x^{2n} + 2cd^2(3bc+2ad)x^{3n} \right. \\ &\qquad \left. + d^3(4bc+ad)x^{4n} + bd^4x^{5n}\right) \, dx \\ &= ac^4x + \frac{c^3(bc+4ad)x^{1+n}}{1+n} + \frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n} \\ &\quad + \frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n} + \frac{d^3(4bc+ad)x^{1+4n}}{1+4n} + \frac{bd^4x^{1+5n}}{1+5n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \frac{bx(c+dx^n)^5 - (bc-ad(1+5n))x\left(c^4 + \frac{4c^3dx^n}{1+n} + \frac{6c^2d^2x^{2n}}{1+2n} + \frac{4cd^3x^{3n}}{1+3n} + \frac{d^4x^{4n}}{1+4n}\right)}{d+5dn}$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n) +
 (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*n)
)/(1 + 4*n)))/(d + 5*d*n)
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^4x + \frac{b\,d^4x\,x^{5n}}{5n+1} + \frac{c^3(4ad+bc)x\,x^n}{1+n} + \frac{d^3(ad+4bc)x\,x^{4n}}{1+4n} + \frac{2c\,d^2(2ad+3bc)x\,x^{3n}}{1+3n} + \frac{2c^2d(3ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^4x + \frac{b\,d^4x\,\mathrm{e}^{5n\ln(x)}}{5n+1} + \frac{c^3(4ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n} + \frac{d^3(ad+4bc)x\,\mathrm{e}^{4n\ln(x)}}{1+4n} + \frac{2c\,d^2(2ad+3bc)x\,\mathrm{e}^{3n\ln(x)}}{1+3n} + \frac{2c^2d(3ad+2bc)x}{1+2n}$
parallelrisch	$244x\,x^{4n}bc\,d^3n^3+354x\,x^{2n}a\,c^2d^2n^2+236x\,x^{2n}b\,c^3d\,n^2+164x\,x^{4n}bc\,d^3n^2+a\,c^4x+360x\,x^{2n}a\,c^2d^2n^4+240x\,x^{2n}b\,c^3d\,n^4+196x\,x^3$

```
[In] int((a+b*x^n)*(c+d*x^n)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^4*x+b*d^4/(5*n+1)*x*(x^n)^5+c^3*(4*a*d+b*c)/(1+n)*x*x^n+d^3*(a*d+4*b*c)
/(1+4*n)*x*(x^n)^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*(x^n)^3+2*c^2*d*(3*a*d+2
*b*c)/(1+2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(132) = 264$.

Time = 0.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.99

$$\int (a+bx^n)(c+dx^n)^4\,dx = \frac{(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4)xx^{5\,n}+(4\,bcd^3+ad^4+30\,(4\,bcd^3+ad^4)n^4+61\,(4\,bcd^3}{}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n
) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4
)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) +
2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2
*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2
+ 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*
a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^
2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d
 + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 +
4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c
^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 2
74*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2744 vs. $2(124) = 248$.

Time = 0.73 (sec) , antiderivative size = 2744, normalized size of antiderivative = 20.79

$$\int (a+bx^n)(c+dx^n)^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqr
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
```

```
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
*4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
*2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
*2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
 + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c*d**3*x*x**(3*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x
**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 11*a
*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
20*b*c**4*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 154*b*c**4*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 71*b*c**4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 236*b*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4
```

```
+ 225*n**3 + 85*n**2 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*b*c**2*d**2*n*
*3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 294*b
*c**2*d**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 72*b*c**2*d**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c*d**3*n**2*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 44*b*c*d**3*n*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**
(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**
4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 10*b*d**4*n*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + b*d**4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^4\,dx = ac^4x + \frac{bd^4x^{5n+1}}{5n+1} + \frac{4\,bcd^3x^{4n+1}}{4n+1} + \frac{ad^4x^{4n+1}}{4n+1} + \frac{6\,bc^2d^2x^{3n+1}}{3n+1} + \frac{4\,acd^3x^{3n+1}}{3n+1} + \frac{4\,bc^3dx^{2n+1}}{2n+1} + \frac{6\,ac^2d^2x^{2n+1}}{2n+1} + \frac{bc^4x^{n+1}}{n+1} + \frac{4\,ac^3dx^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) + a
*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^3*
x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^(2*
n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(132) = 264.

Time = 0.29 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.61

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \frac{120\,ac^4n^5x + 24\,bd^4n^4xx^{5\,n} + 120\,bcd^3n^4xx^{4\,n} + 30\,ad^4n^4xx^{4\,n} + 240\,bc^2d^2n^4xx^{3\,n} + 160\,acd^3n^4xx^{3\,n} + 240}{}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30*
a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(3*
n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4*n^
4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5*n)
+ 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n^3*x*
x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642*a*c^2
*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 225*a*c^
4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a*d^4*n^2
*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*n) + 236*
b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^2*x*x^n +
284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) + 44*b*c*d^3*
n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) + 48*a*c*d^3*
n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n) + 14*b*c^4*
n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n) + 4*b*c*d^3*x
*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d^3*x*x^(3*n) +
4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n + 4*a*c^3*d*x*x^n
 + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (a+bx^n)(c+dx^n)^4 \, dx = a\,c^4\,x + \frac{x\,x^n\,(b\,c^4+4\,a\,d\,c^3)}{n+1} + \frac{x\,x^{4\,n}\,(a\,d^4+4\,b\,c\,d^3)}{4\,n+1} + \frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1} + \frac{2\,c^2\,d\,x\,x^{2\,n}\,(3\,a\,d+2\,b\,c)}{2\,n+1} + \frac{2\,c\,d^2\,x\,x^{3\,n}\,(2\,a\,d+3\,b\,c)}{3\,n+1}$$

```
[In] int((a + b*x^n)*(c + d*x^n)^4,x)
```

```
[Out] a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*d
^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d +
2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

3.285 $\int (a+bx^n)(c+dx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int (a+bx^n)(c+dx^n)^3\,dx = ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}$$

```
[Out] a*c^3*x+c^2*(3*a*d+b*c)*x^(1+n)/(1+n)+3*c*d*(a*d+b*c)*x^(1+2*n)/(1+2*n)+d^2
*(a*d+3*b*c)*x^(1+3*n)/(1+3*n)+b*d^3*x^(1+4*n)/(1+4*n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^n)(c+dx^n)^3\,dx = \frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 +
 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1
+ 4*n))/(1 + 4*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^3 + c^2(bc+3ad)x^n + 3cd(bc+ad)x^{2n} + d^2(3bc+ad)x^{3n} + bd^3x^{4n}\right) dx \\ &= ac^3x + \frac{c^2(bc+3ad)x^{1+n}}{1+n} + \frac{3cd(bc+ad)x^{1+2n}}{1+2n} + \frac{d^2(3bc+ad)x^{1+3n}}{1+3n} + \frac{bd^3x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int (a+bx^n)(c+dx^n)^3 \, dx = \frac{bx(c+dx^n)^4 - (bc-ad(1+4n))x\left(c^3 + \frac{3c^2dx^n}{1+n} + \frac{3cd^2x^{2n}}{1+2n} + \frac{d^3x^{3n}}{1+3n}\right)}{d+4dn}$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n) +
 (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^3x + \frac{b\,d^3x\,x^{4n}}{1+4n} + \frac{c^2(3ad+bc)x\,x^n}{1+n} + \frac{d^2(ad+3bc)x\,x^{3n}}{1+3n} + \frac{3cd(ad+bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^3x + \frac{b\,d^3x\,\mathrm{e}^{4n\ln(x)}}{1+4n} + \frac{c^2(3ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n} + \frac{d^2(ad+3bc)x\,\mathrm{e}^{3n\ln(x)}}{1+3n} + \frac{3cd(ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\underline{21x\,x^{3n}bc\,d^2n+57x\,x^{2n}ac\,d^2n^2+a\,c^3x+9x\,x^nb\,c^3n+3x\,x^na\,c^2d+78x\,x^na\,c^2d\,n^2+27x\,x^na\,c^2dn+24x\,x^{2n}ac\,d^2n+24x\,x^nb\,c^3n^3+}$

```
[In] int((a+b*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^3*x+b*d^3/(1+4*n)*x*(x^n)^4+c^2*(3*a*d+b*c)/(1+n)*x*x^n+d^2*(a*d+3*b*c)
/(1+3*n)*x*(x^n)^3+3*c*d*(a*d+b*c)/(1+2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(99) = 198$.

Time = 0.25 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.22

$$\int (a+bx^n)(c+dx^n)^3\,dx = \frac{(6\,bd^3n^3+11\,bd^3n^2+6\,bd^3n+bd^3)xx^{4\,n}+(3\,bcd^2+ad^3+8\,(3\,bcd^2+ad^3)n^3+14\,(3\,bcd^2+ad^3)n^2+7\,(3}{}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d
^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^
3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^
3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9
*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2
 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(90) = 180$.

Time = 0.49 (sec) , antiderivative size = 1540, normalized size of antiderivative = 15.56

$$\int (a+bx^n)(c+dx^n)^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n,
 -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x
) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, E
q(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d*
*3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) -
 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d
**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x)
+ 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n*
```

```
*2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 5
7*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*
c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*
c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3
*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*
*3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x
**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^3\,dx = ac^3x + \frac{bd^3x^{4n+1}}{4n+1} + \frac{3\,bcd^2x^{3n+1}}{3n+1} + \frac{ad^3x^{3n+1}}{3n+1} + \frac{3\,bc^2dx^{2n+1}}{2n+1} + \frac{3\,acd^2x^{2n+1}}{2n+1} + \frac{bc^3x^{n+1}}{n+1} + \frac{3\,ac^2dx^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) + a
*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.55

$$\int (a+bx^n)(c+dx^n)^3\,dx = \frac{24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3xx}{}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^
3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*
b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^
(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*
x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*
x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a
*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^
3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2
*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) +
b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n +
 1)
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(c+dx^n)^3\,dx = a\,c^3\,x + \frac{x\,x^n\,(b\,c^3+3\,a\,d\,c^2)}{n+1} + \frac{x\,x^{3\,n}\,(a\,d^3+3\,b\,c\,d^2)}{3\,n+1} + \frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1} + \frac{3\,c\,d\,x\,x^{2\,n}\,(a\,d+b\,c)}{2\,n+1}$$

```
[In] int((a + b*x^n)*(c + d*x^n)^3,x)
```

```
[Out] a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*d
^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*c)
)/(2*n + 1)
```

3.286 $\int (a+bx^n)(c+dx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)(c+dx^n)^2\,dx=ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}$$

```
[Out] a*c^2*x+c*(2*a*d+b*c)*x^(1+n)/(1+n)+d*(a*d+2*b*c)*x^(1+2*n)/(1+2*n)+b*d^2*x
^(1+3*n)/(1+3*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^n)(c+dx^n)^2\,dx=\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ac^2+c(bc+2ad)x^n+d(2bc+ad)x^{2n}+bd^2x^{3n}\right)\,dx\\&=ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(c+dx^n)^2\,dx=\frac{bx(c+dx^n)^3-(bc-ad(1+3n))x\left(c^2+\frac{2cdx^n}{1+n}+\frac{d^2x^{2n}}{1+2n}\right)}{d+3dn}$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] (b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) + (
d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^2x+\frac{b\,d^2x\,x^{3n}}{1+3n}+\frac{c(2ad+bc)x\,x^n}{1+n}+\frac{d(ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^2x+\frac{b\,d^2x\,e^{3n\ln(x)}}{1+3n}+\frac{c(2ad+bc)x\,e^{n\ln(x)}}{1+n}+\frac{d(ad+2bc)x\,e^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{2x\,x^{3n}b\,d^2n^2+3x\,x^{3n}b\,d^2n+3x\,x^{2n}a\,d^2n^2+6x\,x^{2n}bcd\,n^2+b\,d^2x\,x^{3n}+4x\,x^{2n}a\,d^2n+8x\,x^{2n}bcdn+12x\,x^nacd\,n^2+6x\,x^nb\,c^2n^2+}{(1+3n)(1+n)(1+2n}$

```
[In] int((a+b*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^2*x+b*d^2/(1+3*n)*x*(x^n)^3+c*(2*a*d+b*c)/(1+n)*x*x^n+d*(a*d+2*b*c)/(1+
2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)(c+dx^n)^2\,dx$$
$$=\frac{(2\,bd^2n^2+3\,bd^2n+bd^2)xx^{3\,n}+(2\,bcd+ad^2+3\,(2\,bcd+ad^2)n^2+4\,(2\,bcd+ad^2)n)xx^{2\,n}+(bc^2+2\,acd+}{6\,n^3+11\,n^2+6\,n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*
d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b
*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^
2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.36 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a + bx^n)(c + dx^n)^2 \, dx$$
$$= \begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2ac}{6n^3+11} \end{cases}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x
 - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x)
 + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c
**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*
d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*
a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d*
*2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(
6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)(c+dx^n)^2 \, dx = ac^2x + \frac{bd^2x^{3n+1}}{3n+1} + \frac{2bcdx^{2n+1}}{2n+1} + \frac{ad^2x^{2n+1}}{2n+1} + \frac{bc^2x^{n+1}}{n+1} + \frac{2acdx^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*d
^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n +
 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)(c+dx^n)^2\,dx = \frac{6\,ac^2n^3x + 2\,bd^2n^2xx^{3\,n} + 6\,bcdn^2xx^{2\,n} + 3\,ad^2n^2xx^{2\,n} + 6\,bc^2n^2xx^n + 12\,acdn^2xx^n + 11\,ac^2n^2x + 3\,bd^2nx}{}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^
2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b
*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*
x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n)
+ a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)(c+dx^n)^2\,dx = a\,c^2\,x + \frac{x\,x^{2\,n}\,(a\,d^2 + 2\,b\,c\,d)}{2\,n+1} + \frac{x\,x^n\,(b\,c^2 + 2\,a\,d\,c)}{n+1} + \frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

```
[In] int((a + b*x^n)*(c + d*x^n)^2,x)
```

```
[Out] a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*d
))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

3.287 $\int (a+bx^n)(c+dx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int (a+bx^n)(c+dx^n)\,dx = acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}$$

```
[Out] a*c*x+(a*d+b*c)*x^(1+n)/(1+n)+b*d*x^(1+2*n)/(1+2*n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(ac+(bc+ad)x^n+bdx^{2n}\right)\,dx$$

$$= acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (a+bx^n)(c+dx^n)\,dx = x\left(ac+\frac{(bc+ad)x^n}{1+n}+\frac{bdx^{2n}}{1+2n}\right)$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
risch	$acx+\frac{(ad+bc)x\,x^n}{1+n}+\frac{bdx\,x^{2n}}{1+2n}$	39
norman	$acx+\frac{(ad+bc)x\,e^{n\ln(x)}}{1+n}+\frac{bdx\,e^{2n\ln(x)}}{1+2n}$	43
parallelrisch	$\frac{x\,x^{2n}bdn+bdx\,x^{2n}+2x\,x^nadn+2x\,x^nbcn+2xac\,n^2+x\,x^nad+x\,x^nbc+3xacn+acx}{(1+n)(1+2n)}$	84

```
[In] int((a+b*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+(a*d+b*c)/(1+n)*x*x^n+b*d/(1+2*n)*x*(x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{(bdn+bd)xx^{2n}+(bc+ad+2(bc+ad)n)xx^n+(2acn^2+3acn+ac)x}{2n^2+3n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2
 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.90

$$\int (a + bx^n)(c + dx^n)\ dx$$
$$= \begin{cases} acx + ad\log(x) + bc\log(x) - \frac{bd}{x} & \text{fo} \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log(x) & \text{fo} \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{ot} \end{cases}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n),x)
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*
a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n*
*2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2
*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*
x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n
)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a + bx^n)(c + dx^n)\ dx = acx + \frac{bdx^{2n+1}}{2n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
[Out] a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/(
n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int (a + bx^n)(c + dx^n)\ dx$$
$$= \frac{2\,acn^2x + bdnxx^{2\,n} + 2\,bcnxx^n + 2\,adnxx^n + 3\,acnx + bdxx^{2\,n} + bcxx^n + adxx^n + acx}{2\,n^2 + 3\,n + 1}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
[Out] (2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
+ b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (a+bx^n)\,(c+dx^n)\;dx = a\,c\,x+\frac{x\,x^n\,(a\,d+b\,c)}{n+1}+\frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

```
[In] int((a + b*x^n)*(c + d*x^n),x)
[Out] a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

3.288 $\int \frac{a+bx^n}{c+dx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{bx}{d} - \frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

```
[Out] b*x/d-(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {396, 251}

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{bx}{d} - \frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

```
[In] Int[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^n}\,dx}{d} \\ &= \frac{bx}{d} - \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{x\left(bc+(-bc+ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{cd}$$

```
[In] Integrate[(a + b*x^n)/(c + d*x^n),x]
[Out] (x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(c*d)
```

Maple [F]

$$\int \frac{a+b\,x^n}{c+d\,x^n}dx$$

```
[In] int((a+b*x^n)/(c+d*x^n),x)
[Out] int((a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \int \frac{bx^n+a}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral((b*x^n + a)/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{ac^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{bc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate((a+b*x**n)/(c+d*x**n),x)
```

```
[Out] a*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma
(1/n)/(n**2*gamma(1 + 1/n)) - b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n)*x*lerch
phi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(c*c**(1/n
)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \int \frac{bx^n+a}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

Giac [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \int \frac{bx^n+a}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\int \frac{a+b\,x^n}{c+d\,x^n}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n),x)
[Out] int((a + b*x^n)/(c + d*x^n), x)
```

3.289 $\int \frac{a+bx^n}{(c+dx^n)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{a+bx^n}{(c+dx^n)^2} dx = -\frac{(bc-ad)x}{cdn(c+dx^n)} + \frac{(bc-ad(1-n))x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}$$

```
[Out] -(-a*d+b*c)*x/c/d/n/(c+d*x^n)+(b*c-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],
-d*x^n/c)/c^2/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 251}

$$\int \frac{a+bx^n}{(c+dx^n)^2} dx = \frac{x(bc-ad(1-n)) \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn} - \frac{x(bc-ad)}{cdn(c+dx^n)}$$

```
[In] Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))\int\frac{1}{c+dx^n}\,dx}{cdn}$$
$$= -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int\frac{a+bx^n}{(c+dx^n)^2}\,dx=\frac{x\left(\frac{b}{c+dx^n}-\frac{(bc+ad(-1+n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
[Out] (x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

Maple [F]

$$\int\frac{a+b\,x^n}{(c+d\,x^n)^2}dx$$

```
[In] int((a+b*x^n)/(c+d*x^n)^2,x)
[Out] int((a+b*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx=\int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
[Out] integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.15

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = a\left(\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}nx\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnxx^n\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dxx^n\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right)+b\left(\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}n^2x^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}+\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}nx^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{cc^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}x^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{c^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}dnx^nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{c^{-3-\frac{1}{n}}c^{1+\frac{1}{n}}dx^nx^{n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{cn^3\Gamma\left(2+\frac{1}{n}\right)+dn^3x^n\Gamma\left(2+\frac{1}{n}\right)}\right)$$

```
[In] integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] a*(c*c**(1/n)*c**(-2 - 1/n)*n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*
gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) + c*c**(1/n
)*c**(-2 - 1/n)*n*x*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1
 + 1/n)) - c*c**(1/n)*c**(-2 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1,
 1/n)*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) + c**
(1/n)*c**(-2 - 1/n)*d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*g
amma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) - c**(1/n)*c
**(-2 - 1/n)*d*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)
/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + b*(c*c**(-3 - 1/n)
*c**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**
3*x**n*gamma(2 + 1/n)) - c*c**(-3 - 1/n)*c**(1 + 1/n)*n*x**(n + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n
) + d*n**3*x**n*gamma(2 + 1/n)) + c*c**(-3 - 1/n)*c**(1 + 1/n)*n*x**(n + 1)
*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n)) - c*c*
*(-3 - 1/n)*c**(1 + 1/n)*x**(n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1
 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))
 - c**(-3 - 1/n)*c**(1 + 1/n)*d*n*x**n*x**(n + 1)*lerchphi(d*x**n*exp_polar
(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*g
amma(2 + 1/n)) - c**(-3 - 1/n)*c**(1 + 1/n)*d*x**n*x**(n + 1)*lerchphi(d*x*
*n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d
*n**3*x**n*gamma(2 + 1/n)))
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx=\int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*x
/(c*d^2*n*x^n + c^2*d*n)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx=\int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx=\int \frac{a+b\,x^n}{(c+d\,x^n)^2}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n)^2,x)
[Out] int((a + b*x^n)/(c + d*x^n)^2, x)
```

3.290 $\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad(1-2n))x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3dn}$$

```
[Out] -1/2*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^2+1/2*(b*c-a*d*(1-2*n))*x*hypergeom([2, 1
/n],[1+1/n],-d*x^n/c)/c^3/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 251}

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \frac{x(bc-ad(1-2n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

```
[In] Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hyper
geometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad(1-2n))\int\frac{1}{(c+dx^n)^2}\,dx}{2cdn}\\ &= -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad(1-2n))x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int\frac{a+bx^n}{(c+dx^n)^3}\,dx=\frac{x\left(\frac{b}{(c+dx^n)^2}-\frac{(bc+ad(-1+2n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
[Out] (x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

Maple [F]

$$\int\frac{a+b\,x^n}{(c+d\,x^n)^3}dx$$

```
[In] int((a+b*x^n)/(c+d*x^n)^3,x)
[Out] int((a+b*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
[Out] integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*x**n)/(c+d*x**n)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
[Out] ((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3*
d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*c^
2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{a+b\,x^n}{(c+d\,x^n)^3}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n)^3,x)
[Out] int((a + b*x^n)/(c + d*x^n)^3, x)
```

3.291 $\int \frac{a+bx^n}{(c+dx^n)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{a+bx^n}{(c+dx^n)^4} dx = -\frac{(bc-ad)x}{3cdn(c+dx^n)^3} + \frac{(bc-ad(1-3n))x \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{3c^4dn}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^3+1/3*(b*c-a*d*(1-3*n))*x*hypergeom([3, 1
/n],[1+1/n],-d*x^n/c)/c^4/d/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 251}

$$\int \frac{a+bx^n}{(c+dx^n)^4} dx = \frac{x(bc-ad(1-3n)) \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc-ad)}{3cdn(c+dx^n)^3}$$

```
[In] Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hyper
geometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))\int\frac{1}{(c+dx^n)^3}\,dx}{3cdn}\\ &= -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int\frac{a+bx^n}{(c+dx^n)^4}\,dx=\frac{x\left(\frac{b}{(c+dx^n)^3}-\frac{(bc+ad(-1+3n))\,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
[Out] (x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

Maple [F]

$$\int\frac{a+b\,x^n}{(c+d\,x^n)^4}dx$$

```
[In] int((a+b*x^n)/(c+d*x^n)^4,x)
[Out] int((a+b*x^n)/(c+d*x^n)^4,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
[Out] integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4
*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*x**n)/(c+d*x**n)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
[Out] ((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^3
*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*((((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(2*
n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*x^n
 - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n^3*x
^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^4}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n)^4,x)
[Out] int((a + b*x^n)/(c + d*x^n)^4, x)
```

3.292 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{ad^2(2bd+3ae)x^{1+n}}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{1+3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{1+4n}}{1+4n} + \frac{b^2e^3x^{1+5n}}{1+5n}$$

```
[Out] a^2*d^3*x+a*d^2*(3*a*e+2*b*d)*x^(1+n)/(1+n)+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)
*x^(1+2*n)/(1+2*n)+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)*x^(1+3*n)/(1+3*n)+b*e^2*
(2*a*e+3*b*d)*x^(1+4*n)/(1+4*n)+b^2*e^3*x^(1+5*n)/(1+5*n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \frac{dx^{2n+1}(3a^2e^2+6abde+b^2d^2)}{2n+1} + \frac{ex^{3n+1}(a^2e^2+6abde+3b^2d^2)}{3n+1} + a^2d^3x + \frac{ad^2x^{n+1}(3ae+2bd)}{n+1} + \frac{be^2x^{4n+1}(2ae+3bd)}{4n+1} + \frac{b^2e^3x^{5n+1}}{5n+1}$$

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
```

```
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2d^3 + ad^2(2bd+3ae)x^n + d\left(b^2d^2+6abde+3a^2e^2\right)x^{2n} \right. \\ &\quad \left. + e\left(3b^2d^2+6abde+a^2e^2\right)x^{3n} + be^2(3bd+2ae)x^{4n} + b^2e^3x^{5n}\right)\,dx \\ &= a^2d^3x + \frac{ad^2(2bd+3ae)x^{1+n}}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n} \\ &\quad + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{1+3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{1+4n}}{1+4n} + \frac{b^2e^3x^{1+5n}}{1+5n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2\,(d+ex^n)^3\,dx = x\left(a^2d^3 + \frac{ad^2(2bd+3ae)x^n}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{4n}}{1+4n} + \frac{b^2e^3x^{5n}}{1+5n}\right)$$

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
+ 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^(3
*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^(5*
n))/(1 + 5*n))
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
risch	$x\,a^2d^3+\frac{b^2e^3x\,x^{5n}}{5n+1}+\frac{d(3a^2e^2+6abde+b^2d^2)x\,x^{2n}}{1+2n}+\frac{e(a^2e^2+6abde+3b^2d^2)x\,x^{3n}}{1+3n}+\frac{a\,d^2(3ae+2bd)x\,x^n}{1+n}+\frac{e^2b(2ae-$
norman	$x\,a^2d^3+\frac{b^2e^3x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+\frac{d(3a^2e^2+6abde+b^2d^2)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{e(a^2e^2+6abde+3b^2d^2)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{a\,d^2(3ae+2bd)x\,\mathrm{e}^{n}}{1+n}$
parallelrisch	$3x\,x^na^2d^2e+2x\,x^nab\,d^3+33x\,x^{4n}b^2d\,e^2n+6x\,x^{2n}ab\,d^2e+22x\,x^{4n}ab\,e^3n+82x\,x^{4n}ab\,e^3n^2+180x\,x^{2n}a^2d\,e^2n^4+60x\,x^{4n}ab\,e^3n^4$

```
[In] int((a+b*x^n)^2*(d+e*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^2*d^3+b^2*e^3/(5*n+1)*x*(x^n)^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/(1+2*n)
*x*(x^n)^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(1+3*n)*x*(x^n)^3+a*d^2*(3*a*e+2
*b*d)/(1+n)*x*x^n+e^2*b*(2*a*e+3*b*d)/(1+4*n)*x*(x^n)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(158) = 316$.

Time = 0.26 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.22

$$\int (a+bx^n)^2\,(d+ex^n)^3\,dx$$
$$=\frac{(24\,b^2e^3n^4+50\,b^2e^3n^3+35\,b^2e^3n^2+10\,b^2e^3n+b^2e^3)xx^{5\,n}+(3\,b^2de^2+2\,abe^3+30\,(3\,b^2de^2+2\,abe^3)n^4+}{}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3
)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4 +
 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 11*(
3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^
3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3
*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^2*e +
 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(b^2*d^3
+ 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)
*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 +
 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d
^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)
*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d
^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 +
 15*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3376 vs. 2(151) = 302.

Time = 7.41 (sec) , antiderivative size = 3376, normalized size of antiderivative = 21.37

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*log(x) - 3*a**2*d*e**2/x - a**2*e**3
/(2*x**2) + 2*a*b*d**3*log(x) - 6*a*b*d**2*e/x - 3*a*b*d*e**2/x**2 - 2*a*b*
e**3/(3*x**3) - b**2*d**3/x - 3*b**2*d**2*e/(2*x**2) - b**2*d*e**2/x**3 - b
**2*e**3/(4*x**4), Eq(n, -1)), (a**2*d**3*x + 6*a**2*d**2*e*sqrt(x) + 3*a**
2*d*e**2*log(x) - 2*a**2*e**3/sqrt(x) + 4*a*b*d**3*sqrt(x) + 6*a*b*d**2*e*l
og(x) - 12*a*b*d*e**2/sqrt(x) - 2*a*b*e**3/x + b**2*d**3*log(x) - 6*b**2*d*
*2*e/sqrt(x) - 3*b**2*d*e**2/x - 2*b**2*e**3/(3*x**(3/2)), Eq(n, -1/2)), (a
**2*d**3*x + 9*a**2*d**2*e*x**(2/3)/2 + 9*a**2*d*e**2*x**(1/3) + a**2*e**3*
log(x) + 3*a*b*d**3*x**(2/3) + 18*a*b*d**2*e*x**(1/3) + 6*a*b*d*e**2*log(x)
 - 6*a*b*e**3/x**(1/3) + 3*b**2*d**3*x**(1/3) + 3*b**2*d**2*e*log(x) - 9*b*
*2*d*e**2/x**(1/3) - 3*b**2*e**3/(2*x**(2/3)), Eq(n, -1/3)), (a**2*d**3*x +
 4*a*d**2*x**(3/4)*(3*a*e + 2*b*d)/3 - 4*b**2*e**3/x**(1/4) + 4*b*e**2*(2*a
*e + 3*b*d)*log(x**(1/4)) + 2*d*sqrt(x)*(3*a**2*e**2 + 6*a*b*d*e + b**2*d**
2) + 4*e*x**(1/4)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2), Eq(n, -1/4)), (a**
2*d**3*x + 5*a*d**2*x**(4/5)*(3*a*e + 2*b*d)/4 + 5*b**2*e**3*log(x**(1/5))
+ 5*b*e**2*x**(1/5)*(2*a*e + 3*b*d) + 5*d*x**(3/5)*(3*a**2*e**2 + 6*a*b*d*e
 + b**2*d**2)/3 + 5*e*x**(2/5)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2)/2, Eq(
n, -1/5)), (120*a**2*d**3*n**5*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
+ 15*n + 1) + 274*a**2*d**3*n**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**
2 + 15*n + 1) + 225*a**2*d**3*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 85*a**2*d**3*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 15*a**2*d**3*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + a**2*d**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 360*a**2*d**2*e*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 462*a**2*d**2*e*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 213*a**2*d**2*e*n**2*x*x**n/(120*n**5 + 274*n**
4 + 225*n**3 + 85*n**2 + 15*n + 1) + 42*a**2*d**2*e*n*x*x**n/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 3*a**2*d**2*e*x*x**n/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 180*a**2*d*e**2*n**4*x*x**(2*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 321*a**2*d*e**2*n*
*3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 177*a
**2*d*e**2*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 39*a**2*d*e**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + 3*a**2*d*e**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 40*a**2*e**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 +
```

225*n**3 + 85*n**2 + 15*n + 1) + 78*a**2*e**3*n**3*x*x**(3*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 49*a**2*e**3*n**2*x*x**(3*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 12*a**2*e**3*n*x*x**(3
*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a**2*e**3*x*x**
(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*a*b*d**3*
n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 308*a*b
*d**3*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
42*a*b*d**3*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 28*a*b*d**3*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 2*a*b*d**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 360*a*b*d**2*e*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
2 + 15*n + 1) + 642*a*b*d2*e*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 354*a*b*d**2*e*n**2*x*x**(2*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a*b*d**2*e*n*x*x**(2*n)/(120*n
5 + 274*n4 + 225*n**3 + 85*n**2 + 15*n + 1) + 6*a*b*d**2*e*x*x**(2*n)/(
120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*a*b*d*e**2*n**4*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*a*b*
d*e**2*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1
) + 294*a*b*d*e**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**
2 + 15*n + 1) + 72*a*b*d*e**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3
+ 85*n**2 + 15*n + 1) + 6*a*b*d*e**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*
n**3 + 85*n**2 + 15*n + 1) + 60*a*b*e**3*n**4*x*x**(4*n)/(120*n**5 + 274*n*
*4 + 225*n**3 + 85*n**2 + 15*n + 1) + 122*a*b*e**3*n**3*x*x**(4*n)/(120*n**
5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 82*a*b*e**3*n**2*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 22*a*b*e**3*n*x*x*
*(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 2*a*b*e**3*x
*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 60*b**2*d
3*n4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 107*b**2*d**3*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + 59*b**2*d**3*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 13*b**2*d**3*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*
n**3 + 85*n**2 + 15*n + 1) + b**2*d**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 120*b**2*d**2*e*n**4*x*x**(3*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 234*b**2*d**2*e*n**3*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 147*b**2*d**2*e*n*
*2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 36*b*
*2*d**2*e*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1
) + 3*b**2*d**2*e*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 90*b**2*d*e**2*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 183*b**2*d*e**2*n**3*x*x**(4*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 123*b**2*d*e**2*n**2*x*x**(4*n)/(120*n**
5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 33*b**2*d*e**2*n*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 3*b**2*d*e**2*x*x*
*(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b**2*e**3
*n**4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50

```
*b**2*e**3*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 35*b**2*e**3*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 10*b**2*e**3*n*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + b**2*e**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n*
*3 + 85*n**2 + 15*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.53

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{3b^2de^2x^{4n+1}}{4n+1} + \frac{2abe^3x^{4n+1}}{4n+1} + \frac{3b^2d^2ex^{3n+1}}{3n+1} + \frac{6abde^2x^{3n+1}}{3n+1} + \frac{a^2e^3x^{3n+1}}{3n+1} + \frac{b^2d^3x^{2n+1}}{2n+1} + \frac{6abd^2ex^{2n+1}}{2n+1} + \frac{3a^2de^2x^{2n+1}}{2n+1} + \frac{2abd^3x^{n+1}}{n+1} + \frac{3a^2d^2ex^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1) +
6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*d^3
*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^2*x^
(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n + 1)/(
n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(158) = 316.

Time = 0.30 (sec) , antiderivative size = 962, normalized size of antiderivative = 6.09

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \frac{120a^2d^3n^5x + 24b^2e^3n^4xx^{5n} + 90b^2de^2n^4xx^{4n} + 60abe^3n^4xx^{4n} + 120b^2d^2en^4xx^{3n} + 240abde^2n^4xx^{3n} + \cdots}{\cdots}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] (120*a^2*d^3*n^5*x + 24*b^2*e^3*n^4*x*x^(5*n) + 90*b^2*d*e^2*n^4*x*x^(4*n)
+ 60*a*b*e^3*n^4*x*x^(4*n) + 120*b^2*d^2*e*n^4*x*x^(3*n) + 240*a*b*d*e^2*n^
4*x*x^(3*n) + 40*a^2*e^3*n^4*x*x^(3*n) + 60*b^2*d^3*n^4*x*x^(2*n) + 360*a*b
*d^2*e*n^4*x*x^(2*n) + 180*a^2*d*e^2*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n
+ 360*a^2*d^2*e*n^4*x*x^n + 274*a^2*d^3*n^4*x + 50*b^2*e^3*n^3*x*x^(5*n) +
```

```
183*b^2*d*e^2*n^3*x*x^(4*n) + 122*a*b*e^3*n^3*x*x^(4*n) + 234*b^2*d^2*e*n^3
*x*x^(3*n) + 468*a*b*d*e^2*n^3*x*x^(3*n) + 78*a^2*e^3*n^3*x*x^(3*n) + 107*b
^2*d^3*n^3*x*x^(2*n) + 642*a*b*d^2*e*n^3*x*x^(2*n) + 321*a^2*d*e^2*n^3*x*x^
(2*n) + 308*a*b*d^3*n^3*x*x^n + 462*a^2*d^2*e*n^3*x*x^n + 225*a^2*d^3*n^3*x
 + 35*b^2*e^3*n^2*x*x^(5*n) + 123*b^2*d*e^2*n^2*x*x^(4*n) + 82*a*b*e^3*n^2*
x*x^(4*n) + 147*b^2*d^2*e*n^2*x*x^(3*n) + 294*a*b*d*e^2*n^2*x*x^(3*n) + 49*
a^2*e^3*n^2*x*x^(3*n) + 59*b^2*d^3*n^2*x*x^(2*n) + 354*a*b*d^2*e*n^2*x*x^(2
*n) + 177*a^2*d*e^2*n^2*x*x^(2*n) + 142*a*b*d^3*n^2*x*x^n + 213*a^2*d^2*e*n
^2*x*x^n + 85*a^2*d^3*n^2*x + 10*b^2*e^3*n*x*x^(5*n) + 33*b^2*d*e^2*n*x*x^(
4*n) + 22*a*b*e^3*n*x*x^(4*n) + 36*b^2*d^2*e*n*x*x^(3*n) + 72*a*b*d*e^2*n*x
*x^(3*n) + 12*a^2*e^3*n*x*x^(3*n) + 13*b^2*d^3*n*x*x^(2*n) + 78*a*b*d^2*e*n
*x*x^(2*n) + 39*a^2*d*e^2*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n + 42*a^2*d^2*e*n
*x*x^n + 15*a^2*d^3*n*x + b^2*e^3*x*x^(5*n) + 3*b^2*d*e^2*x*x^(4*n) + 2*a*b
*e^3*x*x^(4*n) + 3*b^2*d^2*e*x*x^(3*n) + 6*a*b*d*e^2*x*x^(3*n) + a^2*e^3*x*
x^(3*n) + b^2*d^3*x*x^(2*n) + 6*a*b*d^2*e*x*x^(2*n) + 3*a^2*d*e^2*x*x^(2*n)
 + 2*a*b*d^3*x*x^n + 3*a^2*d^2*e*x*x^n + a^2*d^3*x)/(120*n^5 + 274*n^4 + 22
5*n^3 + 85*n^2 + 15*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2 d^3 x + \frac{x\,x^{2\,n}\,(3\,a^2\,d\,e^2+6\,a\,b\,d^2\,e+b^2\,d^3)}{2\,n+1} + \frac{x\,x^{3\,n}\,(a^2\,e^3+6\,a\,b\,d\,e^2+3\,b^2\,d^2\,e)}{3\,n+1} + \frac{b^2\,e^3\,x\,x^{5\,n}}{5\,n+1} + \frac{a\,d^2\,x\,x^n\,(3\,a\,e+2\,b\,d)}{n+1} + \frac{b\,e^2\,x\,x^{4\,n}\,(2\,a\,e+3\,b\,d)}{4\,n+1}$$

```
[In] int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

```
[Out] a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) + (
x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x^(
5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*n)*
(2*a*e + 3*b*d))/(4*n + 1)
```

3.293 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{2ad(bd+ae)x^{1+n}}{1+n} + \frac{(b^2d^2+4abde+a^2e^2)\,x^{1+2n}}{1+2n} + \frac{2be(bd+ae)x^{1+3n}}{1+3n} + \frac{b^2e^2x^{1+4n}}{1+4n}$$

```
[Out] a^2*d^2*x+2*a*d*(a*e+b*d)*x^(1+n)/(1+n)+(a^2*e^2+4*a*b*d*e+b^2*d^2)*x^(1+2*
n)/(1+2*n)+2*b*e*(a*e+b*d)*x^(1+3*n)/(1+3*n)+b^2*e^2*x^(1+4*n)/(1+4*n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \frac{x^{2n+1}(a^2e^2+4abde+b^2d^2)}{2n+1} + a^2d^2x + \frac{2adx^{n+1}(ae+bd)}{n+1} + \frac{2bex^{3n+1}(ae+bd)}{3n+1} + \frac{b^2e^2x^{4n+1}}{4n+1}$$

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2d^2+2ad(bd+ae)x^n+\left(b^2d^2+4abde+a^2e^2\right)x^{2n}+2be(bd+ae)x^{3n}+b^2e^2x^{4n}\right)\,dx \\ &= a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+3n}}{1+3n}+\frac{b^2e^2x^{1+4n}}{1+4n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2\,(d+ex^n)^2\,dx = x\left(a^2d^2+\frac{2ad(bd+ae)x^n}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{2n}}{1+2n}+\frac{2be(bd+ae)x^{3n}}{1+3n}+\frac{b^2e^2x^{4n}}{1+4n}\right)$$

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2*
e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^2*
x^(4*n))/(1 + 4*n))
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
risch	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,x^{2n}}{1+2n}+\frac{b^2e^2x\,x^{4n}}{1+4n}+\frac{2ad(ae+bd)x\,x^n}{1+n}+\frac{2be(ae+bd)x\,x^{3n}}{1+3n}$
norman	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2e^2x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{2ad(ae+bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{2be(ae+bd)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\underline{28x\,x^{3n}b^2de\,n^2+14x\,x^{3n}ab\,e^2n+52x\,x^n a^2de\,n^2+35x\,a^2d^2n^2+10x\,a^2d^2n+b^2e^2x\,x^{4n}+18x\,x^n a^2den+4x\,x^{2n}abde+48x\,x^n a^2de}$

```
[In] int((a+b*x^n)^2*(d+e*x^n)^2,x,method=_RETURNVERBOSE)
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*(x^n)^2+b^2*e^2/(1+4*n)*x*(
x^n)^4+2*a*d*(a*e+b*d)/(1+n)*x*x^n+2*b*e*(a*e+b*d)/(1+3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(112) = 224.

Time = 0.25 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.30

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$
$$= \frac{(6\,b^2e^2n^3+11\,b^2e^2n^2+6\,b^2e^2n+b^2e^2)xx^{4\,n}+2\,(b^2de+abe^2+8\,(b^2de+abe^2)n^3+14\,(b^2de+abe^2)n^2+7}{}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b^
2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2 +
7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12*(b
^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^2
+ 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*e + 2
4*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 + a^2*d
*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*a^2*d
^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. 2(104) = 208.

Time = 2.96 (sec) , antiderivative size = 1760, normalized size of antiderivative = 15.71

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(x
) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2/(
3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x) +
 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2*log
(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3*a**2
*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d*e*x**
(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x) - 3*b*
*2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e + b*d)/3
 + 4*b**2*e**2*log(x**(1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - 2*sqrt(x)*(-a**
2*e**2 - 4*a*b*d*e - b**2*d**2), Eq(n, -1/4)), (24*a**2*d**2*n**4*x/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 +
10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a**
2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d*e*n**3*x*x**n
```

```
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
 + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*
*2*e**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*d**2*n
**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b*d**2*n**2*x*x*
*n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*d**2*n*x*x**n/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*d**2*x*x**n/(24*n**4 + 50*n**3 + 3
5*n**2 + 10*n + 1) + 48*a*b*d*e*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 76*a*b*d*e*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 32*a*b*d*e*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 4*a*b*d*e*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16*a*b*e
**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*a*b*e**2*
n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*b*e**2*n*x*
x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*e**2*x*x**(3*n)/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2*d**2*n**3*x*x**(2*n)/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*d**2*n**2*x*x**(2*n)/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*d**2*n*x*x**(2*n)/(24*n**4 + 50*
n**3 + 35*n**2 + 10*n + 1) + b**2*d**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 16*b**2*d*e*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 28*b**2*d*e*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 14*b**2*d*e*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 2*b**2*d*e*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*
e**2*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b**2*e**
2*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e**2*n*
x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*e**2*x*x**(4*n)/
(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{b^2e^2x^{4n+1}}{4n+1} + \frac{2b^2dex^{3n+1}}{3n+1} + \frac{2abe^2x^{3n+1}}{3n+1} + \frac{b^2d^2x^{2n+1}}{2n+1} + \frac{4abdex^{2n+1}}{2n+1} + \frac{a^2e^2x^{2n+1}}{2n+1} + \frac{2abd^2x^{n+1}}{n+1} + \frac{2a^2dex^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1)
 + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*b*
d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^(n
+ 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.81

$$\int (a + bx^n)^2 (d + ex^n)^2 \, dx$$
$$= \frac{24\,a^2d^2n^4x + 6\,b^2e^2n^3xx^{4\,n} + 16\,b^2den^3xx^{3\,n} + 16\,abe^2n^3xx^{3\,n} + 12\,b^2d^2n^3xx^{2\,n} + 48\,abden^3xx^{2\,n} + 12\,a^2e^2n}{}$$

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 6*b^2*e^2*n^3*x*x^(4*n) + 16*b^2*d*e*n^3*x*x^(3*n) + 16
*a*b*e^2*n^3*x*x^(3*n) + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d*e*n^3*x*x^(2*n
) + 12*a^2*e^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 48*a^2*d*e*n^3*x*x^n
+ 50*a^2*d^2*n^3*x + 11*b^2*e^2*n^2*x*x^(4*n) + 28*b^2*d*e*n^2*x*x^(3*n) +
28*a*b*e^2*n^2*x*x^(3*n) + 19*b^2*d^2*n^2*x*x^(2*n) + 76*a*b*d*e*n^2*x*x^(2
*n) + 19*a^2*e^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 52*a^2*d*e*n^2*x*x^
n + 35*a^2*d^2*n^2*x + 6*b^2*e^2*n*x*x^(4*n) + 14*b^2*d*e*n*x*x^(3*n) + 14*
a*b*e^2*n*x*x^(3*n) + 8*b^2*d^2*n*x*x^(2*n) + 32*a*b*d*e*n*x*x^(2*n) + 8*a^
2*e^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 18*a^2*d*e*n*x*x^n + 10*a^2*d^2*n*
x + b^2*e^2*x*x^(4*n) + 2*b^2*d*e*x*x^(3*n) + 2*a*b*e^2*x*x^(3*n) + b^2*d^2
*x*x^(2*n) + 4*a*b*d*e*x*x^(2*n) + a^2*e^2*x*x^(2*n) + 2*a*b*d^2*x*x^n + 2*
a^2*d*e*x*x^n + a^2*d^2*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (a + bx^n)^2 (d + ex^n)^2 \, dx = a^2\,d^2\,x + \frac{x\,x^{2\,n}\,(a^2\,e^2 + 4\,a\,b\,d\,e + b^2\,d^2)}{2\,n + 1} + \frac{b^2\,e^2\,x\,x^{4\,n}}{4\,n + 1} + \frac{2\,b\,e\,x\,x^{3\,n}\,(a\,e + b\,d)}{3\,n + 1} + \frac{2\,a\,d\,x\,x^n\,(a\,e + b\,d)}{n + 1}$$

```
[In] int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

```
[Out] a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e^
2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d*x
*x^n*(a*e + b*d))/(n + 1)
```

3.294 $\int (a+bx^n)^2 (c+dx^n) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)^2 (c+dx^n) \, dx = a^2cx + \frac{a(2bc+ad)x^{1+n}}{1+n} + \frac{b(bc+2ad)x^{1+2n}}{1+2n} + \frac{b^2dx^{1+3n}}{1+3n}$$

```
[Out] a^2*c*x+a*(a*d+2*b*c)*x^(1+n)/(1+n)+b*(2*a*d+b*c)*x^(1+2*n)/(1+2*n)+b^2*d*x
^(1+3*n)/(1+3*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{380\}$

$$\int (a+bx^n)^2 (c+dx^n) \, dx = a^2cx + \frac{ax^{n+1}(ad+2bc)}{n+1} + \frac{bx^{2n+1}(2ad+bc)}{2n+1} + \frac{b^2dx^{3n+1}}{3n+1}$$

```
[In] Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2c + a(2bc+ad)x^n + b(bc+2ad)x^{2n} + b^2dx^{3n}\right) dx \\ &= a^2cx + \frac{a(2bc+ad)x^{1+n}}{1+n} + \frac{b(bc+2ad)x^{1+2n}}{1+2n} + \frac{b^2dx^{1+3n}}{1+3n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2\,(c+dx^n)\;dx=\frac{dx(a+bx^n)^3-(ad-b(c+3cn))x\left(a^2+\frac{2abx^n}{1+n}+\frac{b^2x^{2n}}{1+2n}\right)}{b+3bn}$$

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) + (
b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a^2cx+\frac{a(ad+2bc)x\,x^n}{1+n}+\frac{b(2ad+bc)x\,x^{2n}}{1+2n}+\frac{b^2dx\,x^{3n}}{1+3n}$
norman	$a^2cx+\frac{a(ad+2bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{b(2ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2dx\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{2x\,x^{3n}b^2d\,n^2+3x\,x^{3n}b^2dn+6x\,x^{2n}abd\,n^2+3x\,x^{2n}b^2c\,n^2+b^2dx\,x^{3n}+8x\,x^{2n}abdn+4x\,x^{2n}b^2cn+6x\,x^na^2d\,n^2+12x\,x^nabc\,n^2+6x\,a}{(1+n)(1+2n)(1+3n)}$

```
[In] int((a+b*x^n)^2*(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*x^n+b*(2*a*d+b*c)/(1+2*n)*x*(x^n)^2+b^2*d/(1+
3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)^2\,(c+dx^n)\;dx$$
$$=\frac{(2\,b^2dn^2+3\,b^2dn+b^2d)xx^{3\,n}+(b^2c+2\,abd+3\,(b^2c+2\,abd)n^2+4\,(b^2c+2\,abd)n)xx^{2\,n}+(2\,abc+a^2d+6}{6\,n^3+11\,n^2+6\,n+1}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
+ 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*(2
*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*
c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a + bx^n)^2 (c + dx^n)\, dx = \begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a}{6n^3+1} \end{cases}$$

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x
 - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt
(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**
2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*
c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a
**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*
c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)^2 (c+dx^n)\, dx = a^2cx + \frac{b^2dx^{3n+1}}{3n+1} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2\,abdx^{2n+1}}{2n+1} + \frac{2\,abcx^{n+1}}{n+1} + \frac{a^2dx^{n+1}}{n+1}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*b
*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n +
 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)^2 (c+dx^n)\, dx = \frac{6\,a^2cn^3x + 2\,b^2dn^2xx^{3\,n} + 3\,b^2cn^2xx^{2\,n} + 6\,abdn^2xx^{2\,n} + 12\,abcn^2xx^n + 6\,a^2dn^2xx^n + 11\,a^2cn^2x + 3\,b^2dnxx}{}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n^
2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b
^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*n*x
*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n) +
2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^2 (c+dx^n)\, dx = a^2\,c\,x + \frac{x\,x^{2\,n}\,\left(c\,b^2+2\,a\,d\,b\right)}{2\,n+1} + \frac{x\,x^n\,\left(d\,a^2+2\,b\,c\,a\right)}{n+1} + \frac{b^2\,d\,x\,x^{3\,n}}{3\,n+1}$$

```
[In] int((a + b*x^n)^2*(c + d*x^n),x)
```

```
[Out] a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*c
))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

3.295 $\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)}+\frac{bx(a+bx^n)}{d(1+n)} +\frac{(bc-ad)^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

```
[Out] -b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {427, 396, 251}

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{x(bc-ad)^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2} -\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx(a+bx^n)}{d(n+1)}$$

```
[In] Int[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a + b*x^n))/(d
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*d^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(a+bx^n)}{d(1+n)} + \frac{\int \frac{-a(bc-ad(1+n))-b(bc(1+n)-ad(1+2n))x^n}{c+dx^n}\,dx}{d(1+n)} \\
&= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2\int \frac{1}{c+dx^n}\,dx}{d^2} \\
&= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{a^2x}{c} - \frac{(bc-ad)^2x}{cd^2} + \frac{b^2x^{1+n}}{d(1+n)} + \frac{(-bc+ad)^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] (a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(b
*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d
^2)
```

Maple [F]

$$\int \frac{(a+b\,x^n)^2}{c+d\,x^n}dx$$

[In] int((a+b*x^n)^2/(c+d*x^n),x)

[Out] int((a+b*x^n)^2/(c+d*x^n),x)

Fricas [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")

[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{a^2c^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{2abc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{2b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)}$$

[In] integrate((a+b*x**n)**2/(c+d*x**n),x)

[Out] a**2*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(n**2*gamma(1 + 1/n)) - 2*a*b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n)*x*lerchphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(c*c**(1/n)*n**2*gamma(1 + 1/n)) + 2*b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*gamma(3 + 1/n)) + b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n))

Maxima [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d*
x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

```
[In] int((a + b*x^n)^2/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n), x)
```

3.296 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx = -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{(bc-ad)(ad(1-n)-bc(1+n))x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2d^2n}$$

```
[Out] -b*(a*d-b*c*(1+n))*x/c/d^2/n-(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)+(-a*d+b
*c)*(a*d*(1-n)-b*c*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^2/d^2/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 396, 251}

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx = \frac{x(bc-ad)(ad(1-n)-bc(n+1)) \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2d^2n} - \frac{bx(ad-bc(n+1))}{cd^2n} - \frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^n))/(c*d*n
*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(1+n))x^n}{c+dx^n}\,dx}{cdn}\\ &= -\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}+\frac{((bc-ad)(ad(1-n)-bc(1+n)))\int \frac{1}{c+dx^n}\,dx}{cd^2n}\\ &= -\frac{b(ad-bc(1+n))x}{cd^2n}-\frac{(bc-ad)x(a+bx^n)}{cdn\,(c+dx^n)}\\ &\quad+\frac{(bc-ad)(ad(1-n)-bc(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \frac{x\left(\frac{c(-2abcd+a^2d^2+b^2c(c+cn+dnx^n))}{c+dx^n}-(bc-ad)(ad(-1+n)+bc(1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]

[Out] (x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (b*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*d^2*n)

Maple [F]

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^2}dx$$

[In] int((a+b*x^n)^2/(c+d*x^n)^2,x)

[Out] int((a+b*x^n)^2/(c+d*x^n)^2,x)

Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")

[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)

Sympy [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

[In] integrate((a+b*x**n)**2/(c+d*x**n)**2,x)

[Out] Integral((a + b*x**n)**2/(c + d*x**n)**2, x)

Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n +
 c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d^2
)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^2}\,dx$$

```
[In] int((a + b*x^n)^2/(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^2, x)
```

3.297 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)} - \frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2n^2))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3d^2n^2}$$

```
[Out] -1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2+1/2*(-a*d+b*c)*(a*d*(1-2*n)-b
*c*(1+n))*x/c^2/d^2/n^2/(c+d*x^n)-1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*d^
2*(2*n^2-3*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/d^2/n^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 393, 251}

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = -\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3d^2n^2} + \frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\,(c+dx^n)} - \frac{x(bc-ad)\,(a+bx^n)}{2cdn\,(c+dx^n)^2}$$

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] -1/2*((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*
(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 -
```

```
n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(1+n))x^n}{(c+dx^n)^2}\,dx}{2cdn}\\
&= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)}\\
&\quad -\frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2n^2))\int \frac{1}{c+dx^n}\,dx}{2c^2d^2n^2}\\
&= -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)}\\
&\quad -\frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2n^2))\,x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \frac{x\left(\frac{c^2(bc-ad)^2n}{(c+dx^n)^2}-\frac{c(bc-ad)(ad(-1+2n)+b(c+2cn))}{c+dx^n}+(2abcd(-1+n)+b^2c^2(1+n)+a^2d^2(1-3n+2n^2))\,\text{Hyperge}\right.}{2c^3d^2n^2}$$

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
[Out] (x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2*d
^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)
]))/(2*c^3*d^2*n^2)
```

Maple [F]

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}dx$$

```
[In] int((a+b*x^n)^2/(c+d*x^n)^3,x)
[Out] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
[Out] ((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrate
(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^2*
d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n + 1)
 - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c^4*d
^2*n^2)
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}\,dx$$

```
[In] int((a + b*x^n)^2/(c + d*x^n)^3,x)
[Out] int((a + b*x^n)^2/(c + d*x^n)^3, x)
```

3.298 $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 310

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = -\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+24n^3)+ab^2c^2}{b^4(1+n)(1+2n)(1+3n)} - \frac{d(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))\,x(c+dx^n)}{b^3(1+n)(1+2n)(1+3n)} - \frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2\,(1+5n+6n^2)} + \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{(bc-ad)^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^4}$$

```
[Out] -d*(a^3*d^3*(6*n^3+11*n^2+6*n+1)-b^3*c^3*(24*n^3+18*n^2+7*n+1)-a^2*b*c*d^2*
(24*n^3+38*n^2+19*n+3)+a*b^2*c^2*d*(36*n^3+45*n^2+20*n+3))*x/b^4/(6*n^3+11*
n^2+6*n+1)-d*(2*a*b*c*d*(1+3*n)^2-a^2*d^2*(6*n^2+5*n+1)-b^2*c^2*(18*n^2+7*n
+1))*x*(c+d*x^n)/b^3/(6*n^3+11*n^2+6*n+1)-d*(a*d*(1+3*n)-b*(6*c*n+c))*x*(c+
d*x^n)^2/b^2/(6*n^2+5*n+1)+d*x*(c+d*x^n)^3/b/(1+3*n)+(-a*d+b*c)^4*x*hyperge
om([1, 1/n],[1+1/n],-b*x^n/a)/a/b^4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 542, 396, 251}

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$
$$= -\frac{dx(c+dx^n)\left(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1)\right)}{b^3(n+1)(2n+1)(3n+1)}$$
$$-\frac{dx(a^3d^3(6n^3+11n^2+6n+1)-a^2bcd^2(24n^3+38n^2+19n+3)+ab^2c^2d(36n^3+45n^2+20n+3)-b^3}{b^4(n+1)(2n+1)(3n+1)}$$
$$+\frac{x(bc-ad)^4\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^4}$$
$$-\frac{dx(c+dx^n)^2\left(ad(3n+1)-b(6cn+c)\right)}{b^2(6n^2+5n+1)}+\frac{dx(c+dx^n)^3}{b(3n+1)}$$

```
[In] Int[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 45
*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1 + 3*n))) - (d*(2*a*b*c*d*(1 +
3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x
^n))/(b^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))
*x*(c + d*x^n)^2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n
)) + ((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b^4)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)^2(-c(ad-b(c+3cn))-d(ad(1+3n)-b(c+6cn))x^n)}{a+bx^n}\,dx}{b(1+3n)}$$

$$= -\frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2(1+5n+6n^2)} + \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)(c(a^2d^2(1+3n)-2abcd(1+4n)+b^2c^2(1+5n+6n^2))-d(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))x^n)}{a+bx^n}\,dx}{b^2(1+5n+6n^2)}$$

$$= -\frac{d(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))\,x(c+dx^n)}{b^3(1+n)(1+5n+6n^2)} - \frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2(1+5n+6n^2)} + \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{-c(a^3d^3(1+5n+6n^2)-a^2bcd^2(3+16n+21n^2)+ab^2c^2d(3+17n+26n^2)-b^3c^3(1+6n+11n^2+6n^3))-d(a^3d^3(1+6n+11n^2+6n^3)\cdots}{a+bx^n}}{b^3(1+n)(1+5n+6n^2)}$$

$$= -\frac{d(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3)-a^2bcd^2(3+19n+38n^2+24n^3)\cdots}{b^4(1+n)(1+5n+6n^2)} - \frac{d(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))\,x(c+dx^n)}{b^3(1+n)(1+5n+6n^2)} - \frac{d(ad(1+3n)-b(c+6cn))x(c+dx^n)^2}{b^2(1+5n+6n^2)} + \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{(bc-ad)^4\int \frac{1}{a+bx^n}\,dx}{b^4}$$

$$= -\frac{d(a^3d^3(1+6n+11n^2+6n^3) - b^3c^3(1+7n+18n^2+24n^3) - a^2bcd^2(3+19n+38n^2+24n^3) +}{b^4(1+n)\left(1+5n+6n^2\right)}$$
$$- \frac{d(2abcd(1+3n)^2 - a^2d^2(1+5n+6n^2) - b^2c^2(1+7n+18n^2))\,x(c+dx^n)}{b^3(1+n)\left(1+5n+6n^2\right)}$$
$$- \frac{d(ad(1+3n) - b(c+6cn))x(c+dx^n)^2}{b^2\left(1+5n+6n^2\right)}$$
$$+ \frac{dx(c+dx^n)^3}{b(1+3n)} + \frac{(bc-ad)^4x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$
$$= \frac{x\left(4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right) + 6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right) + 4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right) + d^4x^{4n}\Phi\left(-\frac{bx^n}{a},1,4\right.\right.}{an}$$

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] (x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x^
(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((b*x
^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))/(a*
n)
```

Maple [F]

$$\int \frac{(c+d\,x^n)^4}{a+b\,x^n}dx$$

```
[In] int((c+d*x^n)^4/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = {} & \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} \\ & + \frac{4a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n\Gamma\left(5+\frac{1}{n}\right)} \\ & + \frac{a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n^2\Gamma\left(5+\frac{1}{n}\right)} \\ & + \frac{12a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)} \\ & + \frac{4a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)} \\ & + \frac{12a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} \\ & + \frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} \\ & - \frac{4a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

```
[In] integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] a**(1/n)*a**(-1 - 1/n)*c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(n**2*gamma(1 + 1/n)) + 4*a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*n
```

```
+ 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n*gamma
(5 + 1/n)) + a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n**2*gamma(5 + 1/n)) + 12*a**
(-4 - 1/n)*a**(3 + 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*gamma(4 + 1/n)) + 4*a**(-4 - 1/n)*a**(3 +
 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*ga
mma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 12*a**(-3 - 1/n)*a**(2 + 1/n)*c**2*d**
2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n
)/(n*gamma(3 + 1/n)) + 6*a**(-3 - 1/n)*a**(2 + 1/n)*c**2*d**2*x**(2*n + 1)*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3
 + 1/n)) - 4*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c**3*d*x*lerchphi(a*exp_po
lar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma
(1 + 1/n))
```

Maxima [F]

$$\int \frac{(c + dx^n)^4}{a + bx^n}\,dx = \int \frac{(dx^n + c)^4}{bx^n + a}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*int
egrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (4*
(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (6*(
6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5
*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n
^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c
*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n + 1)*b
^4)
```

Giac [F]

$$\int \frac{(c + dx^n)^4}{a + bx^n}\,dx = \int \frac{(dx^n + c)^4}{bx^n + a}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx^n)^4}{a + bx^n} \, dx = \int \frac{(c + d\, x^n)^4}{a + b\, x^n} \, dx$$

```
[In] int((c + d*x^n)^4/(a + b*x^n),x)
[Out] int((c + d*x^n)^4/(a + b*x^n), x)
```

3.299 $\int \frac{(c+dx^n)^3}{a+bx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(c+dx^n)^3}{a+bx^n} \, dx = \frac{d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))\,x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{(bc-ad)^3x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^3}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)+b^2*c^2*(6*n^2+4*n+1)-a*b*c*d*(6*n^2+7*n+2))*x/b^3
/(2*n^2+3*n+1)-d*(a*d*(1+2*n)-b*(4*c*n+c))*x*(c+d*x^n)/b^2/(2*n^2+3*n+1)+d*
x*(c+d*x^n)^2/b/(1+2*n)+(-a*d+b*c)^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)
/a/b^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 542, 396, 251}

$$\int \frac{(c+dx^n)^3}{a+bx^n} \, dx = \frac{dx(a^2d^2(2n^2+3n+1)-abcd(6n^2+7n+2)+b^2c^2(6n^2+4n+1))}{b^3(n+1)(2n+1)} + \frac{x(bc-ad)^3 \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^3} - \frac{dx(c+dx^n)\left(ad(2n+1)-b(4cn+c)\right)}{b^2(n+1)(2n+1)} + \frac{dx(c+dx^n)^2}{b(2n+1)}$$

```
[In] Int[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*
n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))
*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n))
 + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b^3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{(c+dx^n)(-c(ad-b(c+2cn))-d(ad(1+2n)-b(c+4cn))x^n)}{a+bx^n}\,dx}{b(1+2n)}$$

$$= -\frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{c(a^2d^2(1+2n)-abcd(2+5n)+b^2c^2(1+3n+2n^2))+d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))x^n}{a+bx^n}\,dx}{b^2(1+n)(1+2n)}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))\,x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{(bc-ad)^3\int\frac{1}{a+bx^n}\,dx}{b^3}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)+b^2c^2(1+4n+6n^2)-abcd(2+7n+6n^2))\,x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{(bc-ad)^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{x\left(3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(2
*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLerch
Phi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(
-1)]))/(a*n)
```

Maple [F]

$$\int \frac{(c+d\,x^n)^3}{a+b\,x^n}dx$$

```
[In] int((c+d*x^n)^3/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c + dx^n)^3}{a + bx^n}\,dx = \int \frac{(dx^n + c)^3}{bx^n + a}\,dx$$

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \frac{(c + dx^n)^3}{a + bx^n}\,dx = {} & \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} \\ & + \frac{3a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)} \\ & + \frac{a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)} \\ & + \frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} \\ & + \frac{3a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} \\ & - \frac{3a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

```
[In] integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] a**(1/n)*a**(-1 - 1/n)*c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(n**2*gamma(1 + 1/n)) + 3*a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*n
+ 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*gamma
(4 + 1/n)) + a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 6*a**(
-3 - 1/n)*a**(2 + 1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*gamma(3 + 1/n)) + 3*a**(-3 - 1/n)*a**(2 +
1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gam
ma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 3*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c
**2*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/
n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^3*
(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*a*b
*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

Giac [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^3}{a+b\,x^n}\,dx$$

```
[In] int((c + d*x^n)^3/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^3/(a + b*x^n), x)
```

3.300 $\int \frac{(c+dx^n)^2}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)} + \frac{dx(c+dx^n)}{b(1+n)} + \frac{(bc-ad)^2 x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2}$$

```
[Out] -d*(a*d*(1+n)-b*(2*c*n+c))*x/b^2/(1+n)+d*x*(c+d*x^n)/b/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {427, 396, 251}

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = \frac{x(bc-ad)^2 \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2} - \frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)} + \frac{dx(c+dx^n)}{b(n+1)}$$

```
[In] Int[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c + d*x^n))/(b
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*b^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(c+dx^n)}{b(1+n)}+\frac{\int \frac{-c(ad-bc(1+n))-d(ad(1+n)-b(c+2cn))x^n}{a+bx^n}\,dx}{b(1+n)}\\
&=-\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2\int \frac{1}{a+bx^n}\,dx}{b^2}\\
&=-\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx=\frac{x\left(2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] (x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n)/
a), 1, n^(-1)]))/(a*n)
```

Maple [F]

$$\int \frac{(c+d\,x^n)^2}{a+b\,x^n}dx$$

[In] int((c+d*x^n)^2/(a+b*x^n),x)

[Out] int((c+d*x^n)^2/(a+b*x^n),x)

Fricas [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx=\int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")

[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx=\frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{2a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}+\frac{a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)}-\frac{2a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

[In] integrate((c+d*x**n)**2/(a+b*x**n),x)

[Out] a**(1/n)*a**(-1 - 1/n)*c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(n**2*gamma(1 + 1/n)) + 2*a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*n
+ 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*gamma
(3 + 1/n)) + a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 2*a**(
1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n),
1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))

Maxima [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n} \, dx = \int \frac{(dx^n+c)^2}{bx^n+a} \, dx$$

[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")

[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2*x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))

Giac [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n} \, dx = \int \frac{(dx^n+c)^2}{bx^n+a} \, dx$$

[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")

[Out] integrate((d*x^n + c)^2/(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{a+bx^n} \, dx = \int \frac{(c+d\,x^n)^2}{a+b\,x^n} \, dx$$

[In] int((c + d*x^n)^2/(a + b*x^n),x)

[Out] int((c + d*x^n)^2/(a + b*x^n), x)

3.301 $\int \frac{c+dx^n}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{dx}{b}+\frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

```
[Out] d*x/b+(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {396, 251}

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

```
[In] Int[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^n}\,dx}{b}\\ &= \frac{dx}{b} + \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{x\left(ad+(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{ab}$$

```
[In] Integrate[(c + d*x^n)/(a + b*x^n),x]
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
]))/(a*b)
```

Maple [F]

$$\int \frac{c+d\,x^n}{a+b\,x^n}dx$$

```
[In] int((c+d*x^n)/(a+b*x^n),x)
[Out] int((c+d*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
[Out] integral((d*x^n + c)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.62

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate((c+d*x**n)/(a+b*x**n),x)
[Out] a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma
(1/n)/(n**2*gamma(1 + 1/n)) - a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*d*x*lerch
phi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n
)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
[Out] (b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

Giac [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{dx^n+c}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \int \frac{c+d\,x^n}{a+b\,x^n}\,dx$$

```
[In] int((c + d*x^n)/(a + b*x^n),x)
[Out] int((c + d*x^n)/(a + b*x^n), x)
```

3.302 $\int \frac{1}{(a+bx^n)(c+dx^n)} dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)} dx = \frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {400, 251}

$$\int \frac{1}{(a+bx^n)(c+dx^n)} dx = \frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{1}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{x\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)\ (c+d\,x^n)}\,dx$$

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.303 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx = -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{d(bc(1-2n)-ad(1-n))x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^2n}$$

```
[Out] -d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/
(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/
c)/c^2/(-a*d+b*c)^2/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {425, 536, 251}

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx = \frac{b^2x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{dx(bc(1-2n)-ad(1-n)) \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2} - \frac{dx}{cn(bc-ad)(c+dx^n)}$$

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1
 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c
- a*d)^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{c(bc-ad)n\left(c+dx^n\right)}+\frac{\int \frac{bcn+a(d-dn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{c(bc-ad)n}\\
&= -\frac{dx}{c(bc-ad)n\left(c+dx^n\right)}+\frac{b^2\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^2}-\frac{(d(ad(1-n)-b(c-2cn)))\int \frac{1}{c+dx^n}\,dx}{c(bc-ad)^2n}\\
&= -\frac{dx}{c(bc-ad)n\left(c+dx^n\right)}+\frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}\\
&\quad+\frac{d(bc(1-2n)-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2(bc-ad)^2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \frac{x\left(b^2c^2n(c+dx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\left(c(-bc+ad)+(ad(-1+n)+b(c-2cn))\right.\right.}{ac^2(bc-ad)^2n\,(c+dx^n)}$$

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] (x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n)*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c - a*d
)^2*n*(c + d*x^n))
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)(c+d\,x^n)^2}dx$$

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2*
a*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b^2
*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n + a
^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x
^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^2}\,dx$$

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^2),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^2), x)
```

3.304 $\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 210

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx = -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} - \frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^3} - \frac{d(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2))\,x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\right.}{2c^3(bc-ad)^3n^2}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2/
(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a
*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(6*n
^2-5*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/n^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {425, 541, 536, 251}

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx = -\frac{dx(a^2d^2(2n^2-3n+1)-2abcd(3n^2-4n+1)+b^2c^2(6n^2-5n+1)) \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\right.}{2c^3n^2(bc-ad)^3} + \frac{b^3x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^3} + \frac{dx(bc(1-4n)-ad(1-2n))}{2c^2n^2(bc-ad)^2(c+dx^n)} - \frac{dx}{2cn(bc-ad)(c+dx^n)^2}$$

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*n*(c + d*x^n)^2) + (d*(b*c*(1 - 4*n) - a*d*(1 - 2
*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1,
 n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*
n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n
^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx}{2c(bc-ad)n\left(c+dx^n\right)^2} + \frac{\int \frac{2bcn+a(d-2dn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{2c(bc-ad)n}$$

$$= -\frac{dx}{2c(bc-ad)n\left(c+dx^n\right)^2} - \frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2\left(c+dx^n\right)} + \frac{\int \frac{2b^2c^2n^2+a^2d^2(1-3n+2n^2)-abcd(1-5n+4n^2)-bd(bc(1-4n)-ad(1-2n))(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{2c^2(bc-ad)^2n^2}$$

$$= -\frac{dx}{2c(bc-ad)n\left(c+dx^n\right)^2} - \frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2\left(c+dx^n\right)} + \frac{b^3\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^3} - \frac{\left(d\left(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2)\right)\right)\int \frac{1}{c+dx^n}\,dx}{2c^2(bc-ad)^3n^2}$$

$$= -\frac{dx}{2c(bc-ad)n\left(c+dx^n\right)^2} - \frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2\left(c+dx^n\right)} + \frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3} - \frac{d\left(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3(bc-ad)^3n^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)\left(c+dx^n\right)^3}\,dx = \frac{x\left(-ac^2d(bc-ad)^2n+acd(bc-ad)(ad(-1+2n)+b(c-4cn))\left(c+dx^n\right)+2b^3c^3n^2\left(c+dx^n\right)^2 \text{Hypergeo}\right.}{}$$

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] (x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c -
 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d
*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c +
 d*x^n)^2)
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)\left(c+d\,x^n\right)^3}dx$$

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} \, dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3} \, dx$$

[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")

[Out] integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3} \, dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3} \, dx$$

[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")

[Out] -b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 + (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^2 - 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)*a^2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(2*n - 1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 - 2*a*b*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^2)*x^n)

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^3}\,dx$$

[In] int(1/((a + b*x^n)*(c + d*x^n)^3),x)

[Out] int(1/((a + b*x^n)*(c + d*x^n)^3), x)

3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 341

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+a^2bcd^2(3+15n}{ab^4n(1+n)(1+2n)} - \frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} - \frac{(bc-ad)^3(bc(1-n)-ad(1+3n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^4n}$$

```
[Out] -d*(b^3*c^3*(2*n^2+3*n+1)-a^3*d^3*(6*n^3+11*n^2+6*n+1)-a*b^2*c^2*d*(12*n^3+
17*n^2+12*n+3)+a^2*b*c*d^2*(16*n^3+26*n^2+15*n+3))*x/a/b^4/n/(2*n^2+3*n+1)-
d*(b^2*c^2*(2*n^2+3*n+1)-2*a*b*c*d*(5*n^2+4*n+1)+a^2*d^2*(6*n^2+5*n+1))*x*(
c+d*x^n)/a/b^3/n/(2*n^2+3*n+1)-d*(-3*a*d*n+2*b*c*n-a*d+b*c)*x*(c+d*x^n)^2/a
/b^2/n/(1+2*n)+(-a*d+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(b*c*(
1-n)-a*d*(1+3*n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^4/n
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {424, 542, 396, 251}

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$$
$$= -\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1)) \text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^2b^4n}$$
$$- \frac{dx(c+dx^n)\left(a^2d^2(6n^2+5n+1)-2abcd(5n^2+4n+1)+b^2c^2(2n^2+3n+1)\right)}{ab^3n(n+1)(2n+1)}$$
$$- \frac{dx(-a^3d^3(6n^3+11n^2+6n+1)+a^2bcd^2(16n^3+26n^2+15n+3)-ab^2c^2d(12n^3+17n^2+12n+3)+}{ab^4n(n+1)(2n+1)}$$
$$+ \frac{dx(c+dx^n)^2\left(ad(3n+1)-b(2cn+c)\right)}{ab^2n(2n+1)} + \frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

```
[In] Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - a*b^
2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 + 15*n + 26*n^2 + 16*
n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b^2*c^2*(1 + 3*n + 2*n^2) - 2*a
*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3
*n*(1 + n)*(1 + 2*n)) + (d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)
/(a*b^2*n*(1 + 2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) -
((b*c - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)^2(c(ad-bc(1-n))+d(ad(1+3n)-b(c+2cn))x^n)}{a+bx^n}\,dx}{abn}$$

$$= \frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(2abcd(1+2n)-a^2d^2(1+3n)-b^2c^2(1+n-2n^2))-d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))x^n)}{a+bx^n}\,dx}{ab^2n(1+2n)}$$

$$= -\frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{c(3ab^2c^2d(1+3n+2n^2)+a^3d^3(1+5n+6n^2)-a^2bcd^2(3+12n+13n^2)-b^3c^3(1+2n-n^2-2n^3))-d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+\cdots}{a+bx^n}}{ab^3n(1+n)(1+2n)}$$

$$= -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+a^2bcd^2(3\cdots}{ab^4n(1+n)(1+2n)} - \frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} - \frac{((bc-ad)^3(bc(1-n)-ad(1+3n)))\int \frac{1}{a+bx^n}\,dx}{ab^4n}$$

$$= -\frac{d(b^3c^3(1+3n+2n^2)-a^3d^3(1+6n+11n^2+6n^3)-ab^2c^2d(3+12n+17n^2+12n^3)+a^2bcd^2}{ab^4n(1+n)(1+2n)}$$
$$-\frac{d(b^2c^2(1+3n+2n^2)-2abcd(1+4n+5n^2)+a^2d^2(1+5n+6n^2))\,x(c+dx^n)}{ab^3n(1+n)(1+2n)}$$
$$+\frac{d(ad(1+3n)-b(c+2cn))x(c+dx^n)^2}{ab^2n(1+2n)}+\frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)}$$
$$-\frac{(bc-ad)^3(bc(1-n)-ad(1+3n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^4n}$$

Mathematica [A] (verified)

Time = 6.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$
$$= \frac{x\left(\frac{4ab^3c^3d-6a^2b^2c^2d^2+4a^3bcd^3-a^4d^4+b^4c^4(-1+n)}{a^2n}+\frac{(-bc+ad)^3(bc(-1+n)+ad(1+3n))}{a^2n}+\frac{2bd^3(2bc-ad)x^n}{1+n}+\frac{b^2d^4x^{2n}}{1+2n}+\frac{(bc-ad)^4}{an(a+bx^n)}\right.}{b^4}$$

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] (x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4*
(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^2*
n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) + (b
*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n
))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b^4
```

Maple [F]

$$\int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}dx$$

```
[In] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

```
[In] integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**4/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1) -
 b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n), x)
 + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2 + n
)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*n^3
+ 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((2*n^2
 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*n^2 +
4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 + (6*n^3
 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (2*n^3 +
3*n^2 + n)*a^2*b^4)
```

Giac [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}\,dx$$

```
[In] int((c + d*x^n)^4/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)^4/(a + b*x^n)^2, x)
```

3.306 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx = -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} - \frac{(bc-ad)^2(bc(1-n)-ad(1+2n))x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^3n}$$

```
[Out] -d*(b^2*c^2*(1+n)+a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(3*n^2+4*n+2))*x/a/b^3/n/(1
+n)-d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n)/a/b^2/n/(1+n)+(-a*d+b*c)*x*(c+d*x
^n)^2/a/b/n/(a+b*x^n)-(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([1,
1/n],[1+1/n],-b*x^n/a)/a^2/b^3/n
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{424, 542, 396, 251\}$

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$$
$$= -\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^3n}$$
$$-\frac{dx(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2)+b^2c^2(n+1))}{ab^3n(n+1)}$$
$$-\frac{dx(c+dx^n)\,(bc(n+1)-ad(2n+1))}{ab^2n(n+1)}+\frac{x(bc-ad)\,(c+dx^n)^2}{abn\,(a+bx^n)}$$

```
[In] Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^
2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 + 2*n))*x*(c + d*x^n))
/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) - ((
b*c - a*d)^2*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
```

```
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(ad-bc(1-n))-d(bc(1+n)-ad(1+2n))x^n)}{a+bx^n}\,dx}{abn}$$

$$= -\frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{c(2abcd(1+n)-a^2d^2(1+2n)-b^2c^2(1-n^2))-d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))x^n}{a+bx^n}\,dx}{ab^2n(1+n)}$$

$$= -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} - \frac{((bc-ad)^2(bc(1-n)-ad(1+2n)))\int \frac{1}{a+bx^n}\,dx}{ab^3n}$$

$$= -\frac{d(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-abcd(2+4n+3n^2))\,x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} - \frac{(bc-ad)^2(bc(1-n)-ad(1+2n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^3n}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 4.83 (sec) , antiderivative size = 2050, normalized size of antiderivative = 10.25

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] (x*(3*a*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + n)^4 + 3*c^2*d*(1 +
 4*n + 6*n^2 + 2*n^3 + n^4)*x^n + 3*c*d^2*(1 + n)^4*x^(2*n) + d^3*(1 + n)^4
*x^(3*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] - 3*a*(1 + 10*n + 35
*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + 2*n)^4 + 3*c^2*d*(1 + 2*n)^4*x^n + 3*c*d^
```

2*(1 + 8*n + 24*n^2 + 34*n^3 + 18*n^4)*x^(2*n) + d^3*(1 + 2*n)^4*x^(3*n))*H
urwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + a*c^3*HurwitzLerchPhi[-((b*x^
n)/a), 1, 3 + n^(-1)] + 22*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 209*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1118*a*
c^3*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3675*a*c^3*n^4*Hurwi
tzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7578*a*c^3*n^5*HurwitzLerchPhi[-(
(b*x^n)/a), 1, 3 + n^(-1)] + 9531*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1
, 3 + n^(-1)] + 6642*a*c^3*n^7*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)]
 + 1944*a*c^3*n^8*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c^2*d*
x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c^2*d*n*x^n*Hurwitz
LerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c^2*d*n^2*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c^2*d*n^3*x^n*HurwitzLerchPhi[-((b*x
^n)/a), 1, 3 + n^(-1)] + 11025*a*c^2*d*n^4*x^n*HurwitzLerchPhi[-((b*x^n)/a)
, 1, 3 + n^(-1)] + 22734*a*c^2*d*n^5*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3
 + n^(-1)] + 28593*a*c^2*d*n^6*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 19926*a*c^2*d*n^7*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
 5832*a*c^2*d*n^8*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c*
d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c*d^2*n*x^(
2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c*d^2*n^2*x^(2*n)
*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c*d^2*n^3*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 11025*a*c*d^2*n^4*x^(2*n)*Hurw
itzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 22734*a*c*d^2*n^5*x^(2*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 28593*a*c*d^2*n^6*x^(2*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 19926*a*c*d^2*n^7*x^(2*n)*HurwitzLer
chPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5832*a*c*d^2*n^8*x^(2*n)*HurwitzLerchP
hi[-((b*x^n)/a), 1, 3 + n^(-1)] + a*d^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a
), 1, 3 + n^(-1)] + 22*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 +
 n^(-1)] + 209*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1
)] + 1112*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
3603*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7248*
a*d^3*n^5*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 8811*a*d^3
*n^6*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5898*a*d^3*n^7*
x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1656*a*d^3*n^8*x^(3*
n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] - a*c^3*HurwitzLerchPhi[-((
b*x^n)/a), 1, n^(-1)] - 10*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 35*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 56*a*c^3*n^3*Hur
witzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 78*a*c^3*n^4*HurwitzLerchPhi[-((b*x
^n)/a), 1, n^(-1)] - 150*a*c^3*n^5*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 90*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 156*a*c^3*n^7*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 144*a*c^3*n^8*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] - 3*a*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 30*a*c^2*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 105*a*c^2*
d*n^2*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a*c^2*d*n^3*x^n*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c^2*d*n^4*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, n^(-1)] - 3*a*c*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),

```
1, n^(-1)] - 30*a*c*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 105*a*c*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a
*c*d^2*n^3*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c*d^2*n^
4*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - a*d^3*x^(3*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, n^(-1)] - 10*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*
x^n)/a), 1, n^(-1)] - 35*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1,
 n^(-1)] - 50*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] -
24*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 6*b*c^3*n^8
*x^n*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((
b*x^n)/a)] - 18*b*c^2*d*n^8*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-
1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] - 18*b*c*d^2*n^8*x^(3*n)*Hypergeo
metricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] -
6*b*d^3*n^8*x^(4*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5
 + n^(-1)}, -((b*x^n)/a)]))/(6*a^3*n^5*(1 + n)*(1 + 2*n)*(1 + 3*n)*(1 + 4*n
))
```

Maple [F]

$$\int \frac{(c+d\,x^n)^3}{(a+b\,x^n)^2}dx$$

[In] `int((c+d*x^n)^3/(a+b*x^n)^2,x)`

[Out] `int((c+d*x^n)^3/(a+b*x^n)^2,x)`

Fricas [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2}\,dx$$

[In] `integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")`

[Out] `integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)`

Sympy [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx$$

```
[In] integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
[Out] Integral((c + d*x**n)**3/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2} \, dx$$

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
[Out] (a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2*
d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (3*
(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1)*a
^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*d*(n
 + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

Giac [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2} \, dx$$

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(c+d\,x^n)^3}{(a+b\,x^n)^2} \, dx$$

```
[In] int((c + d*x^n)^3/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)^3/(a + b*x^n)^2, x)
```

3.307 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = -\frac{d(bc-ad(1+n))x}{ab^2n}+\frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^2n}$$

```
[Out] -d*(b*c-a*d*(1+n))*x/a/b^2/n+(-a*d+b*c)*x*(c+d*x^n)/a/b/n/(a+b*x^n)-(-a*d+b
*c)*(b*c*(1-n)-a*d*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^2/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 396, 251}

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = -\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^2n} - \frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\,(a+bx^n)}$$

```
[In] Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^n))/(a*b*n
*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^2*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)}+\frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(1+n))x^n}{a+bx^n}\,dx}{abn}\\
&= -\frac{d(bc-ad(1+n))x}{ab^2n}+\frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)}-\frac{((bc-ad)(bc(1-n)-ad(1+n)))\int \frac{1}{a+bx^n}\,dx}{ab^2n}\\
&= -\frac{d(bc-ad(1+n))x}{ab^2n}+\frac{(bc-ad)x(c+dx^n)}{abn\,(a+bx^n)}\\
&\quad-\frac{(bc-ad)(bc(1-n)-ad(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.83 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.79

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \frac{x\left(-2a(1+6n+11n^2+6n^3)\left(c^2(1+n)^3+2cd(1+3n+4n^2+n^3)\,x^n+d^2(1+n)^3x^{2n}\right)\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)\right.}{}$$

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] (x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^2
 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n
^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)^3*
x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a),
 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*c^2*
n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerchPhi[-(
(b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c^2*n^5*H
urwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*a*c*d*n^2*x
^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 1
1*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n^3*
x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*n^6*x^n*Hypergeo
metricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 4*b*c*
d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)},
-((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}
, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 + 6*n + 11*n^2 + 6*n^3)
)
```

Maple [F]

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}dx$$

```
[In] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx=\int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x
)
```

Sympy [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx=\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$

```
[In] integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx=\int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n +
 a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c*d
)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx=\int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}\,dx$$

```
[In] int((c + d*x^n)^2/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)^2/(a + b*x^n)^2, x)
```

3.308 $\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}$$

```
[Out] (-a*d+b*c)*x/a/b/n/(a+b*x^n)+(a*d-b*c*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-
b*x^n/a)/a^2/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 251}

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\frac{x(ad-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

```
[In] Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))\int\frac{1}{a+bx^n}\,dx}{abn}\\ &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int\frac{c+dx^n}{(a+bx^n)^2}\,dx=\frac{x\left(\frac{d}{a+bx^n}-\frac{(ad+bc(-1+n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

```
[In] Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
[Out] (x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

Maple [F]

$$\int\frac{c+d\,x^n}{(a+b\,x^n)^2}dx$$

```
[In] int((c+d*x^n)/(a+b*x^n)^2,x)
[Out] int((c+d*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int\frac{c+dx^n}{(a+bx^n)^2}\,dx=\int\frac{dx^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.29

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = c\left(\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}+\frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}-\frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right)+d\left(\frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}n^2x^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}+\frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}nx^{n+1}\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{aa^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{a^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}bnx^nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}-\frac{a^{-3-\frac{1}{n}}a^{1+\frac{1}{n}}bx^nx^{n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}{an^3\Gamma\left(2+\frac{1}{n}\right)+bn^3x^n\Gamma\left(2+\frac{1}{n}\right)}\right)$$

```
[In] integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*
gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1/n
)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1
 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a**
(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**(1/n)*a
```

```
**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)
/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*(a*a**(-3 - 1/n)
*a**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**
3*x**n*gamma(2 + 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*n*x**(n + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n
) + b*n**3*x**n*gamma(2 + 1/n)) + a*a**(-3 - 1/n)*a**(1 + 1/n)*n*x**(n + 1)
*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n)) - a*a*
*(-3 - 1/n)*a**(1 + 1/n)*x**(n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))
 - a**(-3 - 1/n)*a**(1 + 1/n)*b*n*x**n*x**(n + 1)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*g
amma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*x**n*x**(n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b
*n**3*x**n*gamma(2 + 1/n)))
```

Maxima [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
[Out] (b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*x
/(a*b^2*n*x^n + a^2*b*n)
```

Giac [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \int \frac{c+d\,x^n}{(a+b\,x^n)^2}\,dx$$

```
[In] int((c + d*x^n)/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)/(a + b*x^n)^2, x)
```

3.309 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {425, 536, 251}

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx(ad(1-2n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{a(bc-ad)n\,(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n} \\
&= \frac{bx}{a(bc-ad)n\,(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n} \\
&= \frac{bx}{a(bc-ad)n\,(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} \\
&\quad + \frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.310 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\begin{aligned}\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx &= \frac{d(bc+ad)x}{ac(bc-ad)^2n\,(c+dx^n)}+\frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)}\\&\quad+\frac{b^2(ad(1-3n)-b(c-cn))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^3n}\\&\quad-\frac{d^2(bc(1-3n)-ad(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^3n}\end{aligned}$$

```
[Out] d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c+
d*x^n)+b^2*(a*d*(1-3*n)-b*(-c*n+c))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/
a^2/(-a*d+b*c)^3/n-d^2*(b*c*(1-3*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n]
,-d*x^n/c)/c^2/(-a*d+b*c)^3/n
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{425, 541, 536, 251\}$

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{b^2x(ad(1-3n)-b(c-cn))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3} - \frac{d^2x(bc(1-3n)-ad(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3} + \frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)} + \frac{dx(ad+bc)}{acn(bc-ad)^2(c+dx^n)}$$

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)*
n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^3*n) - (d^2
*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((d*x^n)/c)])/(c^2*(b*c - a*d)^3*n)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)}-\frac{\int \frac{adn+b(c-cn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{a(bc-ad)n}\\
&= \frac{d(bc+ad)x}{ac(bc-ad)^2n\left(c+dx^n\right)}+\frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)}\\
&\quad -\frac{\int \frac{n\left(b^2c^2(1-n)+a^2d^2(1-n)+2abcdn\right)+bd(bc+ad)(1-n)nx^n}{(a+bx^n)(c+dx^n)}\,dx}{ac(bc-ad)^2n^2}\\
&= \frac{d(bc+ad)x}{ac(bc-ad)^2n\left(c+dx^n\right)}+\frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)}\\
&\quad +\frac{\left(d^2(ad(1-n)-b(c-3cn))\right)\int \frac{1}{c+dx^n}\,dx}{c(bc-ad)^3n}\\
&\quad +\frac{\left(b^2(ad(1-3n)-b(c-cn))\right)\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^3n}\\
&= \frac{d(bc+ad)x}{ac(bc-ad)^2n\left(c+dx^n\right)}+\frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)}\\
&\quad +\frac{b^2(ad(1-3n)-b(c-cn))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^3n}\\
&\quad -\frac{d^2(bc(1-3n)-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2(bc-ad)^3n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+bx^n\right)^2\left(c+dx^n\right)^2}\,dx = \frac{x\left(\frac{b^2(bc-ad)}{a(a+bx^n)}+\frac{d^2(bc-ad)}{c(c+dx^n)}+\frac{b^2(ad(1-3n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2(-ad(-1+n)+bc(-1+3n))\,\text{Hypergeome}}{c^2}\right.}{(bc-ad)^3n}$$

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n)) +
 (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{2}\,(c+d\,x^{n})^{2}}dx$$

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^{n})^{2}\,(c+dx^{n})^{2}}\,dx = \int \frac{1}{(bx^{n}+a)^{2}(dx^{n}+c)^{2}}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
[Out] integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^{n})^{2}\,(c+dx^{n})^{2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^{n})^{2}\,(c+dx^{n})^{2}}\,dx = \int \frac{1}{(bx^{n}+a)^{2}(dx^{n}+c)^{2}}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
[Out] (a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^2
*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n +
 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*(n
- 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a^3*c
^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*c*d^4
*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2
*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^2*c^2*d
^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^3*b*c^2*
d^2*n + a^4*c*d^3*n)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^2}\,dx$$

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^2),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

3.311 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n)x}{2ac^2(bc-ad)^3n^2(c+dx^n)} + \frac{b^3(ad(1-4n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^4n} + \frac{d^2(a^2d^2(1-3n+2n^2)-2abcd(1-5n+4n^2)+b^2c^2(1-7n+12n^2))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{2c^3(bc-ad)^4n^2}$$

```
[Out] 1/2*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b*
x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/c^
2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1, 1/
n],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-2*a*
b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c^3/(-a*d+b*c)^4/n^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{425, 541, 536, 251\}$

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx$$
$$= \frac{b^3x(ad(1-4n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4}$$
$$-\frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2(bc-ad)^3\,(c+dx^n)}$$
$$+\frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n}\right.}{2c^3n^2(bc-ad)^4}$$
$$+\frac{bx}{an(bc-ad)\,(a+bx^n)\,(c+dx^n)^2}+\frac{dx(ad+2bc)}{2acn(bc-ad)^2\,(c+dx^n)^2}$$

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)/(a*(b*c -
 a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2
*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)) + (b^3*(a*d*(
1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*(b*c - a*d)^4*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*
(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)^2} - \frac{\int \frac{adn+b(c-cn)+bd(1-3n)x^n}{(a+bx^n)(c+dx^n)^3}\,dx}{a(bc-ad)n}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n\left(c+dx^n\right)^2} + \frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)^2} - \frac{\int \frac{n\left(a^2d^2(1-2n)+2b^2c^2(1-n)+4abcdn\right)+bd(2bc+ad)(1-2n)nx^n}{(a+bx^n)(c+dx^n)^2}\,dx}{2ac(bc-ad)^2n^2}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n\left(c+dx^n\right)^2} + \frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)^2} - \frac{d\left(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n\right)x}{2ac^2(bc-ad)^3n^2\left(c+dx^n\right)} - \frac{\int \frac{n\left(2b^3c^3(1-n)n+6ab^2c^2dn^2+a^3d^3\left(1-3n+2n^2\right)-a^2bcd^2\left(1-7n+6n^2\right)\right)-bd(1-n)n\left(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n\right)x^n}{(a+bx^n)(c+dx^n)}\,dx}{2ac^2(bc-ad)^3n^3}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n\left(c+dx^n\right)^2} + \frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)^2} - \frac{d\left(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n\right)x}{2ac^2(bc-ad)^3n^2\left(c+dx^n\right)} + \frac{\left(b^3(ad(1-4n)-bc(1-n))\right)\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^4n} + \frac{\left(d^2\left(a^2d^2\left(1-3n+2n^2\right)-2abcd\left(1-5n+4n^2\right)+b^2c^2\left(1-7n+12n^2\right)\right)\right)\int \frac{1}{c+dx^n}\,dx}{2c^2(bc-ad)^4n^2}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n\left(c+dx^n\right)^2}+\frac{bx}{a(bc-ad)n\left(a+bx^n\right)\left(c+dx^n\right)^2}$$
$$-\frac{d\left(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n\right)x}{2ac^2(bc-ad)^3n^2\left(c+dx^n\right)}$$
$$+\frac{b^3(ad(1-4n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^4n}$$
$$+\frac{d^2\left(a^2d^2\left(1-3n+2n^2\right)-2abcd\left(1-5n+4n^2\right)+b^2c^2\left(1-7n+12n^2\right)\right)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3(bc-ad)^4n^2}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+bx^n\right)^2\left(c+dx^n\right)^3}\,dx$$
$$= \frac{x\left(\frac{2b^3(bc-ad)n}{a(a+bx^n)}+\frac{d^2(bc-ad)^2n}{c(c+dx^n)^2}+\frac{d^2(-bc+ad)(ad(-1+2n)+b(c-6cn))}{c^2(c+dx^n)}+\frac{2b^3(ad(1-4n)+bc(-1+n))n\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}\right.}{2(bc-ad)^4n^2}$$

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c + d
*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c +
d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*
b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^n\right)^2\left(c+d\,x^n\right)^3}dx$$

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n) +
 (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$$

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 -
3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*b^
2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 - 4*
a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c^2*d
^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^2*b^4
*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n + a^6*d^
4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4*b^2*c*
d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b*d^4*(2*
n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a^3*d^4*(2
*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2*(7*n - 1)
 - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^3*b^2*c^6*d
*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n^2 - 3*a^2*b
^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^(3*n) + (2*a*
b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2 + a^4*b*c^3*d
^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3*c^6*d*n^2 - 3*
a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n^2)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^3}\,dx$$

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

3.312 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n,-p,-q,1+1/n,-b*x^n/a,-d*x^n/c)/((1+b
*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p(c+dx^n)^q\,dx$$

$$=\left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\right)\int\left(1+\frac{bx^n}{a}\right)^p\left(1+\frac{dx^n}{c}\right)^q dx$$

$$=x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(81) = 162$.

Time = 0.45 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int(a+bx^n)^p(c+dx^n)^q\,dx$$

$$=\frac{ac(1+n)x(a+bx^n)^p(c+dx^n)^q\,\text{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n}\right.}{bcnpx^n\,\text{AppellF1}\left(1+\frac{1}{n},1-p,-q,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+adnqx^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,1-q,2+\frac{1}{n},-\frac{bx^n}{a},\right.}$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(
-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
-q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n^(-
1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Appell
F1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(c+d\,x^{n})^{q}\,dx$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^q,x)
[Out] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^q\,\,dx=\int (bx^n+a)^p(dx^n+c)^q\,dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p\,(c+dx^n)^q\,\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p\,(c+dx^n)^q\,\,dx=\int (bx^n+a)^p(dx^n+c)^q\,dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^q \, dx$$

```
[In] int((a + b*x^n)^p*(c + d*x^n)^q,x)
[Out] int((a + b*x^n)^p*(c + d*x^n)^q, x)
```

3.313 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 402

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$

$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+p)+n^2(11+6p+p^2)))\,x(a+\cdots}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

$$-\frac{d(ad(1+2n)-bc(1+n(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))}+\frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+3n+np)}$$

$$-\frac{(a^3d^3(1+3n+2n^2)-3a^2bcd^2(1+n)(1+n(3+p))+3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2))-b^3c\cdots}{b^3(1+\cdots}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3+
p)+n^2*(p^2+6*p+11)))*x*(a+b*x^n)^(p+1)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p
))-d*(a*d*(1+2*n)-b*c*(1+n*(5+p)))*x*(a+b*x^n)^(p+1)*(c+d*x^n)/b^2/(1+n*(2+
p))/(1+n*(3+p))+d*x*(a+b*x^n)^(p+1)*(c+d*x^n)^2/b/(n*p+3*n+1)-(a^3*d^3*(2*n
^2+3*n+1)-3*a^2*b*c*d^2*(1+n)*(1+n*(3+p))+3*a*b^2*c^2*d*(1+n*(5+2*p)+n^2*(p
^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11)+n^3*(p^3+6*p^2+11*p+6))
)*x*(a+b*x^n)^p*hypergeom([1/n, -p],[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/(1+n*(2
+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {427, 542, 396, 252, 251}

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= \frac{dx(a+bx^n)^{p+1} \left(a^2d^2(2n^2+3n+1) - abcd(n^2(p+7)+n(2p+9)+2) + b^2c^2(n^2(p^2+6p+11)+2n(p+\right.}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}$$
$$- \frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(a^3d^3(2n^2+3n+1) - 3a^2bcd^2(n+1)(n(p+3)+1) + 3ab^2c^2d(n^2(p^2+5p+6)\right.}{b^3(np+}$$
$$- \frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b^2(n(p+2)+1)(n(p+3)+1)} + \frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b
^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x^n)^(1 + p))/(b^
3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))) - (d*(a*d*(1 + 2*n) - b*(c
 + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1
 + n*(3 + p))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))
) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3
*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2
+ p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3)))*x*(a + b*x^
n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b^3*(1 + n +
 n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1 + (b*x^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} + \frac{\int (a+bx^n)^p(c+dx^n)(-c(ad-b(c+cn(3+p)))-d(ad(1+2n)-b(c+cn(5+p)))x^n)\,dx}{b(1+n(3+p))}$$

$$= -\frac{d(ad(1+2n)-b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} + \frac{\int (a+bx^n)^p\left(c(a^2d^2(1+2n)-abcd(2+n(7+2p))+b^2c^2(1+n(5+2p)+n^2(6+5p+p^2)))-\right.}{b^2(1+n}$$

$$= \frac{d(a^2d^2(1+3n+2n^2)-abcd(2+n^2(7+p)+n(9+2p))+b^2c^2(1+2n(3+p)+n^2(11+6p+p^2}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))} - \frac{d(ad(1+2n)-b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} - \frac{(a^3d^3(1+3n+2n^2)-3a^2bcd^2(1+n)(1+n(3+p))+3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2}{b^3(1+n+np)(1+n(2}$$

$$= \frac{d(a^2d^2(1+3n+2n^2) - abcd(2+n^2(7+p)+n(9+2p)) + b^2c^2(1+2n(3+p)+n^2(11+6p+p^2))}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$
$$- \frac{d(ad(1+2n) - b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))}$$
$$+ \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))}$$
$$- \frac{\left((a^3d^3(1+3n+2n^2) - 3a^2bcd^2(1+n)(1+n(3+p)) + 3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2}{b^3(1+n+n}$$

$$= \frac{d(a^2d^2(1+3n+2n^2) - abcd(2+n^2(7+p)+n(9+2p)) + b^2c^2(1+2n(3+p)+n^2(11+6p+p^2))}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$
$$- \frac{d(ad(1+2n) - b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))}$$
$$+ \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))}$$
$$- \frac{(a^3d^3(1+3n+2n^2) - 3a^2bcd^2(1+n)(1+n(3+p)) + 3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2)}{b^3(1+n}$$

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.42

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{3c^2dx^n \text{ Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+n} + \frac{3cd^2x^{2n} \text{ Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+2n} + \frac{d^3x^{3n} \text{ Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+3n} + c^3 \text{ Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1)
, -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1), -
p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1[3
+ n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric2F1[
n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^3\,dx$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^3,x)
[Out] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^3\;dx=\int (dx^n+c)^3(bx^n+a)^p\,dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p,
 x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.60

$$\int (a+bx^n)^p\,(c+dx^n)^3\;dx=\frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}c^3x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{3a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}c^2dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}+\frac{3a^{2+\frac{1}{n}}a^{p-2-\frac{1}{n}}cd^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}+\frac{a^{3+\frac{1}{n}}a^{p-3-\frac{1}{n}}d^3x^{3n+1}\Gamma\left(3+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,3+\frac{1}{n}\\4+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] a**(1/n)*a**(p - 1/n)*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 3*a**(1 + 1/n)*a**(p - 1 - 1/n)*c*
*2*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**(2 + 1/n)*a**(p - 2 - 1/n)*c*d**2*
x**(2*n + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**(3 + 1/n)*a**(p - 3 - 1/n)*d**3*x**(3*n
 + 1)*gamma(3 + 1/n)*hyper((-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_polar(I*pi
)/a)/(n*gamma(4 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (dx^n+c)^3 (bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,3,4,3,0]%%%}+%%%{6
,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^3 \, dx$$

```
[In] int((a + b*x^n)^p*(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n)^3, x)
```

3.314 $\int (a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 202

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{d(ad(1+n)-bc(1+n(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+2n+np)} - \frac{(bc(1+n+np)(ad-bc(1+n(2+p)))-ad(ad(1+n)-bc(1+n(3+p))))x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \mathrm{H}}{b^2(1+n+np)(1+n(2+p))}$$

```
[Out] -d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^(p+1)/b^2/(n*p+n+1)/(1+n*(2+p))+
d*x*(a+b*x^n)^(p+1)*(c+d*x^n)/b/(n*p+2*n+1)-(b*c*(n*p+n+1)*(a*d-b*c*(1+n*(2
+p)))-a*d*(a*d*(1+n)-b*c*(1+n*(3+p))))*x*(a+b*x^n)^p*hypergeom([1/n, -p],[1
+1/n],-b*x^n/a)/b^2/(n*p+n+1)/(1+n*(2+p))/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 396, 252, 251}

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b^2(np+n+1)(n(p+2)+1)} - \frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c(ad-b(cn(p+2)+c))-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}\right) \text{Hypergeometric2F1}\left(\frac{1}{n},-\right.}{b(n(p+2)+1)} + \frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b^2*(1 + n
 + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((c*(a*d - b*(c + c*n*(2 + p))) - (a*d*(a*d*(1 + n) - b*(c + c
*n*(3 + p))))/(b*(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1),
-p, 1 + n^(-1), -((b*x^n)/a)])/(b*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

integral

$$= \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} + \frac{\int (a+bx^n)^p \left(-c(ad-b(c+cn(2+p)))-d(ad(1+n)-b(c+cn(3+p)))x^n\right) dx}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(ad-b(c+cn(2+p))) - \frac{ad(ad(1+n)-b(c+cn(3+p)))}{b(1+n+np)}\right)\int (a+bx^n)^p\,dx}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(\left(c(ad-b(c+cn(2+p))) - \frac{ad(ad(1+n)-b(c+cn(3+p)))}{b(1+n+np)}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \left(1+\frac{bx^n}{a}\right)^p dx}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(ad-b(c+cn(2+p))) - \frac{ad(ad(1+n)-b(c+cn(3+p)))}{b(1+n+np)}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(1+n(2+p))}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.69

$$\int (a+bx^n)^p\,(c+dx^n)^2\,dx = \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(2cd(1+2n)x^n\,\text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)+(1+n)\left(d^2x^{2n}\,\text{H}\right.\right.}{(1+n)(1}$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 +
 n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
 -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1), -p
, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^2\,dx$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (dx^n+c)^2 (bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \frac{a^{\frac{1}{n}} a^{p-\frac{1}{n}} c^2 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^{1+\frac{1}{n}} a^{p-1-\frac{1}{n}} c d x^{n+1} \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{a^{2+\frac{1}{n}} a^{p-2-\frac{1}{n}} d^2 x^{2n+1} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] a**(1/n)*a**(p - 1/n)*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 2*a**(1 + 1/n)*a**(p - 1 - 1/n)*c*
d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**(2 + 1/n)*a**(p - 2 - 1/n)*d**2*x**(2*n
 + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*pi
)/a)/(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (dx^n+c)^2(bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,2,0]%%%}+%%%
{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^2 \,dx$$

```
[In] int((a + b*x^n)^p*(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n)^2, x)
```

3.315 $\int (a+bx^n)^p (c+dx^n)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \frac{(ad-bc(1+n+np))x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(1+n+np)}$$

```
[Out] d*x*(a+b*x^n)^(p+1)/b/(n*p+n+1)-(a*d-b*c*(n*p+n+1))*x*(a+b*x^n)^p*hypergeom
([1/n, -p],[1+1/n],-b*x^n/a)/b/(n*p+n+1)/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {396, 252, 251}

$$\int (a+bx^n)^p (c+dx^n)\, dx = x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c - \frac{ad}{bnp+bn+b}\right) \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right) + \frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p)
)*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(-c+\frac{ad}{b+bn+bnp}\right)\int (a+bx^n)^p\,dx\\ &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(\left(-c+\frac{ad}{b+bn+bnp}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p dx\\ &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c-\frac{ad}{b+bn+bnp}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^p\,(c+dx^n)\,dx = \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(d(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p+(-ad+bc(1+n+np))\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1-\right.\right.}{b(1+n+np)}$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n +
 n*p)*(1 + (b*x^n)/a)^p)
```

Maple [F]

$$\int (a+b\,x^{n})^{p} (c+d\,x^{n})\, dx$$

[In] int((a+b*x^n)^p*(c+d*x^n),x)

[Out] int((a+b*x^n)^p*(c+d*x^n),x)

Fricas [F]

$$\int (a+bx^{n})^{p} (c+dx^{n})\ dx = \int (dx^{n}+c)(bx^{n}+a)^{p}\, dx$$

[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")

[Out] integral((d*x^n + c)*(b*x^n + a)^p, x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a+bx^{n})^{p} (c+dx^{n})\ dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{n}, -p\\ 1+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_{2}F_{1}\left(\begin{matrix}-p, 1+\frac{1}{n}\\ 2+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

[In] integrate((a+b*x**n)**p*(c+d*x**n),x)

[Out] a**(1/n)*a**(p - 1/n)*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(1 + 1/n)*a**(p - 1 - 1/n)*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%%{1
,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n) \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n) \, dx$$

```
[In] int((a + b*x^n)^p*(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n), x)
```

3.316 $\int (a+bx^n)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

```
[Out] x*(a+b*x^n)^p*hypergeom([1/n, -p],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

```
[In] Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p\,dx\\ &= x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p\,dx = x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

```
[In] Integrate[(a + b*x^n)^p,x]
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Maple [F]

$$\int (a+b\,x^n)^p\,dx$$

```
[In] int((a+b*x^n)^p,x)
[Out] int((a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a+bx^n)^p\,dx = \int (bx^n+a)^p\,dx$$

```
[In] integrate((a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p \, dx = \frac{a^{\frac{1}{n}} a^{p-\frac{1}{n}} x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate((a+b*x**n)**p,x)
[Out] a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p, x)
```

Giac [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

```
[In] integrate((a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int (a+bx^n)^p \, dx = \frac{x\,(a+b\,x^n)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

```
[In] int((a + b*x^n)^p,x)
[Out] (x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1)
^p
```

3.317 $\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/((1+b*x^n/a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

```
[In] Int[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^n}{a}\right)^p}{c+dx^n}\,dx$$

$$=\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int\frac{(a+bx^n)^p}{c+dx^n}\,dx$$

$$=\frac{ac(1+n)x(a+bx^n)^p\,\text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a}\ldots\right)}{(c+dx^n)\left(bcnpx^n\,\text{AppellF1}\left(1+\frac{1}{n},1-p,1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},-p,2,2+\frac{1}{n},-\ldots\right)\ldots\right)}$$

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1), -
p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1)
, -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]

$$\int\frac{(a+b\,x^n)^p}{c+d\,x^n}dx$$

```
[In] int((a+b*x^n)^p/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

```
[In] int((a + b*x^n)^p/(c + d*x^n),x)
[Out] int((a + b*x^n)^p/(c + d*x^n), x)
```

3.318 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/((1+b*x^n/a)^p
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} \, dx = \frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^2}\,dx$$

$$= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Mathematica [B] (warning: unable to verify)

Leave count is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int\frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$$

$$= \frac{ac(1+n)x(a+bx^n)^p\operatorname{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\right.}{(c+dx^n)^2\left(bcnpx^n\operatorname{AppellF1}\left(1+\frac{1}{n},1-p,2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},-p,3,2+\frac{1}{n},\right.\right.}$$

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]

$$\int\frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

```
[In] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2} dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2} dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2} dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

```
[In] int((a + b*x^n)^p/(c + d*x^n)^2,x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^2, x)
```

3.319 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,3,1+1/n,-b*x^n/a,-d*x^n/c)/c^3/((1+b*x^n/a)^p
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} \, dx = \frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^3}\,dx$$
$$=\frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int\frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$
$$=\frac{ac(1+n)x(a+bx^n)^p\operatorname{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^3\left(bcnpx^n\operatorname{AppellF1}\left(1+\frac{1}{n},1-p,3,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},-p,4,2+\frac{1}{n},\ldots\right)\ldots\right)}$$

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F]

$$\int\frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}dx$$

```
[In] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx=\int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
 x)
```

Sympy [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx=\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx=\int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx=\int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

```
[In] int((a + b*x^n)^p/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^p/(c + d*x^n)^3, x)
```

3.320 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^(-1/n-p)*hypergeom([1/n, -p],[1+1/n],-(-a*d+b*c)*x^
n/a/(c+d*x^n))/c/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {388}

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
+ n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*(((c*(a + b*x^n))/(a*(c
+ d*x^n)))^p)
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
```

```
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^{-\frac{1+np}{n}} \left(1+\frac{dx^n}{c}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{c}$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
[Out] (x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)^(
(1 + n*p)/n))
```

Maple [F]

$$\int (a+b\,x^n)^p \,(c+d\,x^n)^{-1-\frac{1}{n}-p} \, dx$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+1}} \, dx$$

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1), x)
```

3.321 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2 (1+5n+6n^2)} + \frac{6a^2n^2x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c^3(1+n)(1+2n)(1+3n)} + \frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(1+n)(1+2n)(1+3n)}$$

```
[Out] x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^(-
2-1/n)/c^2/(6*n^2+5*n+1)+6*a^2*n^2*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(6*n^
3+11*n^2+6*n+1)+6*a^3*n^3*x/c^4/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {386, 197}

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)} + \frac{6a^2n^2x(a+bx^n) (c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-2}}{c^2 (6n^2+5n+1)} + \frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

```
[In] Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+bx^n)^3\,(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{(3an)\int (a+bx^n)^2\,(c+dx^n)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}\\
&= \frac{x(a+bx^n)^3\,(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c^2\,(1+5n+6n^2)}\\
&\quad+\frac{(6a^2n^2)\int (a+bx^n)\,(c+dx^n)^{-2-\frac{1}{n}}\,dx}{c^2\,(1+5n+6n^2)}\\
&= \frac{x(a+bx^n)^3\,(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c^2\,(1+5n+6n^2)}\\
&\quad+\frac{6a^2n^2x(a+bx^n)\,(c+dx^n)^{-1-\frac{1}{n}}}{c^3(1+n)\,(1+5n+6n^2)}+\frac{(6a^3n^3)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c^3(1+n)\,(1+5n+6n^2)}\\
&= \frac{x(a+bx^n)^3\,(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c^2\,(1+5n+6n^2)}\\
&\quad+\frac{6a^2n^2x(a+bx^n)\,(c+dx^n)^{-1-\frac{1}{n}}}{c^3(1+n)\,(1+5n+6n^2)}+\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(1+n)\,(1+5n+6n^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx$$
$$= \frac{x(c+dx^n)^{-3-\frac{1}{n}} (b^3c^3(1+3n+2n^2) x^{3n} + 3ab^2c^2(1+n)x^{2n}(c+3cn+dnx^n) + 3a^2bcx^n(c^2(1+5n+6n^2)}{c^4(1}$$

```
[In] Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*c
^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n + 6*
n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n + 11*
n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*x^(2
*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(174) = 348.

Time = 5.02 (sec) , antiderivative size = 1288, normalized size of antiderivative = 7.24

method	result	size
parallelrisch	Expression too large to display	1288

```
[In] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

```
[Out] (6*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a^3*d^4*n^3+2*x*(x^n)^3*(c+d*x^n)^(-(1+
4*n)/n)*b^3*c^4*n^2+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d+3*x*(x^n)^3*
(c+d*x^n)^(-(1+4*n)/n)*b^3*c^4*n+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c
^4+x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d+3*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^
2*b*c^4+2*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d*n^2+24*x*(x^n)^3*(c+d*
x^n)^(-(1+4*n)/n)*a^3*c*d^3*n^3+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*
d*n+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^3*c*d^3*n^2+36*x*(x^n)^2*(c+d*x^n)
^(-(1+4*n)/n)*a^3*c^2*d^2*n^3+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d
^2*n^2+9*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^4*n^2+24*x*x^n*(c+d*x^n)^
(-(1+4*n)/n)*a^3*c^3*d*n^3+3*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^3*d+3
*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d^2*n+12*x*(x^n)^2*(c+d*x^n)^(-(1
+4*n)/n)*a*b^2*c^4*n+26*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*n^2+18*x*x^n
*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^4*n^2+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^
2*b*c^3*d+9*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*n+6*x*(x^n)^4*(c+d*x^n)^
(-(1+4*n)/n)*a^2*b*c*d^3*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^2*d
^2*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^2*d^2*n+24*x*(x^n)^3*(c+d
*x^n)^(-(1+4*n)/n)*a^2*b*c^2*d^2*n^2+12*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*
```

```
b^2*c^3*d*n^2+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^2*d^2*n+15*x*(x^n)
^3*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^3*d*n+36*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)
*a^2*b*c^3*d*n^2+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^3*d*n+15*x*x^n
*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^4*n+x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^
4+6*x*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^4*n^3+11*x*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^
4*n^2+x*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^4+6*x*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^4*n
)/c^4/(6*n^3+11*n^2+6*n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(174) = 348$.

Time = 0.27 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.69

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$$
$$= \frac{(6\,a^3d^4n^3 + b^3c^3d + (2\,b^3c^3d + 3\,ab^2c^2d^2 + 6\,a^2bcd^3)n^2 + 3\,(b^3c^3d + ab^2c^2d^2)n)xx^{4\,n} + (24\,a^3cd^3n^3 + b^3c^4}{}$$

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 + b
^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + 3*
a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n)
 + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*
b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*
n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*b*c^4
 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*a^3*c^4*
n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 +
 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2822 vs. $2(160) = 320$.

Time = 29.87 (sec) , antiderivative size = 2822, normalized size of antiderivative = 15.85

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] 6*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(
d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n
) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n)
```

```
+ 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**
(1/n)*gamma(4 + 1/n)) + 11*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma(1/n)
/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(
1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n
**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x*
*(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**3*c**3*c**(1/n)*c**(-
4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)
+ 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + a**3*
c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)
**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n
)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*
gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(
4 + 1/n)) + 18*a**3*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x**n*gamma(1/n)/(c**
3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*
n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x
**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n
)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 15*a**3*c**2*c**(1/n)*c**(-4 -
1/n)*d*n**2*x**n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gam
ma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1
/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) +
 3*a**3*c**2*c**(1/n)*c**(-4 - 1/n)*d*n*x**n*gamma(1/n)/(c**3*d**(1/n)*n**4
*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d
*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**3*c*c**(1/n)*c**(-4 - 1/n)*d**2*n**3*x**
(2*n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)
 + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c
*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*
d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**3*c*c
**(1/n)*c**(-4 - 1/n)*d**2*n**2*x**(2*n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n)
 + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)*
*(1/n)*gamma(4 + 1/n)) + 6*a**3*c**(1/n)*c**(-4 - 1/n)*d**3*n**3*x**(3*n)*g
amma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*
*2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d
**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n
)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 18*a**2*b*c**2*c*
*(-4 - 1/n)*c**(1 + 1/n)*n**2*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(
c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(
4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 15*a**2*b*c**2
*c**(-4 - 1/n)*c**(1 + 1/n)*n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(
```

```
c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(
4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 3*a**2*b*c**2*
c**(-4 - 1/n)*c**(1 + 1/n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**
2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 +
 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 18*a**2*b*c*c**(-
4 - 1/n)*c**(1 + 1/n)*d*n**2*x**n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/
n)/(c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*ga
mma(4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 6*a**2*b*c
*c**(-4 - 1/n)*c**(1 + 1/n)*d*n*x**n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 +
 1/n)/(c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n
*gamma(4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 6*a**2*
b*c**(-4 - 1/n)*c**(1 + 1/n)*d**2*n**2*x**(2*n)*(c/(d*x**n) + 1)**(-1 - 1/n
)*gamma(1 + 1/n)/(c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n
)*n**3*x**n*gamma(4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)
) + 9*a*b**2*c*c**(-4 - 1/n)*c**(2 + 1/n)*n*(c/(d*x**n) + 1)**(-2 - 1/n)*ga
mma(2 + 1/n)/(c*d**(2 + 1/n)*n**2*gamma(4 + 1/n) + d*d**(2 + 1/n)*n**2*x**n
*gamma(4 + 1/n)) + 3*a*b**2*c*c**(-4 - 1/n)*c**(2 + 1/n)*(c/(d*x**n) + 1)**
(-2 - 1/n)*gamma(2 + 1/n)/(c*d**(2 + 1/n)*n**2*gamma(4 + 1/n) + d*d**(2 + 1
/n)*n**2*x**n*gamma(4 + 1/n)) + 3*a*b**2*c**(-4 - 1/n)*c**(2 + 1/n)*d*n*x**
n*(c/(d*x**n) + 1)**(-2 - 1/n)*gamma(2 + 1/n)/(c*d**(2 + 1/n)*n**2*gamma(4
+ 1/n) + d*d**(2 + 1/n)*n**2*x**n*gamma(4 + 1/n)) + b**3*c**(-4 - 1/n)*c**(
3 + 1/n)*d**(-3 - 1/n)*(c/(d*x**n) + 1)**(-3 - 1/n)*gamma(3 + 1/n)/(n*gamma
(4 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \int (bx^n+a)^3(dx^n+c)^{-\frac{1}{n}-4} dx$$

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%{54
,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx = \int \frac{(a+b\,x^n)^3}{(c+d\,x^n)^{\frac{1}{n}+4}}\, dx$$

```
[In] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
[Out] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

3.322 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx = \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(1+n)(1+2n)}$$

```
[Out] x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c/(1+2*n)+2*a*n*x*(a+b*x^n)*(c+d*x^n)^(-1-
1/n)/c^2/(2*n^2+3*n+1)+2*a^2*n^2*x/c^3/(2*n^2+3*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {386, 197}

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx = \frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n) (c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{(2an)\int (a+bx^n)\,(c+dx^n)^{-2-\frac{1}{n}}\,dx}{c(1+2n)} \\
&= \frac{x(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)\,(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{(2a^2n^2)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c^2(1+n)(1+2n)} \\
&= \frac{x(a+bx^n)^2\,(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)\,(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(1+n)(1+2n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int (a+bx^n)^2\,(c+dx^n)^{-3-\frac{1}{n}}\,dx$$
$$= \frac{x(c+dx^n)^{-2-\frac{1}{n}}\,(b^2c^2(1+n)x^{2n}+2abcx^n(c+2cn+dnx^n)+a^2(c^2(1+3n+2n^2)+2cdn(1+2n)x^n+2d^2}{c^3(1+n)(1+2n)}$$

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2*
c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d^2
*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(116) = 232.

Time = 4.51 (sec) , antiderivative size = 588, normalized size of antiderivative = 5.07

method	result
parallelrisch	$\frac{2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}a^{2}d^{3}n^{2}+2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}abc\,d^{2}n+x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}b^{2}c^{2}dn+6x\,x^{2n}(c+d\,x^{n})^{-\frac{1+3n}{n}}a^{2}c\,d^{2}n}{}$

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)
```

```
[Out] (2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a^2*d^3*n^2+2*x*(x^n)^3*(c+d*x^n)^(-(1+
3*n)/n)*a*b*c*d^2*n+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^2*d*n+6*x*(x^n)^
2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*b^2
*c^2*d+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n+6*x*(x^n)^2*(c+d*x^n)
^(-(1+3*n)/n)*a*b*c^2*d*n+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3*n+6*x*x^
n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a
*b*c^2*d+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3+5*x*x^n*(c+d*x^n)^(-(1+3*
n)/n)*a^2*c^2*d*n+4*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*b*c^3*n+2*x*(c+d*x^n)^(-
(1+3*n)/n)*a^2*c^3*n^2+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d+2*x*x^n*(c+d*
x^n)^(-(1+3*n)/n)*a*b*c^3+3*x*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^3*n+x*(c+d*x^n)^
(-(1+3*n)/n)*a^2*c^3)/(1+n)/(1+2*n)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx$$
$$=\frac{(2\,a^2d^3n^2+b^2c^2d+(b^2c^2d+2\,abcd^2)n)xx^{3\,n}+(6\,a^2cd^2n^2+b^2c^3+2\,abc^2d+(b^2c^3+6\,abc^2d+2\,a^2cd^2)n)}{(2\,c^3n^2+3\,c^3n+c^3)(d}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*a
^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d^2
)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 + 5*
a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^
2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(104) = 208$.

Time = 12.92 (sec) , antiderivative size = 1035, normalized size of antiderivative = 8.92

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] 2*a**2*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n
/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 +
 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a**2*c
**2*c**(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*
gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**
2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a**2*c**2*c**(1/n)*
c**(-3 - 2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)
+ 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)
*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a**2*c*c**(1/n)*c**(-3 - 2/n)*d*
n**2*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*
c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1
+ d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c*c**(1/n)*c**(-3 - 2/n)*d*n*x*
x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**
3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c**(1/n)*c**(-3 - 2/n)*d**2*n**2*x*x**
(2*n)*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n*
*3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x*
*n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*b*c*c**(-3 - 1/n)*c**(1 + 1/n)*n*(c/(d*x
**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) +
d*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + 2*a*b*c*c**(-3 - 1/n)*c**(1 + 1/
n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3
 + 1/n) + d*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + 2*a*b*c**(-3 - 1/n)*c*
*(1 + 1/n)*d*n*x**n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 +
1/n)*n**2*gamma(3 + 1/n) + d*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + b**2*
c**(-3 - 1/n)*c**(2 + 1/n)*d**(-2 - 1/n)*(c/(d*x**n) + 1)**(-2 - 1/n)*gamma
(2 + 1/n)/(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} dx = \int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-3} dx$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6,[1
,0,4,1
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+3}} \,dx$$

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

3.323 $\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \frac{x(a+bx^n) (c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

```
[Out] x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c/(1+n)+a*n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {386, 197}

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \frac{x(a+bx^n) (c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)} + \frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{(an)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$$
$$=\frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(bcx^n+a(1+n)(c+dx^n)\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{c^2(1+n)}$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F
1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^
((1 + n)/n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(58) = 116.

Time = 4.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.43

method	result
parallelrisch	$\frac{x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}a\,d^2n+x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}bcd+2x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}acdn+x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}acd+x\,x^n(c+d\,x^n)^{-}}{c^2(1+n)}$

```
[In] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)*a*d^2*n+x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)*
b*c*d+2*x*x^n*(c+d*x^n)^(-(1+2*n)/n)*a*c*d*n+x*x^n*(c+d*x^n)^(-(1+2*n)/n)*a
*c*d+x*x^n*(c+d*x^n)^(-(1+2*n)/n)*b*c^2+x*(c+d*x^n)^(-(1+2*n)/n)*a*c^2*n+x*
(c+d*x^n)^(-(1+2*n)/n)*a*c^2)/c^2/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{(ad^2n+bcd)xx^{2n}+(2\,acdn+bc^2+acd)xx^n+(ac^2n+ac^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2n+1}{n}}}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n
 + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(48) = 96.

Time = 2.21 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.36

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} \\ &+ \frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} \\ &+ \frac{ac^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} \\ &+ \frac{bc^{-2-\frac{1}{n}}c^{1+\frac{1}{n}}d^{-1-\frac{1}{n}}\left(\frac{cx^{-n}}{d}+1\right)^{-1-\frac{1}{n}}\Gamma\left(1+\frac{1}{n}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}\end{aligned}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] a*c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)**
(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(2
 + 1/n)) + a*c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**
n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n
)*gamma(2 + 1/n)) + a*c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d**(1/n
)*n**2*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*
x**n) + 1)**(1/n)*gamma(2 + 1/n)) + b*c**(-2 - 1/n)*c**(1 + 1/n)*d**(-1 - 1
/n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,[0,
0,2,1,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^{\frac{1}{n}+2}}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

3.324 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[Out] x/c/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {197}

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[In] Int[(c + d*x^n)^(-1 - n^(-1)),x]
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{x(c+dx^n)^{-1/n}}{c}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 4.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

method	result	size
parallelrisch	$\frac{x\,x^{n}(c+d\,x^{n})^{-\frac{1+n}{n}}d+x(c+d\,x^{n})^{-\frac{1+n}{n}}c}{c}$	47
norman	$x\,\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}+\frac{dx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}}{c}$	53

```
[In] int((c+d*x^n)^(-1-1/n),x,method=_RETURNVERBOSE)
[Out] (x*x^n*(c+d*x^n)^(-(1+n)/n)*d+x*(c+d*x^n)^(-(1+n)/n)*c)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{dxx^n+cx}{(dx^n+c)^{\frac{n+1}{n}}c}$$

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int (c + dx^n)^{-1-\frac{1}{n}}\, dx = \frac{c^{\frac{1}{n}} c^{-1-\frac{1}{n}} d^{-\frac{1}{n}} \left(\frac{cx^{-n}}{d} + 1\right)^{-\frac{1}{n}} \Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

```
[In] integrate((c+d*x**n)**(-1-1/n),x)
[Out] c**(1/n)*c**(-1 - 1/n)*gamma(1/n)/(d**(1/n)*n*(c/(d*x**n) + 1)**(1/n)*gamma
(1 + 1/n))
```

Maxima [F]

$$\int (c + dx^n)^{-1-\frac{1}{n}}\, dx = \int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

Giac [F]

$$\int (c + dx^n)^{-1-\frac{1}{n}}\, dx = \int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.17

$$\int (c + dx^n)^{-1-\frac{1}{n}}\, dx = \frac{d\, x^{n+1} \left(\frac{c}{d\, x^n} - \left(\frac{c}{d\, x^n} + 1\right)^{\frac{n+1}{n}} + 1\right)}{c\, n\, \left(\frac{n+1}{n} - 1\right)\, \left(c + d\, x^n\right)^{\frac{n+1}{n}}}$$

```
[In] int(1/(c + d*x^n)^(1/n + 1),x)
[Out] (d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/n
 - 1)*(c + d*x^n)^((n + 1)/n))
```

3.325 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/((c+d*x^n)^(1/n
))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {387}

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{x(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a}$$

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*
x^n))])/(a*(c + d*x^n)^n^(-1))
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{a+b\,x^n}dx$$

```
[In] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
[Out] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{1/n}}\,dx$$

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)), x)
```

3.326 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

```
[Out] c*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/((c+d*x^n)^
(1/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {387}

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d
*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, \frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + 
d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2} dx$$

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2} dx$$

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}\,dx$$

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2,x)
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2, x)
```

3.327 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{c^2x(c+dx^n)^{-1/n}\, \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

```
[Out] c^2*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^n
)^(1/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {387}

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{c^2x(c+dx^n)^{-1/n}\, \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
 d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{c^2 x (c + dx^n)^{-1/n} \ {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{(c + dx^n)^{2-\frac{1}{n}}}{(a + bx^n)^3} \, dx = \frac{c^2 x (c + dx^n)^{-1/n} \, \text{Hypergeometric2F1}\left(3, \frac{1}{n}, 1 + \frac{1}{n}, \frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
+ d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

Maple [F]

$$\int \frac{(c + d\, x^n)^{2-\frac{1}{n}}}{(a + b\, x^n)^3} dx$$

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(c + dx^n)^{2-\frac{1}{n}}}{(a + bx^n)^3} \, dx = \int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3} \, dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3,x)
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3, x)
```

3.328 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 193

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{(bc+(bc-ad)n(1+p))x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-1-p}(c+dx^n)^{-1-\frac{1}{n}-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{n},-1-p,\right.}{ac(bc-ad)n(1+p)}$$

```
[Out] -b*x*(a+b*x^n)^(p+1)*(c+d*x^n)^(-1-1/n-p)/a/(-a*d+b*c)/n/(p+1)+(b*c+(-a*d+b
*c)*n*(p+1))*x*(a+b*x^n)^(p+1)*(c*(a+b*x^n)/a/(c+d*x^n))^(-1-p)*(c+d*x^n)^(
-1-1/n-p)*hypergeom([1/n, -1-p],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/c/(-
a*d+b*c)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {390, 388}

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$$
$$= \frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{b}{n(p+1)(bc-ad)}+\frac{1}{c}\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}\operatorname{Hypergeometric2F1}\left(\frac{1}{n},-p-1,1+\frac{1}{n},\right.}{a} - \frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n*
(1 + p))) + ((c^(-1) + b/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((c
*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyper
geometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)
))])/a
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} \\
&\quad+\frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)\int (a+bx^n)^{1+p}(c+dx^n)^{-2-\frac{1}{n}-p}\,dx}{a} \\
&= -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} \\
&\quad+\frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-1-p}(c+dx^n)^{-1-\frac{1}{n}-p}\,{}_2F_1\left(\frac{1}{n},-1-p;1+\frac{1}{n};-\frac{(bc-}{a(c+}\right.}{ac}
\end{aligned}
$$

Mathematica [F]

$$\int (a+bx^n)^p(c+dx^n)^{-2-\frac{1}{n}-p}\,dx=\int (a+bx^n)^p(c+dx^n)^{-2-\frac{1}{n}-p}\,dx$$

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
[Out] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p), x]
```

Maple [F]

$$\int (a + b\,x^{n})^{p} (c + d\,x^{n})^{-2-\frac{1}{n}-p}\,dx$$

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

Fricas [F]

$$\int (a + bx^{n})^{p} (c + dx^{n})^{-2-\frac{1}{n}-p}\,dx = \int (bx^{n} + a)^{p}(dx^{n} + c)^{-p-\frac{1}{n}-2}\,dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a + bx^{n})^{p} (c + dx^{n})^{-2-\frac{1}{n}-p}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a + bx^{n})^{p} (c + dx^{n})^{-2-\frac{1}{n}-p}\,dx = \int (bx^{n} + a)^{p}(dx^{n} + c)^{-p-\frac{1}{n}-2}\,dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Giac [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-2} \, dx$$

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+2}} \, dx$$

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2), x)
```

3.329 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$

Optimal result

Integrand size = 69, antiderivative size = 57

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

```
[Out] x*(c+d*x^n)^(a*d/(-a*d+b*c)/n)/a/c/((a+b*x^n)^(b*c/(-a*d+b*c)/n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {389}

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}} (c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
 b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}}(c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

```
[In] Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^(
(a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a*
d*n)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(58) = 116.

Time = 8.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 5.86

method	result
parallelrisch	$\frac{x\,x^{2n}(a+b\,x^n)^{\frac{-adn+bc(1+n)}{(ad-bc)n}}(c+d\,x^n)^{-\frac{adn-bcn+ad}{(ad-bc)n}}b^2d^2+x\,x^n(a+b\,x^n)^{\frac{-adn+bc(1+n)}{(ad-bc)n}}(c+d\,x^n)^{-\frac{adn-bcn+ad}{(ad-bc)n}}ab\,d^2+x\,x^n(a+}{abcd}$

```
[In] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(x^n)^2*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b*
c*n+a*d)/(a*d-b*c)/n)*b^2*d^2+x*x^n*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*c)
/n)*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/(a*d-b*c)/n)*a*b*d^2+x*x^n*(a+b*x^n)^((-a
*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/(a*d-b*c)/n)*b^2
*c*d+x*(a+b*x^n)^((-a*d*n+b*c*(1+n))/(a*d-b*c)/n)*(c+d*x^n)^(-(a*d*n-b*c*n+
a*d)/(a*d-b*c)/n)*a*b*c*d)/a/b/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\frac{(bdxx^{2\,n}+acx+(bc+ad)xx^n)(dx^n+c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n+a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d)
*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))*a
*c)
```

Sympy [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int (a+bx^n)^{\frac{adn-bc(n+1)}{n(-ad+bc)}}(c+dx^n)^{\frac{adn+ad-bcn}{-adn+bcn}}\,dx$$

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Integral((a + b*x**n)**((a*d*n - b*c*(n + 1))/(n*(-a*d + b*c)))*(c + d*x**n
)**((a*d*n + a*d - b*c*n)/(-a*d*n + b*c*n)), x)
```

Maxima [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Giac [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx = \int \frac{1}{(a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(n+1)}{n\,(a\,d-b\,c)}}\,(c+d\,x^n)^{\frac{a\,d+a\,d\,n-b\,c\,n}{a\,d\,n-b\,c\,n}}}\,dx$$

```
[In] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
[Out] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

3.330 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx$

Optimal result

Integrand size = 25, antiderivative size = 327

$$\begin{aligned}\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx = & -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} \\ & -\frac{(3adn-b(c+3cn))x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(bc-ad)n(1+3n)} \\ & -\frac{(3adn-b(c+3cn))x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(bc-ad)(1+5n+6n^2)} \\ & -\frac{2an(3adn-b(c+3cn))x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^3(bc-ad)(1+n)(1+2n)(1+3n)} \\ & -\frac{2a^2n^2(3adn-b(c+3cn))x(c+dx^n)^{-1/n}}{c^4(bc-ad)(1+n)(1+2n)(1+3n)}\end{aligned}$$

```
[Out] -1/3*b*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/(-a*d+b*c)/n-1/3*(3*a*d*n-b*(3*c*
n+c))*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/c/(-a*d+b*c)/n/(1+3*n)-(3*a*d*n-b*
(3*c*n+c))*x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c^2/(-a*d+b*c)/(6*n^2+5*n+1)-2*
a*n*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(-a*d+b*c)/(6*
n^3+11*n^2+6*n+1)-2*a^2*n^2*(3*a*d*n-b*(3*c*n+c))*x/c^4/(-a*d+b*c)/(6*n^3+1
1*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {390, 386, 197}

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx = -\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} - \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)} - \frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -1/3*(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(a*(b*c - a*d)*n) - ((3*
a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b
*c - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*
x^n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n -
 b*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(
1 + n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*
(b*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
```

```
Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)\int(a+bx^n)^3\left(c+dx^n\right)^{-4-\frac{1}{n}}\,dx}{3a}$$

$$= -\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{\left(n\left(3+\frac{bc}{bcn-adn}\right)\right)\int(a+bx^n)^2\left(c+dx^n\right)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}$$

$$= -\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^2\left(c+dx^n\right)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{\left(2an^2\left(3+\frac{bc}{bcn-adn}\right)\right)\int(a+bx^n)\left(c+dx^n\right)^{-2-\frac{1}{n}}\,dx}{c^2\left(1+5n+6n^2\right)}$$

$$= -\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^2\left(c+dx^n\right)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{2an^2\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)\left(c+dx^n\right)^{-1-\frac{1}{n}}}{c^3(1+n)\left(1+5n+6n^2\right)} + \frac{\left(2a^2n^3\left(3+\frac{bc}{bcn-adn}\right)\right)\int\left(c+dx^n\right)^{-1-\frac{1}{n}}\,dx}{c^3(1+n)\left(1+5n+6n^2\right)}$$

$$= -\frac{bx(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3a(bc-ad)n}+\frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^2\left(c+dx^n\right)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{2an^2\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)\left(c+dx^n\right)^{-1-\frac{1}{n}}}{c^3(1+n)\left(1+5n+6n^2\right)} + \frac{2a^2n^3\left(3+\frac{bc}{bcn-adn}\right)x\left(c+dx^n\right)^{-1/n}}{c^4(1+n)\left(1+5n+6n^2\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.42

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$$
$$= \frac{x(c+dx^n)^{-1/n} \left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}} \left(b^2c^2 \operatorname{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right) - (bc-ad)\left(2bc \operatorname{Hypergeom}\right.\right.}{c^4d^2}$$

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
+ n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1),
n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 + n^
(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. 2(319) = 638.

Time = 4.52 (sec) , antiderivative size = 1059, normalized size of antiderivative = 3.24

method	result	size
parallelrisch	Expression too large to display	1059

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

```
[Out] (24*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n^3+3*x*(x^n)^2*(c+d*x^n)^(-(1+4
*n)/n)*a^2*c^2*d^2*n+26*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n^2+12*x*x^n
*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^4*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*
c^3*d+9*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d*n+10*x*x^n*(c+d*x^n)^(-(1+4*
n)/n)*a*b*c^4*n+16*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n^2+4*x*(x^
n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b*c*d^3*n^2+24*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/
n)*a^2*c*d^3*n^3+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^2*d^2*n+6*x*(x^n)^3
*(c+d*x^n)^(-(1+4*n)/n)*a^2*c*d^3*n^2+4*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^
2*c^3*d*n^2+4*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n+24*x*(x^n)^2*(
c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d*n^2+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^
2*d^2*n^2+14*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d*n+36*x*(x^n)^2*(c+d
*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n^3+5*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c
^3*d*n+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n^2+6*x*(x^n)^4*(c+d
*x^n)^(-(1+4*n)/n)*a^2*d^4*n^3+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^4*n
^2+x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^3*d+4*x*(x^n)^2*(c+d*x^n)^(-(1+4*
n)/n)*b^2*c^4*n+x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d+2*x*x^n*(c+d*x^n)^(-
(1+4*n)/n)*a*b*c^4+x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4+6*x*(c+d*x^n)^(-(1+4*n)
```

```
/n)*a^2*c^4*n^3+x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^4+11*x*(c+d*x^n)^(-(
1+4*n)/n)*a^2*c^4*n^2+6*x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4*n)/(2*n^2+3*n+1)/(
1+3*n)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.22

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx = \frac{(6a^2d^4n^3 + b^2c^2d^2n + (b^2c^2d^2 + 4abcd^3)n^2)xx^{4n} + (24a^2cd^3n^3 + b^2c^3d + 2(2b^2c^3d + 8abc^2d^2 + 3a^2cd^3)n}{}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*n
) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^2*
c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^2*n
^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2
+ (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n
^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4
 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4*n +
a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1
)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. 2(282) = 564.

Time = 13.32 (sec) , antiderivative size = 2746, normalized size of antiderivative = 8.40

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] 6*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(
d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n
) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n)
+ 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**
(1/n)*gamma(4 + 1/n)) + 11*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma(1/n)
/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(
1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n
**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x*
```

```
*(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**2*c**3*c**(1/n)*c**(-
4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)
+ 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + a**2*
c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)
**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n
)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*
gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(
4 + 1/n)) + 18*a**2*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x**n*gamma(1/n)/(c**
3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*
n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x
**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n
)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 15*a**2*c**2*c**(1/n)*c**(-4 -
1/n)*d*n**2*x**n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gam
ma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1
/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) +
 3*a**2*c**2*c**(1/n)*c**(-4 - 1/n)*d*n*x**n*gamma(1/n)/(c**3*d**(1/n)*n**4
*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d
*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**2*c*c**(1/n)*c**(-4 - 1/n)*d**2*n**3*x**
(2*n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)
 + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c
*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*
d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**2*c*c
**(1/n)*c**(-4 - 1/n)*d**2*n**2*x**(2*n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n)
 + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)*
*(1/n)*gamma(4 + 1/n)) + 6*a**2*c**(1/n)*c**(-4 - 1/n)*d**3*n**3*x**(3*n)*g
amma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*
*2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d
**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n
)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 12*a*b*c**2*c**(-
4 - 1/n)*c**(1 + 1/n)*n**2*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**
2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 +
 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 10*a*b*c**2*c**(-
4 - 1/n)*c**(1 + 1/n)*n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**2*d
**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 + 1/
n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 2*a*b*c**2*c**(-4 -
1/n)*c**(1 + 1/n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**2*d**(1 +
 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 + 1/n) + d
**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 12*a*b*c*c**(-4 - 1/n)*c**
```

```
(1 + 1/n)*d*n**2*x**n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**2*d**
(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 + 1/n)
 + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 4*a*b*c*c**(-4 - 1/n)*
c**(1 + 1/n)*d*n*x**n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c**2*d**
(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(4 + 1/n)
 + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 4*a*b*c**(-4 - 1/n)*c*
*(1 + 1/n)*d**2*n**2*x**(2*n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(
c**2*d**(1 + 1/n)*n**3*gamma(4 + 1/n) + 2*c*d*d**(1 + 1/n)*n**3*x**n*gamma(
4 + 1/n) + d**2*d**(1 + 1/n)*n**3*x**(2*n)*gamma(4 + 1/n)) + 3*b**2*c*c**(-
4 - 1/n)*c**(2 + 1/n)*n*(c/(d*x**n) + 1)**(-2 - 1/n)*gamma(2 + 1/n)/(c*d**(
2 + 1/n)*n**2*gamma(4 + 1/n) + d*d**(2 + 1/n)*n**2*x**n*gamma(4 + 1/n)) + b
**2*c*c**(-4 - 1/n)*c**(2 + 1/n)*(c/(d*x**n) + 1)**(-2 - 1/n)*gamma(2 + 1/n
)/(c*d**(2 + 1/n)*n**2*gamma(4 + 1/n) + d*d**(2 + 1/n)*n**2*x**n*gamma(4 +
1/n)) + b**2*c**(-4 - 1/n)*c**(2 + 1/n)*d*n*x**n*(c/(d*x**n) + 1)**(-2 - 1/
n)*gamma(2 + 1/n)/(c*d**(2 + 1/n)*n**2*gamma(4 + 1/n) + d*d**(2 + 1/n)*n**2
*x**n*gamma(4 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-4}\,dx$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{9,[
1,0,4,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

3.331 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx=-\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)}+\frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)}+\frac{n(bc+2adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}$$

```
[Out] -(-a*d+b*c)*x*(c+d*x^n)^(-2-1/n)/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^(-1-
1/n)/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)^(
1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {393, 198, 197}

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx=\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)\int (c+dx^n)^{-2-\frac{1}{n}}\,dx}{cd(1+2n)} \\
&= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} \\
&\quad + \frac{(n(bc+2adn))\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c^2d(1+n)(1+2n)} \\
&= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

$$
\int (a+bx^n)\,(c+dx^n)^{-3-\frac{1}{n}}\,dx
$$

$$
= \frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(bc\,\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)+(-bc+ad)\,\text{Hypergeometric}\right.}{c^3d}
$$

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^
(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1),
1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(127) = 254$.

Time = 4.40 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.42

method	result
parallelrisch	$\frac{2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}a\,d^{3}n^{2}+x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}bc\,d^{2}n+6x\,x^{2n}(c+d\,x^{n})^{-\frac{1+3n}{n}}ac\,d^{2}n^{2}+2x\,x^{2n}(c+d\,x^{n})^{-\frac{1+3n}{n}}ac\,d^{2}n+3x}{}$

[In] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)

[Out] (2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a*d^3*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*b*c*d^2*n+6*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n+3*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b*c^2*d*n+6*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n^2+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b*c^2*d+5*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n+2*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*b*c^3*n+2*x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3*n^2+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*b*c^3+3*x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3*n+x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3)/c^3/(2*n^2+3*n+1)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int (a+bx^n)\,(c+dx^n)^{-3-\frac{1}{n}}\,dx = \frac{(2\,ad^3n^2+bcd^2n)xx^{3\,n}+(6\,acd^2n^2+bc^2d+(3\,bc^2d+2\,acd^2)n)xx^{2\,n}+(6\,ac^2dn^2+bc^3+ac^2d+(2\,bc^3+5}{(2\,c^3n^2+3\,c^3n+c^3)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")

[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 + 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(105) = 210$.

Time = 2.48 (sec) , antiderivative size = 959, normalized size of antiderivative = 7.55

$$\int (a + bx^n) (c + dx^n)^{-3-\frac{1}{n}} \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] 2*a*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)
**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/
n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a*c**2*c*
*(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(
3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3
*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a*c**2*c**(1/n)*c**(-3 -
2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n
**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x
**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*c*c**(1/n)*c**(-3 - 2/n)*d*n**2*x*x**n*
gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**
n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)*
*(1/n)*gamma(3 + 1/n)) + 2*a*c*c**(1/n)*c**(-3 - 2/n)*d*n*x*x**n*gamma(1/n)
/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x
**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gam
ma(3 + 1/n)) + 2*a*c**(1/n)*c**(-3 - 2/n)*d**2*n**2*x*x**(2*n)*gamma(1/n)/(
c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma
(3 + 1/n)) + 2*b*c*c**(-3 - 1/n)*c**(1 + 1/n)*n*(c/(d*x**n) + 1)**(-1 - 1/n
)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) + d*d**(1 + 1/n)*n**2*
x**n*gamma(3 + 1/n)) + b*c*c**(-3 - 1/n)*c**(1 + 1/n)*(c/(d*x**n) + 1)**(-1
 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) + d*d**(1 + 1/n)
*n**2*x**n*gamma(3 + 1/n)) + b*c**(-3 - 1/n)*c**(1 + 1/n)*d*n*x**n*(c/(d*x*
*n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) + d
*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a + bx^n) (c + dx^n)^{-3-\frac{1}{n}} \, dx = \int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-3} \, dx$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + bx^n) (c + dx^n)^{-3-\frac{1}{n}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,[0,
0,2,1,
```

Mupad [F(-1)]

Timed out.

$$\int (a + bx^n) (c + dx^n)^{-3-\frac{1}{n}} dx = \int \frac{a + b\,x^n}{(c + d\,x^n)^{\frac{1}{n}+3}}\,dx$$

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)
```

3.332 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

```
[Out] x*(c+d*x^n)^(-1-1/n)/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {198, 197}

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

```
[In] Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{n\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}$$
$$= \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}$$

```
[In] Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

Maple [F]

$$\int (c+d\,x^n)^{-2-\frac{1}{n}}\,dx$$

```
[In] int((c+d*x^n)^(-2-1/n),x)
[Out] int((c+d*x^n)^(-2-1/n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{d^2nxx^{2n} + (2\,cdn+cd)xx^n + (c^2n+c^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2n+1}{n}}}$$

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*
(d*x^n + c)^((2*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(39) = 78.

Time = 0.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.00

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}$$

```
[In] integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)**(1
/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(2 +
 1/n)) + c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) +
 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*ga
mma(2 + 1/n)) + c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d**(1/n)*n**2
*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n)
+ 1)**(1/n)*gamma(2 + 1/n))
```

Maxima [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

Giac [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = -\frac{x^{1-2\,n}\,\left(\frac{c}{d\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,\left(c+d\,x^n\right)^{1/n}}$$

```
[In] int(1/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] -(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n)))
/(2*d^2*n*(c + d*x^n)^(1/n))
```

3.333 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{bx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}$$

```
[Out] -d*x/c/(-a*d+b*c)/((c+d*x^n)^(1/n))+b*x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b
*c)*x^n/a/(c+d*x^n))/a/(-a*d+b*c)/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {390, 387}

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \frac{bx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

```
[In] Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c +
 d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}+\frac{b\int\frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{bc-ad}\\ &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}+\frac{bx(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 6.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int\frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(\frac{a(c+dx^n)}{c(a+bx^n)}+\frac{bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)}{a}+\frac{b(-bc+ad)nx^{2n}\,\text{Hypergeometric2F1}\left(2,2+\frac{1}{n},3+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2(1+2n)(c+dx^n)}\right)}{a}$$

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*d
)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*Hyp
ergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^
n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{-1-\frac{1}{n}}}{a+b\,x^n}dx$$

[In] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)

[Out] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)

Fricas [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx=\int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")

[Out] integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

[In] integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx=\int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")

[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)

Giac [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{\frac{1}{n}+1}}\,dx$$

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

3.334 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx = \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n(a+bx^n)} - \frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)/((c+d*x^n)^((1-n)/n))-(b*c*(1-n)+a*d*n)*x*hype
rgeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/(-a*d+b*c)/n/((c+d*
x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {390, 387}

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx = \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)} - \frac{x(c+dx^n)^{-1/n}(adn+bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n)
 + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a
*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc-(bc-ad)n)\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{a(bc-ad)n} \\ &= \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \frac{x(c+dx^n)^{\frac{-1+n}{n}}\left(bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+(an+b(-1+n)x^n)\,\Phi\left(\frac{(-bc+a}{a(c+a}\right.\right.}{n\,(a+bx^n)\left(-b(bc-ad)x^{2n}\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+a\,(c+dx^n)\left(n\,(a+bx^n)-bx^{n}\right.\right.}$$

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^((-1 + n)/n)*(b*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*
(c + d*x^n)), 1, 1 + n^(-1)] + (a*n + b*(-1 + n)*x^n)*HurwitzLerchPhi[((-(b
*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)]))/(n*(a + b*x^n)*(-(b*(b*c - a*
d)*x^(2*n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(
-1)]) + a*(c + d*x^n)*(n*(a + b*x^n) - b*x^n*HurwitzLerchPhi[((-(b*c) + a*d
)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{(a+b\,x^n)^2}dx$$

```
[In] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
[Out] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
[Out] integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Giac [F]

$$\int \frac{(c + dx^n)^{-1/n}}{(a + bx^n)^2} \, dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)^{\left(\frac{1}{n}\right)}} \, dx$$

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx^n)^{-1/n}}{(a + bx^n)^2} \, dx = \int \frac{1}{(a + b\,x^n)^2\,(c + d\,x^n)^{1/n}} \, dx$$

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)
```

3.335 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}$$

```
[Out] 1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^2-1/2*c*(b*c*(1-2*n)+2*a
*d*n)*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d+b
*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {390, 387}

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2} - \frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n)) \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c*
(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a
*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{(bc-2(bc-ad)n)\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx}{2a(bc-ad)n}$$

$$= \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n}\ {}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.80 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.54

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx =$$

$$-\frac{x(c+dx^n)^{2-\frac{1}{n}}\left((2an+b(-1+2n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)-bx^n\Phi\right.}{2n\,(a+bx^n)^2\left(-a\,(c+dx^n)\,(an+b(-1+n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)+x^n\left(-b(bc-ad)x^n\Phi\left(\frac{(-bc}{a(c}\right.\right.\right.}$$

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] -1/2*(x*(c + d*x^n)^(2 - n^(-1))*((2*a*n + b*(-1 + 2*n)*x^n)*HurwitzLerchPh
i[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)] - b*x^n*HurwitzLerc
```

```
hPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)] - 2*(a*n + b*(-1
+ n)*x^n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])
)/(n*(a + b*x^n)^2*(-(a*(c + d*x^n)*(a*n + b*(-1 + n)*x^n)*HurwitzLerchPhi[
((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)]) + x^n*(-(b*(b*c - a*
d)*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)]
) + (a^2*d*n - b^2*c*(-1 + n)*x^n + a*b*(-(c*(1 + n)) + d*(-2 + n)*x^n))*Hu
rwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

Maple [F]

$$\int \frac{\left(c+d\,x^{n}\right)^{1-\frac{1}{n}}}{\left(a+b\,x^{n}\right)^{3}}dx$$

[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)

[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)

Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")

[Out] integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3,x)
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3, x)
```

3.336 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx = \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}$$

```
[Out] 1/3*b*x*(c+d*x^n)^(3-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^3-1/3*c^2*(b*c*(1-3*n)+3
*a*d*n)*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^4/(-a*d
+b*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {390, 387}

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx = \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3} - \frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b*
c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 387

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{(bc-3(bc-ad)n)\int\frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx}{3a(bc-ad)n} \\ &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 36.96 (sec) , antiderivative size = 6405, normalized size of antiderivative = 48.16

$$\int\frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}dx$$

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4,x)
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4, x)
```

3.337 $\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\frac{c^4\left(bc^2+ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^8}+\frac{c^2\left(3bc^2+2ad^2\right)(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^8}+\frac{\left(3bc^2+ad^2\right)(-c+dx)^{7/2}(c+dx)^{7/2}}{7d^8}+\frac{b(-c+dx)^{9/2}(c+dx)^{9/2}}{9d^8}$$

```
[Out] 1/3*c^4*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^8+1/5*c^2*(2*a*d^2+3*b*
c^2)*(d*x-c)^(5/2)*(d*x+c)^(5/2)/d^8+1/7*(a*d^2+3*b*c^2)*(d*x-c)^(7/2)*(d*x
+c)^(7/2)/d^8+1/9*b*(d*x-c)^(9/2)*(d*x+c)^(9/2)/d^8
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {471, 102, 12, 75}

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{21d^4}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{315d^8}+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

```
[In] Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315*d^8) + (4
*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) +
((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(21*d^4) + (b*x^
6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{-c+dx}\sqrt{c+dx}\,dx \\
&= \frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} \\
&\quad + \frac{(2bc^2+3ad^2)\int 4c^2x^3\sqrt{-c+dx}\sqrt{c+dx}\,dx}{21d^4}
\end{aligned}$$

$$= \frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{(4c^2(2bc^2+3ad^2))\int x^3\sqrt{-c+dx}\sqrt{c+dx}\,dx}{21d^4}$$

$$= \frac{4c^2(2bc^2+3ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{(4c^2(2bc^2+3ad^2))\int 2c^2x\sqrt{-c+dx}\sqrt{c+dx}\,dx}{105d^6}$$

$$= \frac{4c^2(2bc^2+3ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{(8c^4(2bc^2+3ad^2))\int x\sqrt{-c+dx}\sqrt{c+dx}\,dx}{105d^6}$$

$$= \frac{8c^4(2bc^2+3ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8} + \frac{4c^2(2bc^2+3ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(2bc^2+3ad^2)\,x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\;dx = \frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(3ad^2(8c^4+12c^2d^2x^2+15d^4x^4)+b(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6))}{315d^8}$$

```
[In] Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4
*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x^6)))/(315*d^
8)
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)}{315d^8}$	92
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6)}{315d^8}$	104
risch	$\frac{\sqrt{dx+c}\,(-35b\,x^8d^8-45a\,d^8x^6+5b\,c^2d^6x^6+9a\,c^2d^6x^4+6b\,c^4d^4x^4+12a\,c^4d^4x^2+8b\,c^6d^2x^2+24a\,c^6d^2+16b\,c^8)(-dx+c)}{315\sqrt{dx-c}\,d^8}$	122

```
[In] int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315/d^8*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d
^4*x^4+36*a*c^2*d^4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{(35\,bd^8x^8-16\,bc^8-24\,ac^6d^2-5\,(bc^2d^6-9\,ad^8)x^6-3\,(2\,bc^4d^4+3\,ac^2d^6)x^4-4\,(2\,bc^6d^2+3\,ac^4d^4)x^2)\sqrt{dx}}{315\,d^8}$$

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^6
 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^8
```

Sympy [F]

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \int x^5\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$\begin{aligned}\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx &= \frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^4}{21\,d^4} \\ &+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}bc^4x^2}{105\,d^6} \\ &+\frac{4\,(d^2x^2-c^2)^{\frac{3}{2}}ac^2x^2}{35\,d^4} \\ &+\frac{16\,(d^2x^2-c^2)^{\frac{3}{2}}bc^6}{315\,d^8}+\frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}ac^4}{105\,d^6}\end{aligned}$$

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4/
d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c
^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(128) = 256$.

Time = 0.45 (sec) , antiderivative size = 621, normalized size of antiderivative = 4.09

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$= \frac{168\left(\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{5\,(dx+c)}{d^5}-\frac{31\,c}{d^5}\right)+\frac{321\,c^2}{d^5}\right)-\frac{451\,c^3}{d^5}\right)(dx+c)+\frac{745\,c^4}{d^5}\right)(dx+c)-\frac{405\,c^5}{d^5}\right)\sqrt{dx}$$

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40320*(168*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 32
1*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)
*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^5)*a*c + 3*(((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x + c)/d^7 - 57*
c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^4/d^7)*(d*x + c
) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23205*c^7/d^7)*sq
rt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)
))/d^7)*b*c + 24*(((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^
6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 633
5*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7
*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*a*d + (((2*((4*(5*(2*(d*x +
c)*(7*(d*x + c)*(8*(d*x + c)/d^8 - 73*c/d^8) + 2073*c^2/d^8) - 9833*c^3/d^8
)*(d*x + c) + 75293*c^4/d^8)*(d*x + c) - 310203*c^5/d^8)*(d*x + c) + 216993
*c^6/d^8)*(d*x + c) - 205275*c^7/d^8)*(d*x + c) + 69615*c^8/d^8)*sqrt(d*x +
 c)*sqrt(d*x - c) + 22050*c^9*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^8)
*b*d)/d
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=-\sqrt{d\,x-c}\left(\frac{\left(16\,b\,c^8+24\,a\,c^6\,d^2\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{b\,x^8\,\sqrt{c+d\,x}}{9}+\frac{x^4\,\left(6\,b\,c^4\,d^4+9\,a\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}+\frac{x^2\,\left(8\,b\,c^6\,d^2+12\,a\,c^4\,d^4\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{x^6\,\left(45\,a\,d^8-5\,b\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}\right)$$

```
[In] int(x^5*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((16*b*c^8 + 24*a*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8) - (
b*x^8*(c + d*x)^(1/2))/9 + (x^4*(9*a*c^2*d^6 + 6*b*c^4*d^4)*(c + d*x)^(1/2)
)/(315*d^8) + (x^2*(12*a*c^4*d^4 + 8*b*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8)
- (x^6*(45*a*d^8 - 5*b*c^2*d^6)*(c + d*x)^(1/2))/(315*d^8))
```

3.338 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$

Optimal result

Integrand size = 31, antiderivative size = 109

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{c^2(bc^2+ad^2)\,(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^6} + \frac{(2bc^2+ad^2)\,(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^6} + \frac{b(-c+dx)^{7/2}(c+dx)^{7/2}}{7d^6}$$

```
[Out] 1/3*c^2*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/5*(a*d^2+2*b*c^2)*(
d*x-c)^(5/2)*(d*x+c)^(5/2)/d^6+1/7*b*(d*x-c)^(7/2)*(d*x+c)^(7/2)/d^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {471, 102, 12, 75}

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}\,(7ad^2+4bc^2)}{105d^6} + \frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}\,(7ad^2+4bc^2)}{35d^4} + \frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

```
[In] Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) + ((
4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(35*d^4) + (b*x^4*
(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} - \frac{1}{7}\left(-7a-\frac{4bc^2}{d^2}\right)\int x^3\sqrt{-c+dx}\sqrt{c+dx}\,dx \\
&= \frac{(4bc^2+7ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} \\
&\quad + \frac{(4bc^2+7ad^2)\int 2c^2x\sqrt{-c+dx}\sqrt{c+dx}\,dx}{35d^4} \\
&= \frac{(4bc^2+7ad^2)\,x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} \\
&\quad + \frac{(2c^2(4bc^2+7ad^2))\int x\sqrt{-c+dx}\sqrt{c+dx}\,dx}{35d^4}
\end{aligned}$$

$$= \frac{2c^2(4bc^2 + 7ad^2)(-c + dx)^{3/2}(c + dx)^{3/2}}{105d^6} + \frac{(4bc^2 + 7ad^2)x^2(-c + dx)^{3/2}(c + dx)^{3/2}}{35d^4} + \frac{bx^4(-c + dx)^{3/2}(c + dx)^{3/2}}{7d^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{-c + dx}\sqrt{c + dx}(a + bx^2)\ dx = \frac{(-c + dx)^{3/2}(c + dx)^{3/2}(7ad^2(2c^2 + 3d^2x^2) + b(8c^4 + 12c^2d^2x^2 + 15d^4x^4))}{105d^6}$$

```
[In] Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2) + b*(8*c^4 +
 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4)}{105d^6}$	68
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4)}{105d^6}$	80
risch	$\frac{\sqrt{dx+c}\,(-15b\,x^6d^6-21a\,d^6x^4+3b\,c^2d^4x^4+7a\,c^2d^4x^2+4b\,c^4d^2x^2+14a\,c^4d^2+8b\,c^6)(-dx+c)}{105\sqrt{dx-c}\,d^6}$	98

```
[In] int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/105/d^6*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d
^2*x^2+14*a*c^2*d^2+8*b*c^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{(15\,bd^6x^6-8\,bc^6-14\,ac^4d^2-3\,(bc^2d^4-7\,ad^6)x^4-(4\,bc^4d^2+7\,ac^2d^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{105\,d^6}$$

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
- (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [F]

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \int x^3\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^4}{7\,d^2} + \frac{4\,(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^2}{35\,d^4} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^2}{5\,d^2} + \frac{8\,(d^2x^2-c^2)^{\frac{3}{2}}bc^4}{105\,d^6} + \frac{2\,(d^2x^2-c^2)^{\frac{3}{2}}ac^2}{15\,d^4}$$

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2/
d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.54

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{70\left(\left(\left((dx+c)\left(2(dx+c)\left(\frac{3(dx+c)}{d^3}-\frac{13c}{d^3}\right)+\frac{43c^2}{d^3}\right)-\frac{39c^3}{d^3}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{18c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)ac}{}$$

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*(70*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d
^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c)))/d^3)*a*c + 7*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)
/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*
x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d
*x + c) + sqrt(d*x - c)))/d^5)*b*c + 14*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x
 + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4
)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^4)*a*d + (((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6
) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335
*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*
log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*b*d)/d
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = -\sqrt{d\,x-c}\,\left(\frac{\left(8\,b\,c^6+14\,a\,c^4\,d^2\right)\,\sqrt{c+d\,x}}{105\,d^6} - \frac{b\,x^6\,\sqrt{c+d\,x}}{7} + \frac{x^2\,\left(4\,b\,c^4\,d^2+7\,a\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6} - \frac{x^4\,\left(21\,a\,d^6-3\,b\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}\right)$$

```
[In] int(x^3*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((8*b*c^6 + 14*a*c^4*d^2)*(c + d*x)^(1/2))/(105*d^6) - (b
*x^6*(c + d*x)^(1/2))/7 + (x^2*(7*a*c^2*d^4 + 4*b*c^4*d^2)*(c + d*x)^(1/2))
/(105*d^6) - (x^4*(21*a*d^6 - 3*b*c^2*d^4)*(c + d*x)^(1/2))/(105*d^6))
```

3.339 $\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$

Optimal result

Integrand size = 29, antiderivative size = 67

$$\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{(bc^2+ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^4} + \frac{b(-c+dx)^{5/2}(c+dx)^{5/2}}{5d^4}$$

```
[Out] 1/3*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/5*b*(d*x-c)^(5/2)*(d*x+
c)^(5/2)/d^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {471, 75}

$$\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \frac{(dx-c)^{3/2}(c+dx)^{3/2}(5ad^2+2bc^2)}{15d^4} + \frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

```
[In] Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (b*x^2*(-
c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2} - \frac{1}{5}\left(-5a - \frac{2bc^2}{d^2}\right)\int x\sqrt{-c+dx}\sqrt{c+dx}\,dx\\ &= \frac{(2bc^2+5ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4} + \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{(-c+dx)^{3/2}(c+dx)^{3/2}\,(2bc^2+5ad^2+3bd^2x^2)}{15d^4}$$

```
[In] Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
[Out] ((-c + d*x)^(3/2)*(c + d*x)^(3/2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2*x^2))/(15*d^
4)
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{(dx-c)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(3b\,d^2x^2+5a\,d^2+2b\,c^2)}{15d^4}$	44
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(-d^2x^2+c^2)(3b\,d^2x^2+5a\,d^2+2b\,c^2)}{15d^4}$	56
risch	$\frac{\sqrt{dx+c}\,(-3b\,d^4x^4-5a\,d^4x^2+b\,c^2d^2x^2+5a\,c^2d^2+2b\,c^4)(-dx+c)}{15\sqrt{dx-c}\,d^4}$	73

```
[In] int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d^4*(d*x-c)^(3/2)*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-(bc^2d^2-5\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^4}$$

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [F]

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \int x\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^2}{5\,d^2} + \frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2}{15\,d^4} + \frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,d^2}$$

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
+ 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(55) = 110.

Time = 0.35 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.39

$$\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$=\frac{5\left(\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bc+\cdots}{\cdots}$$

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*(5*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3
) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c)
 + sqrt(d*x - c)))/d^3)*b*c + 20*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2
*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqr
t(d*x - c)))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c
/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*
sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*d -
60*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) - sqrt(d*x + c)*sqrt(d*x
 - c)*(d*x - 2*c))*a*c/d)/d
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx=\sqrt{d\,x-c}\left(\frac{b\,x^4\,\sqrt{c+d\,x}}{5}-\frac{\left(2\,b\,c^4+5\,a\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}+\frac{x^2\,\left(5\,a\,d^4-b\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}\right)$$

```
[In] int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c +
d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4)
)
```

3.340 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)}{x}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)$$

```
[Out] 1/3*b*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-a*c*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2
)/c)+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {471, 103, 12, 94, 211}

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = -ac\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right) + a\sqrt{dx-c}\sqrt{c+dx}+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} + a\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx \\
&= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - a\int \frac{c^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} \\
&\quad - \left(ac^2d\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)
\end{aligned}
$$

$$= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(-bc^2+3ad^2+bd^2x^2)}{3d^2} - 2ac\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 3*a*d^2 + b*d^2*x^2))/(3*d^2) - 2
*a*c*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(66) = 132.

Time = 4.02 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

method	result
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(b\,d^{2}x^{2}\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}+3\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)a\,c^{2}d^{2}+3\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{2}-b\,c^{2}\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{3\sqrt{d^{2}x^{2}-c^{2}}\,d^{2}\sqrt{-c^{2}}}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*d^2*x^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*x^2
-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = -\frac{6\,acd^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(bd^2x^2-bc^2+3\,ad^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^2}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^2
 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = \int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx = ac\arcsin\left(\frac{c}{d|x|}\right)+\sqrt{d^2x^2-c^2}a+\frac{(d^2x^2-c^2)^{\frac{3}{2}}b}{3\,d^2}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*c*arcsin(c/(d*abs(x))) + sqrt(d^2*x^2 - c^2)*a + 1/3*(d^2*x^2 - c^2)^(3/2
)*b/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$$
$$= 2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)$$
$$+ \frac{1}{3}\,\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^2}-\frac{2\,bc}{d^2}\right)+3\,a\right)$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/3*sqrt(d*x + c)*s
qrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^2 - 2*b*c/d^2) + 3*a)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$$
$$= a\,\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)$$
$$- a\,\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) - \frac{b\,(c^2-d^2\,x^2)\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{3\,d^2}$$
$$- \frac{8\,a\,\sqrt{-c}\,\sqrt{c}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2\,\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}$$

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x,x)
```

```
[Out] a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((
-c)^(1/2) - (d*x - c)^(1/2))) - (b*(c^2 - d^2*x^2)*(c + d*x)^(1/2)*(d*x - c
)^(1/2))/(3*d^2) - (8*a*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(
((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*
x - c)^(1/2))^2 + 1))
```

3.341 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 96

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx = b\sqrt{-c+dx}\sqrt{c+dx} - \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2x^2} - \frac{(2bc^2-ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c}$$

```
[Out] -1/2*(-a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/
2)*(d*x+c)^(1/2)-1/2*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {465, 103, 12, 94, 211}

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx = -\frac{(2bc^2-ad^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c} + \frac{1}{2}\sqrt{dx-c}\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right) + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2)*
(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sq
rt[c + d*x])/c])/(2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2}+\frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int\frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx\\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2}\\
&\quad+\frac{1}{2}\left(-2b+\frac{ad^2}{c^2}\right)\int\frac{c^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} \\
&\quad +\frac{1}{2}\left(-2bc^2+ad^2\right)\int\frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} \\
&\quad -\frac{1}{2}\left(d\left(2bc^2-ad^2\right)\right)\mathrm{Subst}\left(\int\frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\sqrt{c+dx}\right) \\
&= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} \\
&\quad -\frac{\left(2bc^2-ad^2\right)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$
\begin{aligned}
\int\frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^3}\,dx &= \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(-a+2bx^2\right)}{2x^2} \\
&\quad +\left(-2bc+\frac{ad^2}{c}\right)\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)
\end{aligned}
$$

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-a + 2*b*x^2))/(2*x^2) + (-2*b*c + (a*d^2)/c
)*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

method	result
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2x^2\sqrt{dx-c}}-\frac{\left(-b\sqrt{(dx-c)(dx+c)}+\frac{\left(a\,d^2-2b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{2\sqrt{-c^2}}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2-2b\,x^2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}+\sqrt{-c^2}\,\cdots\right.}{2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*(-d*x+c)*(d*x+c)^(1/2)/x^2/(d*x-c)^(1/2)-(-b*((d*x-c)*(d*x+c))^(1/2)+
1/2*(a*d^2-2*b*c^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1
/2))/x))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = -\frac{2\,(2\,bc^2-ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(2\,bcx^2-ac)\sqrt{dx+c}\sqrt{dx-c}}{2\,cx^2}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
[Out] -1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = \int \frac{(a+bx^2)\,\sqrt{-c+dx}\sqrt{c+dx}}{x^3}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = bc\arcsin\left(\frac{c}{d|x|}\right)-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c} + \sqrt{d^2x^2-c^2}b-\frac{\sqrt{d^2x^2-c^2}ad^2}{2\,c^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{2\,c^2x^2}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
[Out] b*c*arcsin(c/(d*abs(x))) - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2
- c^2)*b - 1/2*sqrt(d^2*x^2 - c^2)*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^(3/2)*a/
(c^2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}bd + \frac{(2bc^2d-ad^3)\arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c} + \frac{2\left(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6-4ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2\right)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^4+4c^2\right)^2}}{d}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*x
 + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6
- 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.08

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx = b\sqrt{-c}\sqrt{c}\ln\left(\frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2}+1\right) - \frac{\frac{a\sqrt{-c}d^2}{32c^{3/2}} + \frac{a\sqrt{-c}d^2(\sqrt{c+dx}-\sqrt{c})^2}{16c^{3/2}(\sqrt{-c}-\sqrt{dx-c})^2} - \frac{15a\sqrt{-c}d^2(\sqrt{c+dx}-\sqrt{c})^4}{32c^{3/2}(\sqrt{-c}-\sqrt{dx-c})^4}}{\frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2} + \frac{2(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{-c}-\sqrt{dx-c})^4} + \frac{(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{-c}-\sqrt{dx-c})^6}} - b\sqrt{-c}\sqrt{c}\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right) + \frac{a\sqrt{-c}d^2\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2c^{3/2}} - \frac{a\sqrt{-c}d^2\ln\left(\frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2}+1\right)}{2c^{3/2}} - \frac{a}{32}$$

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^3,x)
```

```
[Out] b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - ((a*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (a*(-c)^(1/2)*d^2*((
c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
- (15*a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/
2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
 - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6)
- b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x -
```

```
 c)^(1/2))) + (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2)
 - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
- c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) - (a*(-c)^(
1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^2) - (8*b*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(((-c)^
(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*
x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))
```

3.342 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx$

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx = -\frac{(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^2x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{d^2(4bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^3}$$

```
[Out] 1/4*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^4+1/8*d^2*(a*d^2+4*b*c^2)*arctan((d
*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3-1/8*(a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/c^2/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {465, 96, 94, 211}

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx = \frac{d^2(ad^2+4bc^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3} - \frac{\sqrt{dx-c}(c+dx)^{3/2}(ad^2+4bc^2)}{8c^3x^2} + \frac{d\sqrt{dx-c}\sqrt{c+dx}(ad^2+4bc^2)}{8c^3x} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((4*b*c^2 +
a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a*(-c + d*x)^(3/2)*(c
 + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/(8*c^3)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{ad^2}{c^2}\right)\int\frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^3}\,dx \\
&= -\frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} \\
&\quad + \frac{1}{8}\left(d\left(4b+\frac{ad^2}{c^2}\right)\right)\int\frac{\sqrt{c+dx}}{x^2\sqrt{-c+dx}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} \\
&\quad + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{8}\left(d^2\left(4b+\frac{ad^2}{c^2}\right)\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} \\
&\quad - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} \\
&\quad + \frac{1}{8}\left(d^3\left(4b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right) \\
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} \\
&\quad + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{d^2(4bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx \\
&= \frac{c\sqrt{-c+dx}\sqrt{c+dx}(-2ac^2-4bc^2x^2+ad^2x^2)+2d^2(4bc^2+ad^2)\,x^4\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^3x^4}
\end{aligned}
$$

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a*c^2 - 4*b*c^2*x^2 + a*d^2*x^2) + 2*d^
2*(4*b*c^2 + a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^3*x^4)
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(-a\,d^2x^2+4b\,c^2x^2+2c^2a\right)}{8x^4c^2\sqrt{dx-c}}-\frac{d^2\left(a\,d^2+4b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)\sqrt{(dx-c)(dx+c)}}{8c^2\sqrt{-c^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4+4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2x^2+4\sqrt{}\right.}{8c^2\sqrt{d^2x^2-c^2}\,x^4\sqrt{-c^2}}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x+c)^(1/2)*(-d*x+c)*(-a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/x^4/c^2/(d*x-c)
^(1/2)-1/8*d^2*(a*d^2+4*b*c^2)/c^2/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(
d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = \frac{2\,(4\,bc^2d^2+ad^4)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-(2\,ac^3+(4\,bc^3-acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^3x^4}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c)
)/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^
3*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = -\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c}-\frac{ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^3}-\frac{\sqrt{d^2x^2-c^2}bd^2}{2\,c^2}-\frac{\sqrt{d^2x^2-c^2}ad^4}{8\,c^4}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}b}{2\,c^2x^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}ad^2}{8\,c^4x^2}+\frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{4\,c^2x^4}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c - 1/8*a*d^4*arcsin(c/(d*abs(x)))/c^3 - 1/
2*sqrt(d^2*x^2 - c^2)*b*d^2/c^2 - 1/8*sqrt(d^2*x^2 - c^2)*a*d^4/c^4 + 1/2*(
d^2*x^2 - c^2)^(3/2)*b/(c^2*x^2) + 1/8*(d^2*x^2 - c^2)^(3/2)*a*d^2/(c^4*x^2
) + 1/4*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(103) = 206$.

Time = 0.36 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx = -\frac{\frac{(4\,bc^2d^3+ad^5)\arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2\,c}\right)}{c^3} - \frac{2\left(4\,bc^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^{14} - ad^5(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 16\,bc^4d^3(\sqrt{dx+c}-\sqrt{dx-c})^{10} + 28\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*x
+ c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10
+ 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x
+ c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^2)
)/d
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 1004, normalized size of antiderivative = 8.30

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$$
$$= \frac{\frac{a\sqrt{-c}\,d^4}{1024\,c^{7/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{11\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{7\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}-\frac{239\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{1024\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{a\,\cdots}{256\cdots}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{6\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}}$$
$$-\frac{\frac{b\sqrt{-c}\,d^2}{32\,c^{3/2}}+\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}+\frac{a\sqrt{-c}\,d^4\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{7/2}}$$
$$+\frac{b\sqrt{-c}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^4\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{8\,c^{7/2}}$$
$$-\frac{b\sqrt{-c}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{3/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{256\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$
$$+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{1024\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^5,x)
```

```
[Out] ((a*(-c)^(1/2)*d^4)/(1024*c^(7/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c
^(1/2))^2)/(128*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (11*a*(-c)^(1/2
)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(7/2)*((-c)^(1/2) - (d*x - c)^(
1/2))^4) + (7*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(7/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^6) - (239*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2)
- c^(1/2))^8)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (a*(-c)^(1/
2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(7/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)^(1/2))^
4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (6
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 + (4*((c +
 d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + ((c + d*x)^(
1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - ((b*(-c)^(1/2)*d^2)
/(32*c^(3/2)) + (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2
)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(1/2)*d^2*((c + d*x)^(1/2)
 - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(
1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) -
c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^
6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(7/2)) + (b*(-c)^(1/2)*d
```

```
^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/(2*c^(3
/2)) - (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d
*x - c)^(1/2))^2 + 1))/(8*c^(7/2)) - (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
 - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) + (a*(-c)^
(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*c^(7/2)*((-c)^(1/2) - (d*x -
c)^(1/2))^2) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(1024*c^(7/
2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) - (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) -
 c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

3.343 $\int x^4\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\ dx$

Optimal result

Integrand size = 31, antiderivative size = 208

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\ dx = \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{c^6(5bc^2+8ad^2)\,\mathrm{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{64d^7}$$

```
[Out] 1/64*c^2*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/48*(8*a*d^2+
5*b*c^2)*x^3*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/8*b*x^5*(d*x-c)^(3/2)*(d*x+c
)^(3/2)/d^2-1/64*c^6*(8*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))
/d^7+1/128*c^4*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used

= {471, 102, 12, 92, 38, 65, 223, 212}

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = -\frac{c^6(8ad^2+5bc^2)\operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7} + \frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6} + \frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4} + \frac{c^4x\sqrt{dx-c}\sqrt{c+dx}(8ad^2+5bc^2)}{128d^6} + \frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

```
[In] Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{1}{8}\left(-8a-\frac{5bc^2}{d^2}\right)\int x^4\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

$$= \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(5bc^2+8ad^2)\int 3c^2x^2\sqrt{-c+dx}\sqrt{c+dx}\,dx}{48d^4}$$

$$= \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(c^2(5bc^2+8ad^2))\int x^2\sqrt{-c+dx}\sqrt{c+dx}\,dx}{16d^4}$$

$$= \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(c^2(5bc^2+8ad^2))\int c^2\sqrt{-c+dx}\sqrt{c+dx}\,dx}{64d^6}$$

$$= \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(c^4(5bc^2+8ad^2))\int \sqrt{-c+dx}\sqrt{c+dx}\,dx}{64d^6}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{(c^6(5bc^2+8ad^2))\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{128d^6}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{(c^6(5bc^2+8ad^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{64d^7}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{(c^6(5bc^2+8ad^2))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{64d^7}$$

$$= \frac{c^4(5bc^2+8ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2(5bc^2+8ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{(5bc^2+8ad^2)\,x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{c^6(5bc^2+8ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{64d^7}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$$
$$=\frac{dx\sqrt{-c+dx}\sqrt{c+dx}(8ad^2(-3c^4-2c^2d^2x^2+8d^4x^4)-b(15c^6+10c^4d^2x^2+8c^2d^4x^4-48d^6x^6))-6c^6(5bc}{384d^7}$$

```
[In] Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(8*a*d^2*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*
x^4) - b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4 - 48*d^6*x^6)) - 6*c^6*(5
*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(384*d^7)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

method	result
risch	$\frac{x\left(-48b\,x^6d^6-64a\,d^6x^4+8b\,c^2d^4x^4+16a\,c^2d^4x^2+10b\,c^4d^2x^2+24a\,c^4d^2+15b\,c^6\right)(-dx+c)\sqrt{dx+c}}{384d^6\sqrt{dx-c}}-\frac{c^6\left(8a\,d^2+5b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2}\right.}{128d^6\sqrt{d^2}\,\sqrt{dx-c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-48\ \mathrm{csgn}(d)b\,d^7x^7\sqrt{d^2x^2-c^2}-64\ \mathrm{csgn}(d)a\,d^7x^5\sqrt{d^2x^2-c^2}+8\ \mathrm{csgn}(d)b\,c^2d^5x^5\sqrt{d^2x^2-c^2}+16\ \mathrm{csgn}(d)a\,c^2d^5x^3\sqrt{d}\right.}{}$

```
[In] int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x*(-48*b*d^6*x^6-64*a*d^6*x^4+8*b*c^2*d^4*x^4+16*a*c^2*d^4*x^2+10*b*c
^4*d^2*x^2+24*a*c^4*d^2+15*b*c^6)*(-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2)-
1/128*c^6*(8*a*d^2+5*b*c^2)/d^6*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(
d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$$
$$=\frac{(48\,bd^7x^7-8\,(bc^2d^5-8\,ad^7)x^5-2\,(5\,bc^4d^3+8\,ac^2d^5)x^3-3\,(5\,bc^6d+8\,ac^4d^3)x)\sqrt{dx+c}\sqrt{dx-c}+3\,(5\,bc}{384\,d^7}$$

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*c
^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) +
3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

Sympy [F]

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\int x^4\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.18

$$\begin{aligned}\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx&=\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}\\&\quad+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}\\&\quad-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{128\,d^7}\\&\quad-\frac{ac^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^5}\\&\quad+\frac{5\,\sqrt{d^2x^2-c^2}bc^6x}{128\,d^6}+\frac{\sqrt{d^2x^2-c^2}ac^4x}{16\,d^4}\\&\quad+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x}{64\,d^6}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x}{8\,d^4}\end{aligned}$$

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3/
d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*d)/d^7 - 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)
/d^5 + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)*a*c
^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2)^(3/
2)*a*c^2*x/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(178) = 356.

Time = 0.40 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.68

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$= \frac{112\left(\left(\left(2\left(dx+c\right)\left(3\left(dx+c\right)\left(\frac{4\left(dx+c\right)}{d^4}-\frac{21\,c}{d^4}\right)+\frac{133\,c^2}{d^4}\right)-\frac{295\,c^3}{d^4}\right)\left(dx+c\right)+\frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{90\,c^5\,\mathrm{lo}}{}}{}$$

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(112*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133
*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x -
c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*a*c + 8*(((2*((4*
(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6) + 661*c^2/d^6) - 4551*c
^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335*c^5/d^6)*(d*x + c) + 283
5*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*log(abs(-sqrt(d*x + c) +
sqrt(d*x - c)))/d^6)*b*c + 56*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5
 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x +
c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x +
 c) + sqrt(d*x - c)))/d^5)*a*d + (((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x
 + c)/d^7 - 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^
4/d^7)*(d*x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23
205*c^7/d^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c)
+ sqrt(d*x - c)))/d^7)*b*d)/d
```

Mupad [B] (verification not implemented)

Time = 49.01 (sec) , antiderivative size = 2314, normalized size of antiderivative = 11.12

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \text{Too large to display}$$

```
[In] int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (a*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*a*c^6*((c + d*x)^(1/2)
```

- c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^21) - (a*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24) - ((5*b*c^8*((c + d*x)^(1/2) - c^(1/2)))/(32*((-c)^(1/2) - (d*x - c)^(1/2))) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^3)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^5)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^7)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^9)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^11)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^13)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^15)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^17)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^19)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^21)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^21) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^23)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^23) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^25)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^25) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^27)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^27) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^29)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^29) + (5*b*c^8*((c + d*x)^(1/2) - c^(1/2))^31)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^31))/(d^7 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (120*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (560*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (1820*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (4368*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (8008*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (12870*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (8008*d^7

```
*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (4368*
d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (18
20*d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24 -
(560*d^7*((c + d*x)^(1/2) - c^(1/2))^26)/((-c)^(1/2) - (d*x - c)^(1/2))^26
+ (120*d^7*((c + d*x)^(1/2) - c^(1/2))^28)/((-c)^(1/2) - (d*x - c)^(1/2))^2
8 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^30)/((-c)^(1/2) - (d*x - c)^(1/2))^
30 + (d^7*((c + d*x)^(1/2) - c^(1/2))^32)/((-c)^(1/2) - (d*x - c)^(1/2))^32
) + (a*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))
))/(4*d^5) + (5*b*c^8*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
- c)^(1/2))))/(32*d^7)
```

3.344 $\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 159

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4} + \frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4} + \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2} - \frac{c^4(bc^2+2ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}$$

```
[Out] 1/8*(2*a*d^2+b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/6*b*x^3*(d*x-c)^(3/
2)*(d*x+c)^(3/2)/d^2-1/8*c^4*(2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^
(1/2))/d^5+1/16*c^2*(2*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {471, 92, 12, 38, 65, 223, 212}

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = -\frac{c^4(2ad^2+bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5} + \frac{c^2x\sqrt{dx-c}\sqrt{c+dx}(2ad^2+bc^2)}{16d^4} + \frac{x(dx-c)^{3/2}(c+dx)^{3/2}\,(2ad^2+bc^2)}{8d^4} + \frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

```
[In] Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{-c+dx}\sqrt{c+dx}\,dx\\
&= \frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}\\
&\quad+\frac{(bc^2+2ad^2)\int c^2\sqrt{-c+dx}\sqrt{c+dx}\,dx}{8d^4}\\
&= \frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}\\
&\quad+\frac{(c^2(bc^2+2ad^2))\int\sqrt{-c+dx}\sqrt{c+dx}\,dx}{8d^4}\\
&= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}\\
&\quad+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}-\frac{(c^4(bc^2+2ad^2))\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{16d^4}\\
&= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}\\
&\quad+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}-\frac{(c^4(bc^2+2ad^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{8d^5}\\
&= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}\\
&\quad+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}-\frac{(c^4(bc^2+2ad^2))\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}\\
&= \frac{c^2(bc^2+2ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{(bc^2+2ad^2)\,x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}\\
&\quad+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}-\frac{c^4(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$=\frac{dx\sqrt{-c+dx}\sqrt{c+dx}(-6ad^2(c^2-2d^2x^2)+b(-3c^4-2c^2d^2x^2+8d^4x^4))-6c^4(bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{48d^5}$$

```
[In] Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-6*a*d^2*(c^2 - 2*d^2*x^2) + b*(-3*c^4 -
 2*c^2*d^2*x^2 + 8*d^4*x^4)) - 6*c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(48*d^5)
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x\left(-8b\,d^4x^4-12a\,d^4x^2+2b\,c^2d^2x^2+6a\,c^2d^2+3b\,c^4\right)(-dx+c)\sqrt{dx+c}}{48d^4\sqrt{dx-c}}-\frac{c^4\left(2a\,d^2+b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{16d^4\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-8\ \mathrm{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}-12\ \mathrm{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+2\ \mathrm{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}+6\ \mathrm{csgn}(d)d^3\sqrt{d^2x^2-c^2}\,a\right.}{48\sqrt{d}}$

```
[In] int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(-8*b*d^4*x^4-12*a*d^4*x^2+2*b*c^2*d^2*x^2+6*a*c^2*d^2+3*b*c^4)*(-d*
x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)-1/16*c^4*(2*a*d^2+b*c^2)/d^4*ln(x*d^2/
(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c
)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$=\frac{(8\,bd^5x^5-2\,(bc^2d^3-6\,ad^5)x^3-3\,(bc^4d+2\,ac^2d^3)x)\sqrt{dx+c}\sqrt{dx-c}+3\,(bc^6+2\,ac^4d^2)\log\left(-dx+\sqrt{dx+c}\right.}{48\,d^5}$$

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^3
)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [F]

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx=\int x^2\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.21

$$\begin{aligned}\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx&=\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^5}\\&\quad-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^3}\\&\quad+\frac{\sqrt{d^2x^2-c^2}bc^4x}{16\,d^4}+\frac{\sqrt{d^2x^2-c^2}ac^2x}{8\,d^2}\\&\quad+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x}{8\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax}{4\,d^2}\end{aligned}$$

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^5 - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 1
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^2 +
 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(135) = 270.

Time = 0.38 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.72

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$$
$$= \frac{40\left(\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)ac+2\left(\left(\left(2\,(dx+c)\left(3\,(dx\right.\right.\right.\right.}{}$$

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*(40*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d^
2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)*a*c +
 2*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4)
- 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^
5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*c + 10*(((d*x + c)*(2*(d*
x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*sqrt(d*x +
c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^3)*a*d
 + (((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 321*c^2/d^5)
 - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)*sqrt(d*x
+ c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^5)*
b*d)/d
```

Mupad [B] (verification not implemented)

Time = 55.45 (sec) , antiderivative size = 1681, normalized size of antiderivative = 10.57

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx = \text{Too large to display}$$

```
[In] int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*x)^(1/2)
 - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*b*c^6*((c + d
*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*b*c^6
*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (3
```

5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^
21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/
2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x -
c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x -
c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x -
c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x -
c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x -
c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x
- c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d
*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) -
(d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2)
- (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2)
- (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2
) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2)
- (d*x - c)^(1/2))^24) - ((a*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2
) - (d*x - c)^(1/2))) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(
1/2) - (d*x - c)^(1/2))^3) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^7
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*a*c^4*((c + d*x)^(1/2) - c^(1
/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*a*c^4*((c + d*x)^(1/2)
- c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35*a*c^4*((c + d*x)
^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (a*c^4*((c +
d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^3 - (8*
d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d
^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^
3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^3
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^3*
((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^3
*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^3
*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^3*(
(c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) + (a*c^4*
atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
+ (b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))
/(4*d^5)

3.345 $\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\ dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\ dx=\frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2}+\frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2}-\frac{c^2\left(bc^2+4ad^2\right)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}$$

```
[Out] 1/4*b*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-1/4*c^2*(4*a*d^2+b*c^2)*arctanh((d*
x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/8*(4*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(
1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {397, 38, 65, 223, 212}

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\ dx=-\frac{c^2\left(4ad^2+bc^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

```
[In] Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{(-bc^2-4ad^2)\int \sqrt{-c+dx}\sqrt{c+dx}\,dx}{4d^2} \\
&= \frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} \\
&\quad + \frac{(c^2(-bc^2-4ad^2))\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^2}
\end{aligned}$$

$$= \frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2}+\frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{(c^2(bc^2+4ad^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{4d^3}$$

$$= \frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2}+\frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{(c^2(bc^2+4ad^2))\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}$$

$$= \frac{(bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2}+\frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2(bc^2+4ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{dx\sqrt{-c+dx}\sqrt{c+dx}(-bc^2+4ad^2+2bd^2x^2)-2c^2(bc^2+4ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^3}$$

```
[In] Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 4*a*d^2 + 2*b*d^2*x^2) - 2*c^
2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^3)
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{x\left(2b\,d^2x^2+4a\,d^2-b\,c^2\right)(-dx+c)\sqrt{dx+c}}{8d^2\sqrt{dx-c}}-\frac{c^2\left(4a\,d^2+b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{8d^2\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(2\,\text{csgn}(d)b\,d^3x^3\sqrt{d^2x^2-c^2}+4\sqrt{d^2x^2-c^2}\,\text{csgn}(d)d^3ax-\sqrt{d^2x^2-c^2}\,\text{csgn}(d)db\,c^2x-4\ln\left(\left(\sqrt{d^2x^2-c^2}\,\text{csgn}(d)+dx\right)\text{csg}\right.\right.}{8\sqrt{d^2x^2-c^2}\,d^3}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(2*b*d^2*x^2+4*a*d^2-b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^2/(d*x-c)^(1/2)
-1/8*c^2*(4*a*d^2+b*c^2)/d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2
)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \frac{(2\,bd^3x^3-(bc^2d-4\,ad^3)x)\sqrt{dx+c}\sqrt{dx-c}+(bc^4+4\,ac^2d^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^3}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (b
*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

Sympy [F]

$$\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = \int (a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int \sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)\,dx = -\frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^3} - \frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d} + \frac{1}{2}\sqrt{d^2x^2-c^2}ax + \frac{\sqrt{d^2x^2-c^2}bc^2x}{8\,d^2} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}bx}{4\,d^2}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] -1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 - 1/2*a*c^2*log(2*d^2
*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^
2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(96) = 192$.

Time = 0.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.53

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$$
$$= \frac{24\left(2\,c\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)+\sqrt{dx+c}\sqrt{dx-c}\right)ac+4\left(\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)\right.\right.}{}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*(2*c*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) + sqrt(d*x + c)*sqrt
(d*x - c))*a*c + 4*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2
 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d
^2)*b*c + (((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^
3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c
) + sqrt(d*x - c)))/d^3)*b*d - 12*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x
- c))) - sqrt(d*x + c)*sqrt(d*x - c)*(d*x - 2*c))*a)/d
```

Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 734, normalized size of antiderivative = 6.44

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{a\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}$$
$$-\frac{\frac{b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{35\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{273\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{715\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{715\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{273\,b\,c^4}{2\,(\sqrt{-}}}{d^3-\frac{8\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28\,d}{(\sqrt{}}}$$
$$-\frac{a\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d}+\frac{b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^3}$$

```
[In] int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (a*x*(c + d*x)^(1/2)*(d*x - c)^(1/2))/2 - ((b*c^4*((c + d*x)^(1/2) - c^(1/2
)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*b*c^4*((c + d*x)^(1/2) - c^(1/
2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*b*c^4*((c + d*x)^(1/2) -
 c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*b*c^4*((c + d*x)^(
1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*b*c^4*((c +
d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*b*c^4*
((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13
) + (b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2)
```

```
)^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(
1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1
/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1
/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(
1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(
1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/
2))^16) - (a*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(1/2)))/(2*d) + (b*c^4
*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
```

3.346 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 104

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{(bc^2-2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x-(-2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/
(d*x+c)^(1/2))/d+1/2*(b-2*a*d^2/c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {465, 38, 65, 223, 212}

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{(bc^2-2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d} + \frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right) + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b - \frac{2ad^2}{c^2}\right) \int \sqrt{-c+dx}\sqrt{c+dx}\,dx$$

$$
\begin{aligned}
&=\frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} \\
&\quad+\frac{1}{2}\left(-bc^2+2ad^2\right)\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&=\frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} \\
&\quad+\frac{(-bc^2+2ad^2)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d} \\
&=\frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} \\
&\quad+\frac{(-bc^2+2ad^2)\,\mathrm{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d} \\
&=\frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} \\
&\quad+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x}-\frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.71

$$
\begin{aligned}
\int\frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx &= \frac{\sqrt{-c+dx}\sqrt{c+dx}(-2a+bx^2)}{2x} \\
&\quad+\left(-\frac{bc^2}{d}+2ad\right)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)
\end{aligned}
$$

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a + b*x^2))/(2*x) + (-((b*c^2)/d) + 2*a*d
)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\,(-b\,x^2+2a)}{2x\sqrt{dx-c}} - \frac{\left(-a\,d^2+\frac{b\,c^2}{2}\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-\operatorname{csgn}(d)bd\,x^2\sqrt{d^2x^2-c^2}-2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^2x+\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right.\right.}{2\sqrt{d^2x^2-c^2}\,xd}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(d*x+c)^(1/2)*(-d*x+c)*(-b*x^2+2*a)/x/(d*x-c)^(1/2)-(-a*d^2+1/2*b*c^2)*
ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/
2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{2\,ad^2x-(bc^2-2\,ad^2)x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-(bdx^2-2\,ad)\sqrt{dx+c}\sqrt{dx-c}}{2\,dx}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
[Out] -1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c
)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d} + ad\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right) + \frac{1}{2}\,\sqrt{d^2x^2-c^2}bx - \frac{\sqrt{d^2x^2-c^2}a}{x}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + a*d*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d) + 1/2*sqrt(d^2*x^2 - c^2)*b*x - sqrt(d^2*x^2 - c^2)*a
/x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = -\frac{\frac{32\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - 2\,((dx+c)b-bc)\sqrt{dx+c}\sqrt{dx-c} - (bc^2-2\,ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{4\,d}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(32*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x +
c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d*x +
 c) - sqrt(d*x - c))^4))/d
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2}\,dx = \frac{a\,d+\frac{5\,a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}} - 4\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) + \frac{b\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2} - \frac{b\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d} + \frac{a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)
```

```
[Out] (a*d + (5*a*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*a*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) + (b*x*(c + d
*x)^(1/2)*(d*x - c)^(1/2))/2 - (b*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(
1/2)))/(2*d) + (a*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)
^(1/2)))
```

3.347 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 84

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4} dx = -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

```
[Out] 1/3*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^3+2*b*d*arctanh((d*x-c)^(1/2)/(d*x+
c)^(1/2))-b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {465, 99, 12, 65, 223, 212}

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4} dx = \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right) - \frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}$$

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b\int \frac{d^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\end{aligned}$$

$$= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (bd^2)\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd)\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)$$

$$= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

$$= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{\sqrt{-c+dx}\sqrt{c+dx}(3bc^2x^2+a(c^2-d^2x^2))}{3c^2x^3} + 2bd\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)$$

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -1/3*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 - d^2*x^2)))/(c^2*
x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.48

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(-a\,d^2x^2+3b\,c^2x^2+c^2a\right)}{3x^3c^2\sqrt{dx-c}}+\frac{b\,d^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(-3\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2d\,x^3-\operatorname{csgn}(d)a\,d^2x^2\sqrt{d^2x^2-c^2}+3\,\operatorname{csgn}(d)b\,c^2x^2\sqrt{d^2x^2-c^2}+\operatorname{csgn}(d)\right.}{3\sqrt{d^2x^2-c^2}\,c^2x^3}$

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(-d*x+c)*(-a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^2/(d*x-c)^(
1/2)+b*d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(
d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)+(3\,bc^2d-ad^3)x^3+(ac^2+(3\,bc^2-ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^2x^3}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^2*x^3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = \text{Exception raised: MellinTransformStripError}$$

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Exception raised: MellinTransformStripError >> Pole inside critical strip?
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = bd\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right) - \frac{\sqrt{d^2x^2-c^2}b}{x} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}a}{3\,c^2x^3}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] b*d*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d) - sqrt(d^2*x^2 - c^2)*b/x + 1/3*
(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(70) = 140.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = -\frac{3\,bd^2\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d^2-16\,ac^4d^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sqr
t(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8 +
 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^4*d
^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx = \frac{b\,d+\frac{5\,b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}} - 4\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) - \frac{\left(\frac{a\,\sqrt{c+d\,x}}{3}-\frac{a\,d^2\,x^2\,\sqrt{c+d\,x}}{3\,c^2}\right)\,\sqrt{d\,x-c}}{x^3} + \frac{b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^4,x)
```

```
[Out] (b*d + (5*b*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*b*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) - (((a*(c + d
*x)^(1/2))/3 - (a*d^2*x^2*(c + d*x)^(1/2))/(3*c^2))*(d*x - c)^(1/2))/x^3 +
(b*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
```

3.348 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\,\text{arccosh}(cx)}{16c^7}$$

```
[Out] 1/16*(6*a*c^2+5*b)*arccosh(c*x)/c^7+1/16*(6*a*c^2+5*b)*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^6+1/24*(6*a*c^2+5*b)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/6*b*
x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {471, 102, 12, 92, 54}

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(6ac^2+5b)\,\text{arccosh}(cx)}{16c^7} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(6ac^2+5b)}{16c^6} + \frac{x^3\sqrt{cx-1}\sqrt{cx+1}(6ac^2+5b)}{24c^4} + \frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} - \frac{1}{6}\left(-6a-\frac{5b}{c^2}\right)\int\frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\int\frac{3x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{24c^4}\end{aligned}$$

$$= \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{8c^4}$$

$$= \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{16c^6}$$

$$= \frac{(5b+6ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6} + \frac{(5b+6ac^2)\,x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2} + \frac{(5b+6ac^2)\cosh^{-1}(cx)}{16c^7}$$

Mathematica [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{cx\sqrt{-1+cx}\sqrt{1+cx}(6ac^2(3+2c^2x^2)+b(15+10c^2x^2+8c^4x^4))+6(5b+6ac^2)\,\mathrm{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{48c^7}$$

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2
*x^2 + 8*c^4*x^4)) + 6*(5*b + 6*a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])
/(48*c^7)
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
risch	$\frac{x\left(8b\,x^4c^4+12a\,c^4x^2+10b\,c^2x^2+18c^2a+15b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{48c^6}+\frac{\left(6c^2a+5b\right)\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx-1)(cx+1)}}{16c^6\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8\ \mathrm{csgn}(c)b\,c^5x^5\sqrt{c^2x^2-1}+12\ \mathrm{csgn}(c)a\,c^5x^3\sqrt{c^2x^2-1}+10\ \mathrm{csgn}(c)c^3\sqrt{c^2x^2-1}\,b\,x^3+18\ \mathrm{csgn}(c)c^3\sqrt{c^2x^2-1}\,ax+15\ \mathrm{cs}\right.}{48c^7\sqrt{c^2x^2-1}}$

```
[In] int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(8*b*c^4*x^4+12*a*c^4*x^2+10*b*c^2*x^2+18*a*c^2+15*b)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^6+1/16*(6*a*c^2+5*b)/c^6*ln(c^2*x/(c^2)^(1/2)+(c^2*x^2-1)^(
1/2))/(c^2)^(1/2)*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(8\,bc^5x^5+2\,(6\,ac^5+5\,bc^3)x^3+3\,(6\,ac^3+5\,bc)x)\sqrt{cx+1}\sqrt{cx-1}-3\,(6\,ac^2+5\,b)\log\left(-cx+\sqrt{cx+1}\sqrt{cx\right.}}{48\,c^7}$$

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sqr
t(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)))/c^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{\sqrt{c^2x^2-1}bx^5}{6\,c^2}+\frac{\sqrt{c^2x^2-1}ax^3}{4\,c^2}+\frac{5\,\sqrt{c^2x^2-1}bx^3}{24\,c^4}\\&\quad+\frac{3\,\sqrt{c^2x^2-1}ax}{8\,c^4}+\frac{3\,a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{8\,c^5}\\&\quad+\frac{5\,\sqrt{c^2x^2-1}bx}{16\,c^6}+\frac{5\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{16\,c^7}\end{aligned}$$

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*sq
rt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5 + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left(\left(2\left((cx+1)\left(4\,(cx+1)\left(\frac{(cx+1)b}{c^6}-\frac{5\,b}{c^6}\right)+\frac{3\,(2\,ac^{38}+15\,bc^{36})}{c^{42}}\right)-\frac{18\,ac^{38}+55\,bc^{36}}{c^{42}}\right)(cx+1)+\frac{54\,ac^{38}+85\,bc^{36}}{c^{42}}\right)(cx+}{48\,c}$$

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((c*x + 1)*(4*(c*x + 1)*((c*x + 1)*b/c^6 - 5*b/c^6) + 3*(2*a*c^38
 + 15*b*c^36)/c^42) - (18*a*c^38 + 55*b*c^36)/c^42)*(c*x + 1) + (54*a*c^38
+ 85*b*c^36)/c^42)*(c*x + 1) - 3*(10*a*c^38 + 11*b*c^36)/c^42)*sqrt(c*x + 1
)*sqrt(c*x - 1) - 6*(6*a*c^2 + 5*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^6)
/c
```

Mupad [B] (verification not implemented)

Time = 45.44 (sec) , antiderivative size = 1154, normalized size of antiderivative = 9.23

$$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*a*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*a*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*a*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*a*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*a*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*a*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*a*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*a*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((311*b*((c*x - 1)^(1/2) - 1i)^5)/(4*((c*x + 1)^(1/2) - 1)^
5) - (175*b*((c*x - 1)^(1/2) - 1i)^3)/(12*((c*x + 1)^(1/2) - 1)^3) + (8361*
b*((c*x - 1)^(1/2) - 1i)^7)/(4*((c*x + 1)^(1/2) - 1)^7) + (42259*b*((c*x -
1)^(1/2) - 1i)^9)/(6*((c*x + 1)^(1/2) - 1)^9) + (25295*b*((c*x - 1)^(1/2) -
```

1i)^11)/(2*((c*x + 1)^(1/2) - 1)^11) + (25295*b*((c*x - 1)^(1/2) - 1i)^13)
/(2*((c*x + 1)^(1/2) - 1)^13) + (42259*b*((c*x - 1)^(1/2) - 1i)^15)/(6*((c*
x + 1)^(1/2) - 1)^15) + (8361*b*((c*x - 1)^(1/2) - 1i)^17)/(4*((c*x + 1)^(1
/2) - 1)^17) + (311*b*((c*x - 1)^(1/2) - 1i)^19)/(4*((c*x + 1)^(1/2) - 1)^1
9) - (175*b*((c*x - 1)^(1/2) - 1i)^21)/(12*((c*x + 1)^(1/2) - 1)^21) + (5*b
*((c*x - 1)^(1/2) - 1i)^23)/(4*((c*x + 1)^(1/2) - 1)^23) + (5*b*((c*x - 1)^
(1/2) - 1i))/(4*((c*x + 1)^(1/2) - 1)))/(c^7 - (12*c^7*((c*x - 1)^(1/2) - 1
i)^2)/((c*x + 1)^(1/2) - 1)^2 + (66*c^7*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1
)^(1/2) - 1)^4 - (220*c^7*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6
 + (495*c^7*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (792*c^7*((
c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (924*c^7*((c*x - 1)^(1/
2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (792*c^7*((c*x - 1)^(1/2) - 1i)^14)
/((c*x + 1)^(1/2) - 1)^14 + (495*c^7*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^
(1/2) - 1)^16 - (220*c^7*((c*x - 1)^(1/2) - 1i)^18)/((c*x + 1)^(1/2) - 1)^1
8 + (66*c^7*((c*x - 1)^(1/2) - 1i)^20)/((c*x + 1)^(1/2) - 1)^20 - (12*c^7*(
(c*x - 1)^(1/2) - 1i)^22)/((c*x + 1)^(1/2) - 1)^22 + (c^7*((c*x - 1)^(1/2)
 - 1i)^24)/((c*x + 1)^(1/2) - 1)^24) + (3*a*atanh(((c*x - 1)^(1/2) - 1i)/((c
*x + 1)^(1/2) - 1)))/(2*c^5) + (5*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)
^(1/2) - 1)))/(4*c^7)

3.349 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2(4b+5ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6} + \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}$$

```
[Out] 2/15*(5*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6+1/15*(5*a*c^2+4*b)*x^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/5*b*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {471, 102, 12, 75}

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^6} + \frac{x^2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^4} + \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} - \frac{1}{5}\left(-5a-\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\
&= \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} + \frac{(4b+5ac^2)\int \frac{2x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{15c^4} \\
&= \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} + \frac{(2(4b+5ac^2))\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{15c^4} \\
&= \frac{2(4b+5ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6} + \frac{(4b+5ac^2)\,x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(5ac^2(2+c^2x^2)+b(8+4c^2x^2+3c^4x^4))}{15c^6}$$

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 + 3
*c^4*x^4)))/(15*c^6)
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10c^2a+8b)}{15c^6}$	57

```
[In] int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*
c^2+8*b)/c^6
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(3\,bc^4x^4+10\,ac^2+(5\,ac^4+4\,bc^2)x^2+8\,b)\sqrt{cx+1}\sqrt{cx-1}}{15\,c^6}$$

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1)
*sqrt(c*x - 1)/c^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.10

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6}$$

```
[In] integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)
), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6)
 + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -
5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^4}{5\,c^2} + \frac{\sqrt{c^2x^2-1}ax^2}{3\,c^2} + \frac{4\,\sqrt{c^2x^2-1}bx^2}{15\,c^4} + \frac{2\,\sqrt{c^2x^2-1}a}{3\,c^4} + \frac{8\,\sqrt{c^2x^2-1}b}{15\,c^6}$$

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*sq
rt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x^2
 - 1)*b/c^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left(\left(\left(cx+1\right)\left(3\left(cx+1\right)\left(\frac{(cx+1)b}{c^5}-\frac{4b}{c^5}\right)+\frac{5ac^{27}+22bc^{25}}{c^{30}}\right)-\frac{10\left(ac^{27}+2bc^{25}\right)}{c^{30}}\right)\left(cx+1\right)+\frac{15\left(ac^{27}+bc^{25}\right)}{c^{30}}\right)\sqrt{cx+1}\sqrt{c\cdots}}{15c}$$

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((c*x + 1)*(3*(c*x + 1)*((c*x + 1)*b/c^5 - 4*b/c^5) + (5*a*c^27 + 22*
b*c^25)/c^30) - 10*(a*c^27 + 2*b*c^25)/c^30)*(c*x + 1) + 15*(a*c^27 + b*c^2
5)/c^30)*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c\,x-1}\,\left(\frac{10\,a\,c^2+8\,b}{15\,c^6}+\frac{b\,x^5}{5\,c}+\frac{b\,x^4}{5\,c^2}+\frac{x^2\,\left(5\,a\,c^4+4\,b\,c^2\right)}{15\,c^6}+\frac{x^3\,\left(5\,a\,c^5+4\,b\,c^3\right)}{15\,c^6}+\frac{x\,\left(10\,a\,c^3+8\,b\,c\right)}{15\,c^6}\right)}{\sqrt{c\,x+1}}$$

```
[In] int((x^3*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((8*b + 10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c^
2) + (x^2*(5*a*c^4 + 4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5 + 4*b*c^3))/(15*c^6
) + (x*(8*b*c + 10*a*c^3))/(15*c^6)))/(c*x + 1)^(1/2)
```

3.350 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\,\operatorname{arccosh}(cx)}{8c^5}$$

```
[Out] 1/8*(4*a*c^2+3*b)*arccosh(c*x)/c^5+1/8*(4*a*c^2+3*b)*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^4+1/4*b*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {471, 92, 54}

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(4ac^2+3b)\,\operatorname{arccosh}(cx)}{8c^5} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(4ac^2+3b)}{8c^4} + \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} - \frac{1}{4}\left(-4a - \frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\
&= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{8c^4} \\
&= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\cosh^{-1}(cx)}{8c^5}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{cx\sqrt{-1+cx}\sqrt{1+cx}(4ac^2+b(3+2c^2x^2)) + (6b+8ac^2)\,\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{8c^5}$$

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (6*b + 8*
a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(8*c^5)
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x\left(2b\,c^{2}x^{2}+4c^{2}a+3b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{8c^{4}}+\frac{\left(4c^{2}a+3b\right)\ln\left(\frac{c^{2}x}{\sqrt{c^{2}}}+\sqrt{c^{2}x^{2}-1}\right)\sqrt{\left(cx-1\right)\left(cx+1\right)}}{8c^{4}\sqrt{c^{2}}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\,\mathrm{csgn}(c)c^{3}\sqrt{c^{2}x^{2}-1}\,b\,x^{3}+4\,\mathrm{csgn}(c)c^{3}\sqrt{c^{2}x^{2}-1}\,ax+3\,\mathrm{csgn}(c)c\sqrt{c^{2}x^{2}-1}\,bx+4\ln\left(\left(\sqrt{c^{2}x^{2}-1}\,\mathrm{csgn}(c)+cx\right)\mathrm{csgn}(c)\right)c}{8c^{5}\sqrt{c^{2}x^{2}-1}}$

[In] int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*x*(2*b*c^2*x^2+4*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/8*(4*a*c^2+3*b)/c^4*ln(c^2*x/(c^2)^(1/2)+(c^2*x^2-1)^(1/2))/(c^2)^(1/2)*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left(2\,bc^3x^3+\left(4\,ac^3+3\,bc\right)x\right)\sqrt{cx+1}\sqrt{cx-1}-\left(4\,ac^2+3\,b\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{8\,c^5}$$

[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*a*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

[In] integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^3}{4\,c^2} + \frac{\sqrt{c^2x^2-1}ax}{2\,c^2} + \frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{2\,c^3} + \frac{3\,\sqrt{c^2x^2-1}bx}{8\,c^4} + \frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{8\,c^5}$$

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3/8
*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\left((cx+1)\left(2\,(cx+1)\left(\frac{(cx+1)b}{c^4}-\frac{3\,b}{c^4}\right)+\frac{4\,ac^{18}+9\,bc^{16}}{c^{20}}\right)-\frac{4\,ac^{18}+5\,bc^{16}}{c^{20}}\right)\sqrt{cx+1}\sqrt{cx-1} - \frac{2\,(4\,ac^2+3\,b)\log(\sqrt{cx+1}-\cdots}{c^4}}{8\,c}$$

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((c*x + 1)*(2*(c*x + 1)*((c*x + 1)*b/c^4 - 3*b/c^4) + (4*a*c^18 + 9*b*
c^16)/c^20) - (4*a*c^18 + 5*b*c^16)/c^20)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(
4*a*c^2 + 3*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^4)/c
```

Mupad [B] (verification not implemented)

Time = 29.41 (sec) , antiderivative size = 720, normalized size of antiderivative = 8.28

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= \frac{\frac{23\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{2\,\left(\sqrt{c\,x+1}-1\right)^3}+\frac{333\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{2\,\left(\sqrt{c\,x+1}-1\right)^5}+\frac{671\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{2\,\left(\sqrt{c\,x+1}-1\right)^7}+\frac{671\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^9}{2\,\left(\sqrt{c\,x+1}-1\right)^9}+\frac{333\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{11}}{2\,\left(\sqrt{c\,x+1}-1\right)^{11}}+\frac{23\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{13}}{2\,\left(\sqrt{c\,x+1}-1\right)^{13}}-}{c^5-\frac{8\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{28\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{56\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{70\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}-\frac{56\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{c\,x+1}-1\right)^{10}}+\frac{28\,c^5\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{c\,x+1}-1\right)^{12}}-}$$

$$-\frac{\frac{14\,a\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,a\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,a\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,a\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}}$$

$$+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}+\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{2\,c^5}$$

```
[In] int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*b*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*b*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*b*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*b*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*b*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*b*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*b*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*b*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((14*a*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 +
(14*a*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*a*((c*x - 1)^(
1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*a*((c*x - 1)^(1/2) - 1i))/((c*x
+ 1)^(1/2) - 1))/(c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) -
 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*(
(c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) -
1i)^8)/((c*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((c*x - 1)^(1/2) - 1i)/((c*x +
 1)^(1/2) - 1)))/c^3 + (3*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))/(2*c^5)
```

3.351 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}$$

```
[Out] 1/3*(3*a*c^2+2*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/3*b*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {471, 75}

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+2b)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2} - \frac{1}{3}\left(-3a-\frac{2b}{c^2}\right)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int\frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{-1+cx}\sqrt{1+cx}(3ac^2+b(2+c^2x^2))}{3c^4}$$

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4)
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(b\,c^2x^2+3c^2a+2b)}{3c^4}$	38

```
[In] int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(bc^2x^2+3\,ac^2+2\,b)\sqrt{cx+1}\sqrt{cx-1}}{3\,c^4}$$

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.11

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$

```
[In] integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/
2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*m
eijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2,
 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c^2x^2-1}bx^2}{3\,c^2} + \frac{\sqrt{c^2x^2-1}a}{c^2} + \frac{2\,\sqrt{c^2x^2-1}b}{3\,c^4}$$

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x^
2 - 1)*b/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{cx+1}\sqrt{cx-1}\left((cx+1)\left(\frac{(cx+1)b}{c^3}-\frac{2\,b}{c^3}\right)+\frac{3\,(ac^{11}+bc^9)}{c^{12}}\right)}{3\,c}$$

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*((c*x + 1)*b/c^3 - 2*b/c^3) + 3*
(a*c^11 + b*c^9)/c^12)/c
```

Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c\,x-1}\,\left(\frac{3\,a\,c^2+2\,b}{3\,c^4}+\frac{b\,x^3}{3\,c}+\frac{b\,x^2}{3\,c^2}+\frac{x\,(3\,a\,c^3+2\,b\,c)}{3\,c^4}\right)}{\sqrt{c\,x+1}}$$

```
[In] int((x*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((2*b + 3*a*c^2)/(3*c^4) + (b*x^3)/(3*c) + (b*x^2)/(3*c^2)
 + (x*(2*b*c + 3*a*c^3))/(3*c^4)))/(c*x + 1)^(1/2)
```

3.352 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2} + \frac{(b+2ac^2)\operatorname{arccosh}(cx)}{2c^3}$$

```
[Out] 1/2*(2*a*c^2+b)*arccosh(c*x)/c^3+1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {397, 54}

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(2ac^2+b)\operatorname{arccosh}(cx)}{2c^3} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
```

```
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2} - \frac{(-b-2ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2c^2} \\ &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2} + \frac{(b+2ac^2)\cosh^{-1}(cx)}{2c^3}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{bcx\sqrt{-1+cx}\sqrt{1+cx}+2(b+2ac^2)\,\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{2c^3}$$

```
[In] Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*(b + 2*a*c^2)*ArcTanh[Sqrt[(-1 + c*
x)/(1 + c*x)]])/(2*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 4.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

method	result
risch	$\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}+\frac{(2c^2a+b)\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx-1)(cx+1)}}{2c^2\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\text{csgn}(c)c\sqrt{c^2x^2-1}\,bx+2\ln\left(\left(\sqrt{c^2x^2-1}\,\text{csgn}(c)+cx\right)\text{csgn}(c)\right)a\,c^2+\ln\left(\left(\sqrt{c^2x^2-1}\,\text{csgn}(c)+cx\right)\text{csgn}(c)\right)b\right)\text{csgn}(c)}{2c^3\sqrt{c^2x^2-1}}$

```
[In] int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2+1/2*(2*a*c^2+b)/c^2*ln(c^2*x/(c^2)^
(1/2)+(c^2*x^2-1)^(1/2))/(c^2)^(1/2)*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{cx+1}\sqrt{cx-1}bcx-(2\,ac^2+b)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{2\,c^3}$$

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/c^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c} + \frac{\sqrt{c^2x^2-1}bx}{2\,c^2} + \frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{2\,c^3}$$

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + 1/2*sqrt(c^2*x^2 - 1)*b*x/c^2 +
1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{cx+1}\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2}-\frac{b}{c^2}\right)-\frac{2\,(2\,ac^2+b)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^2}}{2\,c}$$

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 + b
)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^2)/c
```

Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 6.23

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

```
[In] int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c*
x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) - 1i
)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1
/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 - (4*
c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(
1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((
c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
)^8) - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/(((c*x + 1)^(1/2) - 1)*(-c^2)^(
1/2))))/(-c^2)^(1/2)
```

3.353 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] a*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {471, 94, 211}

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=a\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

```
[In] Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + a\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\
&= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + (ac)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right) \\
&= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + a\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + 2a\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + 2*a*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)
]]
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^2+\sqrt{c^2x^2-1}\,b\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}\,c^2}$	62

```
[In] int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+(c^2*x^2-1)^(1/2)*b)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)^(1/2)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b)/c^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.95 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.52

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

```
[In] integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
```

```
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}b}{c^2}$$

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] -a*arcsin(1/(c*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*x
 - 1)*b/c^2
```

Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{b\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c^2} - a\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1\mathrm{i}$$

```
[In] int((a + b*x^2)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
[Out] (b*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c^2 - a*(log(((c*x - 1)^(1/2) - 1i)^2/(
(c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))*1i
```

3.354 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x}+\frac{b\operatorname{arccosh}(cx)}{c}$$

```
[Out] b*arccosh(c*x)/c+a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {465, 54}

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\operatorname{arccosh}(cx)}{c}$$

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
```

```
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x} + b\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x} + \frac{b\cosh^{-1}(cx)}{c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x} + \frac{2b\,\text{arctanh}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{c}$$

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (2*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)
]])/c
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\operatorname{csgn}(c)c\sqrt{c^2x^2-1}\,a+\ln\left(\left(\sqrt{c^2x^2-1}\,\operatorname{csgn}(c)+cx\right)\operatorname{csgn}(c)\right)bx\right)\operatorname{csgn}(c)}{\sqrt{c^2x^2-1}\,xc}$	77
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\ln\left(\frac{c^2x}{\sqrt{c^2}}+\sqrt{c^2x^2-1}\right)\sqrt{(cx-1)(cx+1)}}{\sqrt{c^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}$	78

```
[In] int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(csgn(c)*c*(c^2*x^2-1)^(1/2)*a+ln(((c^2*x^2-1)^
(1/2)*csgn(c)+c*x)*csgn(c))*b*x)*csgn(c)/(c^2*x^2-1)^(1/2)/x/c
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{ac^2x+\sqrt{cx+1}\sqrt{cx-1}ac-bx\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{cx}$$

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] (a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{acG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iacG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

```
[In] integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3
/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2)*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}c\right)}{c}+\frac{\sqrt{c^2x^2-1}a}{x}$$

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + sqrt(c^2*x^2 - 1)*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4}-b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x + 1
) - sqrt(c*x - 1))^4))/c
```

Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{x}-\frac{4\,b\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

```
[In] int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
[Out] (a*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/x - (4*b*atan((c*((c*x - 1)^(1/2) - 1i)
)/(((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

3.355 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 60

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] 1/2*(a*c^2+2*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*a*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {465, 94, 211}

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{1}{2}\left(ac^2+2b\right)\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 +
c*x]*Sqrt[1 + c*x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(c\left(2b+ac^2\right)\right)\text{Subst}\left(\int\frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\
&= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int\frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\left(2b+ac^2\right)\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + (2*b + a*c^2)*ArcTan[Sqrt[(-1 +
c*x)/(1 + c*x)]]
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^{2}}-\frac{\left(b+\frac{c^{2}a}{2}\right)\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)\sqrt{(cx-1)(cx+1)}}{\sqrt{cx-1}\,\sqrt{cx+1}}$	71
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)a\,c^{2}x^{2}+2\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)b\,x^{2}-\sqrt{c^{2}x^{2}-1}\,a\right)}{2\sqrt{c^{2}x^{2}-1}\,x^{2}}$	84

```
[In] int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-(b+1/2*c^2*a)*arctan(1/(c^2*x^2-1)^(1
/2))*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\,(ac^2+2\,b)x^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}a}{2\,x^2}$$

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(
c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{1}{2}\,ac^2\arcsin\left(\frac{1}{c|x|}\right) - b\arcsin\left(\frac{1}{c|x|}\right) + \frac{\sqrt{c^2x^2-1}a}{2\,x^2}$$

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*a*c^2*arcsin(1/(c*abs(x))) - b*arcsin(1/(c*abs(x))) + 1/2*sqrt(c^2*x^2
 - 1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{(ac^3+2\,bc)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right) + \frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 4.95

$$\begin{aligned}\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{\frac{a\,c^2\,1\mathrm{i}}{32}+\frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,\left(\sqrt{c\,x+1}-1\right)^2}-\frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^4}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}}\\ &\quad - b\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1\mathrm{i}\\ &\quad - \frac{a\,c^2\,\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2}\\ &\quad + \frac{a\,c^2\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{a\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^2}\end{aligned}$$

```
[In] int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - b*(log((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1) - lo
g((((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))*1i - (a*c^2*log((((c*x - 1)
^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log((((c*x - 1)^(
1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1
i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

3.356 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 62

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{(3b+2ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3x}$$

```
[Out] 1/3*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^3+1/3*(2*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {465, 97}

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{\sqrt{cx-1}\sqrt{cx+1}(2ac^2+3b)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{1}{3}\left(3b+2ac^2\right)\int\frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{\left(3b+2ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{3x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int\frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+3bx^2+2ac^2x^2\right)}{3x^3}$$

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3)
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(2a\,c^2x^2+3b\,x^2+a\right)}{3x^3}$	37
risch	$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(2a\,c^2x^2+3b\,x^2+a\right)}{3x^3}$	37
default	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{csgn}(c)^2\left(2a\,c^2x^2+3b\,x^2+a\right)}{3x^3}$	41

```
[In] int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{(2\,ac^3+3\,bc)x^3+((2\,ac^2+3\,b)x^2+a)\sqrt{cx+1}\sqrt{cx-1}}{3\,x^3}$$

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(c
*x - 1))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.35

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bcG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

```
[In] integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1,
5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x*
*2))/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\sqrt{c^2x^2-1}ac^2}{3\,x}+\frac{\sqrt{c^2x^2-1}b}{x}+\frac{\sqrt{c^2x^2-1}a}{3\,x^3}$$

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
1)*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{8\left(3\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^8+24\,ac^4\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+24\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+32\,ac^4+\right.}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^3c}$$

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4 +
48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{c\,x-1}\,\left(\left(\frac{2\,a\,c^3}{3}+b\,c\right)\,x^3+\left(\frac{2\,a\,c^2}{3}+b\right)\,x^2+\frac{a\,c\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{c\,x+1}}$$

```
[In] int((a + b*x^2)/(x^4*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) + (
a*c*x)/3))/(x^3*(c*x + 1)^(1/2))
```

3.357 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 99

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}c^2(4b+3ac^2)\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] 1/8*c^2*(3*a*c^2+4*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*a*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/x^4+1/8*(3*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {465, 105, 12, 94, 211}

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{1}{8}c^2(3ac^2+4b)\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+4b)}{8x^2}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4} + \frac{1}{4}\left(4b+3ac^2\right)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\
&= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2} \\
&\quad + \frac{1}{8}\left(4b+3ac^2\right)\int \frac{c^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx
\end{aligned}
$$

$$= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4} + \frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^2(4b+3ac^2)\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4} + \frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^3(4b+3ac^2)\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)$$

$$= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4} + \frac{(4b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2} + \frac{1}{8}c^2(4b+3ac^2)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{1}{8}\left(\frac{\sqrt{-1+cx}\sqrt{1+cx}(4bx^2+a(2+3c^2x^2))}{x^4} + (8bc^2+6ac^4)\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)\right)$$

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] ((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*b*x^2 + a*(2 + 3*c^2*x^2)))/x^4 + (8*b*c^
2 + 6*a*c^4)*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]])/8
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

method	result	si
risch	$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(3a\,c^2x^2+4b\,x^2+2a)}{8x^4}-\frac{c^2(3c^2a+4b)\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{(cx-1)(cx+1)}}{8\sqrt{cx-1}\,\sqrt{cx+1}}$	9
default	$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)a\,c^4x^4+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)b\,c^2x^4-3\sqrt{c^2x^2-1}\,a\,c^2x^2-4\sqrt{c^2x^2-1}\,b\,x^2-2\sqrt{c^2x^2-1}\,a\right)}{8\sqrt{c^2x^2-1}\,x^4}$	1

```
[In] int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*a*c^2*x^2+4*b*x^2+2*a)/x^4-1/8*c^2*(3*a*
c^2+4*b)*arctan(1/(c^2*x^2-1)^(1/2))*((c*x-1)*(c*x+1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{2\,(3\,ac^4+4\,bc^2)x^4\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+((3\,ac^2+4\,b)x^2+2\,a)\sqrt{cx+1}\sqrt{cx-1}}{8\,x^4}$$

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{3}{8}\,ac^4\arcsin\left(\frac{1}{c|x|}\right)-\frac{1}{2}\,bc^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{3\,\sqrt{c^2x^2-1}ac^2}{8\,x^2}+\frac{\sqrt{c^2x^2-1}b}{2\,x^2}+\frac{\sqrt{c^2x^2-1}a}{4\,x^4}$$

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] -3/8*a*c^4*arcsin(1/(c*abs(x))) - 1/2*b*c^2*arcsin(1/(c*abs(x))) + 3/8*sqrt
(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2 -
1)*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(81) = 162$.

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.71

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = -\frac{(3\,ac^5+4\,bc^3)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(3\,ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4\,bc^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44\,ac^5\left(\sqrt{cx+\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2
*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - sqr
t(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*(sq
rt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))
^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1)
- sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*
x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

Mupad [B] (verification not implemented)

Time = 31.05 (sec) , antiderivative size = 650, normalized size of antiderivative = 6.57

$$\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{\frac{b\,c^2\,1\mathrm{i}}{32}+\frac{b\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,\left(\sqrt{c\,x+1}-1\right)^2}-\frac{b\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^4}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}}$$
$$-\frac{\frac{a\,c^4\,1\mathrm{i}}{1024}-\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,3\mathrm{i}}{128\,\left(\sqrt{c\,x+1}-1\right)^2}-\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,53\mathrm{i}}{512\,\left(\sqrt{c\,x+1}-1\right)^4}+\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6\,87\mathrm{i}}{256\,\left(\sqrt{c\,x+1}-1\right)^6}+\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8\,657\mathrm{i}}{1024\,\left(\sqrt{c\,x+1}-1\right)^8}+\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{10}\,121\mathrm{i}}{256\,\left(\sqrt{c\,x+1}-1\right)^{10}}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{6\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}+\frac{4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{c\,x+1}-1\right)^{10}}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{c\,x+1}-1\right)^{12}}}$$
$$-\frac{a\,c^4\,\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,3\mathrm{i}}{8}-\frac{b\,c^2\,\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2}$$
$$+\frac{a\,c^4\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,3\mathrm{i}}{8}+\frac{b\,c^2\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,7\mathrm{i}}{256\,\left(\sqrt{c\,x+1}-1\right)^2}$$
$$-\frac{a\,c^4\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4\,1\mathrm{i}}{1024\,\left(\sqrt{c\,x+1}-1\right)^4}+\frac{b\,c^2\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,\left(\sqrt{c\,x+1}-1\right)^2}$$

```
[In] int((a + b*x^2)/(x^5*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((b*c^2*1i)/32 + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (b*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - ((a*c^4*1i)/1024 - (a*c^4*((c*x - 1)^(1/2) - 1i)^2*3i)/(128*((c*x
 + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*53i)/(512*((c*x + 1)^
(1/2) - 1)^4) + (a*c^4*((c*x - 1)^(1/2) - 1i)^6*87i)/(256*((c*x + 1)^(1/2)
- 1)^6) + (a*c^4*((c*x - 1)^(1/2) - 1i)^8*657i)/(1024*((c*x + 1)^(1/2) - 1)
^8) + (a*c^4*((c*x - 1)^(1/2) - 1i)^10*121i)/(256*((c*x + 1)^(1/2) - 1)^10)
)/((((c*x - 1)^(1/2) - 1i)^4/((c*x + 1)^(1/2) - 1)^4 + (4*((c*x - 1)^(1/2) -
 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (6*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(
1/2) - 1)^8 + (4*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + ((c*
x - 1)^(1/2) - 1i)^12/((c*x + 1)^(1/2) - 1)^12) - (a*c^4*log(((c*x - 1)^(1/
2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (b*c^2*log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^4*log(((c*x - 1)^(1/2) -
 1i)/((c*x + 1)^(1/2) - 1))*3i)/8 + (b*c^2*log(((c*x - 1)^(1/2) - 1i)/((c*x
 + 1)^(1/2) - 1))*1i)/2 + (a*c^4*((c*x - 1)^(1/2) - 1i)^2*7i)/(256*((c*x +
1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*1i)/(1024*((c*x + 1)^(1/
2) - 1)^4) + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^
2)
```

3.358 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 164

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{c^4(5bc^2+6ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^7}$$

```
[Out] 1/8*c^4*(6*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7+1/16*c^2
*(6*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/24*(6*a*d^2+5*b*c^2)
*x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/6*b*x^5*(d*x-c)^(1/2)*(d*x+c)^(1/2)/
d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {471, 102, 12, 92, 65, 223, 212}

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{c^4(6ad^2+5bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7} + \frac{c^2x\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{16d^6} + \frac{x^3\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{24d^4} + \frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}-\frac{1}{6}\left(-6a-\frac{5bc^2}{d^2}\right)\int\frac{x^4}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\
&= \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}\\
&\quad+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{(5bc^2+6ad^2)\int\frac{3c^2x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{24d^4}\\
&= \frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}\\
&\quad+\frac{(c^2(5bc^2+6ad^2))\int\frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^4}\\
&= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6}+\frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}\\
&\quad+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{(c^2(5bc^2+6ad^2))\int\frac{c^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{16d^6}\\
&= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6}+\frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}\\
&\quad+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{(c^4(5bc^2+6ad^2))\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{16d^6}\\
&= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6}+\frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}\\
&\quad+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{(c^4(5bc^2+6ad^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{8d^7}\\
&= \frac{c^2(5bc^2+6ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6}+\frac{(5bc^2+6ad^2)\,x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4}\\
&\quad+\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{(c^4(5bc^2+6ad^2))\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^7}
\end{aligned}
$$

$$= \frac{c^2(5bc^2 + 6ad^2)\, x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2 + 6ad^2)\, x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{c^4(5bc^2 + 6ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^7}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{x^4(a + bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\, dx = \frac{dx\sqrt{-c+dx}\sqrt{c+dx}(6ad^2(3c^2 + 2d^2x^2) + b(15c^4 + 10c^2d^2x^2 + 8d^4x^4)) + 6c^4(5bc^2 + 6ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c\cdots}}{\sqrt{c+\cdots}}\right.}{48d^7}$$

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4
+ 10*c^2*d^2*x^2 + 8*d^4*x^4)) + 6*c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c
+ d*x]/Sqrt[c + d*x]])/(48*d^7)
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{x\left(8b\,d^4x^4+12a\,d^4x^2+10b\,c^2d^2x^2+18a\,c^2d^2+15b\,c^4\right)(-dx+c)\sqrt{dx+c}}{48d^6\sqrt{dx-c}}+\frac{c^4\left(6a\,d^2+5b\,c^2\right)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{16d^6\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\,\operatorname{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}+12\,\operatorname{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}+10\,\operatorname{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}+18\,\operatorname{csgn}(d)d^3\sqrt{d^2x^2-c^2}\,a\cdots\right.}{48d\cdots}$

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*x*(8*b*d^4*x^4+12*a*d^4*x^2+10*b*c^2*d^2*x^2+18*a*c^2*d^2+15*b*c^4)*(
-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2)+1/16*c^4*(6*a*d^2+5*b*c^2)/d^6*ln(x
*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(
d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.70

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(8\,bd^5x^5+2\,(5\,bc^2d^3+6\,ad^5)x^3+3\,(5\,bc^4d+6\,ac^2d^3)x)\sqrt{dx+c}\sqrt{dx-c}-3\,(5\,bc^6+6\,ac^4d^2)\log\left(-dx+\sqrt{\right.}{48\,d^7}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^2
*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x +
 sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{\sqrt{d^2x^2-c^2}bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}bc^2x^3}{24\,d^4}\\ &\quad+\frac{\sqrt{d^2x^2-c^2}ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{16\,d^7}\\ &\quad+\frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^5}\\ &\quad+\frac{5\,\sqrt{d^2x^2-c^2}bc^4x}{16\,d^6}+\frac{3\,\sqrt{d^2x^2-c^2}ac^2x}{8\,d^4}\end{aligned}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4
+ 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^7 + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5 + 5
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.24

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{5\,bc}{d^6}\right)+\frac{3\,(15\,bc^2d^{36}+2\,ad^{38})}{d^{42}}\right)-\frac{55\,bc^3d^{36}+18\,acd^{38}}{d^{42}}\right)(dx+c)+\frac{85\,bc^4d^{36}+54\,ac^2d}{d^{42}}\right.}{48\,d}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((d*x + c)*(4*(d*x + c)*((d*x + c)*b/d^6 - 5*b*c/d^6) + 3*(15*b*c
^2*d^36 + 2*a*d^38)/d^42) - (55*b*c^3*d^36 + 18*a*c*d^38)/d^42)*(d*x + c) +
 (85*b*c^4*d^36 + 54*a*c^2*d^38)/d^42)*(d*x + c) - 3*(11*b*c^5*d^36 + 10*a*
c^3*d^38)/d^42)*sqrt(d*x + c)*sqrt(d*x - c) - 6*(5*b*c^6 + 6*a*c^4*d^2)*log
(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)/d
```

Mupad [B] (verification not implemented)

Time = 58.97 (sec) , antiderivative size = 1682, normalized size of antiderivative = 10.26

$$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((5*b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (8361*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (
d*x - c)^(1/2))^7) + (42259*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(
1/2) - (d*x - c)^(1/2))^9) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(
2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1
/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (42259*b*c^6*((c + d*x)^(1
/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (8361*b*c^6*((c
+ d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (311*b*
c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/
2))^21) + (5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x -
c)^(1/2))^23))/(d^7 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) -
(d*x - c)^(1/2))^2 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (
d*x - c)^(1/2))^4 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (
d*x - c)^(1/2))^6 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (
```

```
d*x - c)^(1/2))^8 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) -
(d*x - c)^(1/2))^10 + (924*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2)
- (d*x - c)^(1/2))^12 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2
) - (d*x - c)^(1/2))^14 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1
/2) - (d*x - c)^(1/2))^16 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^
(1/2) - (d*x - c)^(1/2))^18 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)
^(1/2) - (d*x - c)^(1/2))^20 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c
)^(1/2) - (d*x - c)^(1/2))^22 + (d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^
(1/2) - (d*x - c)^(1/2))^24) - ((23*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2
*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (3*a*c^4*((c + d*x)^(1/2) - c^(1/2)))/
(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (333*a*c^4*((c + d*x)^(1/2) - c^(1/2))
^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (671*a*c^4*((c + d*x)^(1/2) - c^
(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (671*a*c^4*((c + d*x)^(1/2
) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (333*a*c^4*((c + d*x
)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*a*c^4*((
c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*a*
c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/
(d^5 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^
4 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6
 + (70*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
- (56*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10
 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^1
2 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^1
4 + (d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16)
 - (3*a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)
)))/(2*d^5) - (5*b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
 - c)^(1/2))))/(4*d^7)
```

3.359 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2c^2(4bc^2+5ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6} + \frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}$$

```
[Out] 2/15*c^2*(5*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/15*(5*a*d^2+4*
b*c^2)*x^2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/5*b*x^4*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {471, 102, 12, 75}

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4} + \frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} - \frac{1}{5}\left(-5a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} \\
&\quad + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} + \frac{(4bc^2+5ad^2)\int \frac{2c^2x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{15d^4} \\
&= \frac{(4bc^2+5ad^2)\,x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} \\
&\quad + \frac{(2c^2(4bc^2+5ad^2))\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{15d^4}
\end{aligned}$$

$$= \frac{2c^2(4bc^2+5ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6} + \frac{(4bc^2+5ad^2)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(5ad^2(2c^2+d^2x^2)+b(8c^4+4c^2d^2x^2+3d^4x^4))}{15d^6}$$

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^2
*d^2*x^2 + 3*d^4*x^4)))/(15*d^6)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)}{15d^6}$	68
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)}{15d^6}$	68
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4)}{15d^6\sqrt{dx-c}}$	74

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15/d^6*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x
^2+10*a*c^2*d^2+8*b*c^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+(4\,bc^2d^2+5\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^6}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.03

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{ac^3 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{iac^3 G_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{bc^5 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6} + \frac{ibc^5 G_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 & \\ -\frac{11}{4}, -\frac{9}{4} & -3, -\frac{5}{2}, -\frac{5}{2}, 0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}$$

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-2,
 -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -
7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2
*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2,
1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2
*x**2))/(4*pi**(3/2)*d**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^4}{5\,d^2} + \frac{4\,\sqrt{d^2x^2-c^2}bc^2x^2}{15\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax^2}{3\,d^2} + \frac{8\,\sqrt{d^2x^2-c^2}bc^4}{15\,d^6} + \frac{2\,\sqrt{d^2x^2-c^2}ac^2}{3\,d^4}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4
+ 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6 +
2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\left(\left((dx+c)\left(3\,(dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{4\,bc}{d^5}\right)+\frac{22\,bc^2d^{25}+5\,ad^{27}}{d^{30}}\right)-\frac{10\,(2\,bc^3d^{25}+acd^{27})}{d^{30}}\right)(dx+c)+\frac{15\,(bc^4d^{25}+ac^2d^{27})}{d^{30}}\right)\sqrt{dx+c}\sqrt{dx-c}}{15\,d}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)*b/d^5 - 4*b*c/d^5) + (22*b*c^2*d^2
5 + 5*a*d^27)/d^30) - 10*(2*b*c^3*d^25 + a*c*d^27)/d^30)*(d*x + c) + 15*(b*
c^4*d^25 + a*c^2*d^27)/d^30)*sqrt(d*x + c)*sqrt(d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{8\,b\,c^5+10\,a\,c^3\,d^2}{15\,d^6}+\frac{x^3\,\left(4\,b\,c^2\,d^3+5\,a\,d^5\right)}{15\,d^6}+\frac{x\,\left(8\,b\,c^4\,d+10\,a\,c^2\,d^3\right)}{15\,d^6}+\frac{b\,x^5}{5\,d}+\frac{x^2\,\left(4\,b\,c^3\,d^2+5\,a\,c\,d^4\right)}{15\,d^6}+\frac{b\,c\,x^4}{5\,d^2}\right)}{\sqrt{c+d\,x}}$$

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*c
^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*d)
 + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d*x)
^(1/2)
```

3.360 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{c^2(3bc^2+4ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^5}$$

```
[Out] 1/4*c^2*(4*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5+1/8*(4*a
*d^2+3*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/4*b*x^3*(d*x-c)^(1/2)*(d*
x+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {471, 92, 12, 65, 223, 212}

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{c^2(4ad^2+3bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+3bc^2)}{8d^4}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} - \frac{1}{4}\left(-4a - \frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$=\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{(3bc^2+4ad^2)\int\frac{c^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^4}$$

$$=\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{(c^2(3bc^2+4ad^2))\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^4}$$

$$=\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{(c^2(3bc^2+4ad^2))\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{4d^5}$$

$$=\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{(c^2(3bc^2+4ad^2))\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^5}$$

$$=\frac{(3bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4}+\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{c^2(3bc^2+4ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^5}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

$$\int\frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{dx\sqrt{-c+dx}\sqrt{c+dx}(3bc^2+4ad^2+2bd^2x^2)+(6bc^4+8ac^2d^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8d^5}$$

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + (6*b*
c^4 + 8*a*c^2*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x\left(2b\,d^{2}x^{2}+4a\,d^{2}+3b\,c^{2}\right)\left(-dx+c\right)\sqrt{dx+c}}{8d^{4}\sqrt{dx-c}}+\frac{c^{2}\left(4a\,d^{2}+3b\,c^{2}\right)\ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{\left(dx-c\right)\left(dx+c\right)}}{8d^{4}\sqrt{d^{2}}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\,\mathrm{csgn}\left(d\right)b\,d^{3}x^{3}\sqrt{d^{2}x^{2}-c^{2}}+4\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}\left(d\right)d^{3}ax+3\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}\left(d\right)db\,c^{2}x+4\ln\left(\left(\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}\left(d\right)+dx\right)\right.\right.}{8d^{5}\sqrt{d^{2}x^{2}-c^{2}}}$

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(2*b*d^2*x^2+4*a*d^2+3*b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1/
2)+1/8*c^2*(4*a*d^2+3*b*c^2)/d^4*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/
(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$=\frac{(2\,bd^3x^3+(3\,bc^2d+4\,ad^3)x)\sqrt{dx+c}\sqrt{dx-c}-(3\,bc^4+4\,ac^2d^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^5}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
(3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.20

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^3}{4\,d^2} + \frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^5} + \frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d^3} + \frac{3\,\sqrt{d^2x^2-c^2}bc^2x}{8\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax}{2\,d^2}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/8*
sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^4}-\frac{3\,bc}{d^4}\right)+\frac{9\,bc^2d^{16}+4\,ad^{18}}{d^{20}}\right)-\frac{5\,bc^3d^{16}+4\,acd^{18}}{d^{20}}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{2\,(3\,bc^4+4\,ac^2d^2)\log(}{d}}{8\,d}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16
+ 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(d*
x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))
/d^4)/d
```

Mupad [B] (verification not implemented)

Time = 33.91 (sec) , antiderivative size = 1048, normalized size of antiderivative = 8.88

$$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{\frac{2\,a\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}}$$

$$-\frac{\frac{23\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}-\frac{3\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{333\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{671\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}+\frac{671\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^9}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^9}+\cdots}{d^5-\frac{8\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{28\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{56\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{70\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}-\frac{56\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^{10}}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^{10}}+\cdots}$$

$$-\frac{2\,a\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}-\frac{3\,b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^5}$$

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) - ((23
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3)
- (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2)))
+ (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2
))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c
)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d
*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/
2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((
-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)
/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c + d*x)^(1/2) - c^
(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c + d*x)^(1/2) - c^(
1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c + d*x)^(1/2) - c^(1
/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*((c + d*x)^(1/2) - c^(1/
2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^5*((c + d*x)^(1/2) - c^(1/2
))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^5*((c + d*x)^(1/2) - c^(1/
2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^5*((c + d*x)^(1/2) - c^(1/
2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^5*((c + d*x)^(1/2) - c^(1/2)
)^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) - (2*a*c^2*atanh(((c + d*x)^(1/2)
- c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3 - (3*b*c^4*atanh(((c + d*x)
^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^5)
```

3.361 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 72

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(2bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}$$

```
[Out] 1/3*(3*a*d^2+2*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/3*b*x^2*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {471, 75}

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4} + \frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2} - \frac{1}{3}\left(-3a-\frac{2bc^2}{d^2}\right)\int\frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= \frac{(2bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int\frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(2bc^2+3ad^2+bd^2x^2)}{3d^4}$$

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4}$	43
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4}$	43
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\,(b\,d^2x^2+3a\,d^2+2b\,c^2)}{3d^4\sqrt{dx-c}}$	49

```
[In] int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/d^4*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(bd^2x^2+2\,bc^2+3\,ad^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^4}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.10

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ibc^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), (
)), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1, -1,
 -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi*
*(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4
, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(
3/2)*d**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}bx^2}{3\,d^2} + \frac{2\,\sqrt{d^2x^2-c^2}bc^2}{3\,d^4} + \frac{\sqrt{d^2x^2-c^2}a}{d^2}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sqr
t(d^2*x^2 - c^2)*a/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^3}-\frac{2\,bc}{d^3}\right)+\frac{3\,(bc^2d^9+ad^{11})}{d^{12}}\right)}{3\,d}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^3 - 2*b*c/d^3) +
3*(b*c^2*d^9 + a*d^11)/d^12)/d
```

Mupad [B] (verification not implemented)

Time = 7.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{2\,b\,c^3+3\,a\,c\,d^2}{3\,d^4}+\frac{b\,x^3}{3\,d}+\frac{x\,\left(2\,b\,c^2\,d+3\,a\,d^3\right)}{3\,d^4}+\frac{b\,c\,x^2}{3\,d^2}\right)}{\sqrt{c+d\,x}}$$

```
[In] int((x*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((2*b*c^3 + 3*a*c*d^2)/(3*d^4) + (b*x^3)/(3*d) + (x*(3*a*d
^3 + 2*b*c^2*d))/(3*d^4) + (b*c*x^2)/(3*d^2)))/(c + d*x)^(1/2)
```

3.362 $\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2}+\frac{(bc^2+2ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

```
[Out] (2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/2*b*x*(d*x-c)^(1
/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {397, 65, 223, 212}

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{(2ad^2+bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

```
[In] Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 397

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*(n*(p + 1) + 1))), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} - \frac{(-bc^2-2ad^2)\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2d^2} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int\frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bdx\sqrt{-c+dx}\sqrt{c+dx}+2(bc^2+2ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2d^3}$$

```
[In] Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] + 2*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(2*d^3)
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(-dx+c)\sqrt{dx+c}\,bx}{2d^2\sqrt{dx-c}}+\frac{(2a\,d^2+b\,c^2)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{2d^2\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\operatorname{csgn}(d)d\sqrt{d^2x^2-c^2}\,bx+\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2+2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^2\right)\operatorname{cs}}{2d^3\sqrt{d^2x^2-c^2}}$

[In] int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(-d*x+c)*(d*x+c)^(1/2)*b*x/d^2/(d*x-c)^(1/2)+1/2*(2*a*d^2+b*c^2)/d^2*l
n(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)*((d*x-c)*(d*x+c))^(1/2
)/(d*x-c)^(1/2)/(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}bdx-(bc^2+2\,ad^2)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{2\,d^3}$$

[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^3

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d^3} + \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d} + \frac{\sqrt{d^2x^2-c^2}bx}{2\,d^2}$$

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + a*log(2*d^2*x + 2*sq
rt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{dx+c}\sqrt{dx-c}\left(\frac{(dx+c)b}{d^2}-\frac{bc}{d^2}\right)-\frac{2\,(bc^2+2\,ad^2)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}}{2\,d}$$

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*b/d^2 - b*c/d^2) - 2*(b*c^2 + 2
*a*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.13

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\frac{2\,b\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,b\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}} + \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}} - \frac{2\,b\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}$$

```
[In] int((a + b*x^2)/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*b*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*b*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*b*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*b*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) + (4*a
*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))))/(-d^2)^(1/2) - (2*b*c^2*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^
(1/2) - (d*x - c)^(1/2))))/d^3
```

3.363 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+\frac{a\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}$$

```
[Out] a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {471, 94, 211}

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

```
[In] Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + a\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + (ad)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right) \\
&= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + \frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + \frac{2a\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{c}$$

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (2*a*ArcTan[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/c
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(48) = 96$.

Time = 4.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

method	result	size
default	$\frac{\left(-\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)a\,d^{2}+b\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)\sqrt{dx-c}\,\sqrt{dx+c}}{\sqrt{d^{2}x^{2}-c^{2}}\,\sqrt{-c^{2}}\,d^{2}}$	108

```
[In] int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^2
*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/(-c^2)^(1/
2)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\,ad^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}bc}{cd^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sqr
t(d*x - c)*b*c)/(c*d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.18

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bcG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c
**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), (
)), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c} + \frac{\sqrt{d^2x^2-c^2}b}{d^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2 - c^2)*b/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqrt
(d*x - c)*b/d^2
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{b\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{d^2} - \frac{a\,\sqrt{-c}\,\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}$$

```
[In] int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (b*(c + d*x)^(1/2)*(d*x - c)^(1/2))/d^2 - (a*(-c)^(1/2)*(log(((c + d*x)^(1/
2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2)
```

3.364 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}} dx$

Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}} dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{2b\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^
2/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {465, 65, 223, 212}

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}} dx = \frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{2b\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + b\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d} \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d} \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2x+\operatorname{csgn}(d)d\sqrt{d^2x^2-c^2}\,a\right)\operatorname{csgn}(d)}{c^2\sqrt{d^2x^2-c^2}\,xd}$	97
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{c^2x\sqrt{dx-c}}+\frac{b\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)\sqrt{(dx-c)(dx+c)}}{\sqrt{d^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	98

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(
d))*b*c^2*x+csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^2*x^2-c^2)^(1/2)/x/
d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-ad^2x-\sqrt{dx+c}\sqrt{dx-c}\,cad}{c^2dx}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c)
*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.89

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3
/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4
, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d} + \frac{\sqrt{d^2x^2-c^2}a}{c^2x}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + sqrt(d^2*x^2 - c^2)*a/(c^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}} + \frac{a\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{c^2\,x}$$

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
[Out] (4*b*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2)
 - c^(1/2)))))/(-d^2)^(1/2) + (a*(c + d*x)^(1/2)*(d*x - c)^(1/2))/(c^2*x)
```

3.365 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{(2bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

```
[Out] 1/2*(a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+1/2*a*(d*x-c)
^(1/2)*(d*x+c)^(1/2)/c^2/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {465, 94, 211}

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{(ad^2+2bc^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2} + \frac{1}{2}\left(2b+\frac{ad^2}{c^2}\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2} + \frac{1}{2}\left(d\left(2b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right) \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2} + \frac{(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\frac{ac\sqrt{-c+dx}\sqrt{c+dx}}{x^2} + 2(2bc^2+ad^2)\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{2c^3}$$

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] ((a*c*Sqrt[-c + d*x]*Sqrt[c + d*x])/x^2 + 2*(2*b*c^2 + a*d^2)*ArcTan[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/(2*c^3)
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

method	result	size
risch	$-\frac{a(-dx+c)\sqrt{dx+c}}{2c^2x^2\sqrt{dx-c}}-\frac{(a\,d^2+2b\,c^2)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)\sqrt{(dx-c)(dx+c)}}{2c^2\sqrt{-c^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	123
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^2x^2+2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2x^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\right)}{2c^2\sqrt{d^2x^2-c^2}\,x^2\sqrt{-c^2}}$	158

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(-d*x+c)*(d*x+c)^(1/2)/c^2/x^2/(d*x-c)^(1/2)-1/2/c^2*(a*d^2+2*b*c^2)
/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*(d
*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\,(2\,bc^2+ad^2)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}ac}{2\,c^3x^2}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)
 + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c} - \frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3} + \frac{\sqrt{d^2x^2-c^2}a}{2\,c^2x^2}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arcsin(c/(d*abs(x)))/c - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c^3 + 1/2*sqrt(d
^2*x^2 - c^2)*a/(c^2*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{\frac{(2\,bc^2d+ad^3)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3} + \frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 +
 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) -
sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 457, normalized size of antiderivative = 6.01

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}} - \frac{b\,\sqrt{-c}\,\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}} - \frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}} - \frac{\frac{a\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{a\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}} + \frac{a\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (a*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))/(2*c^(9/2)) - (b*(-c)^(1/2)*(log(((c + d*x)^(1/2) - c^(1/2)
)^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2) - c^(1/2))
/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2) - (a*(-c)^(3/2)*d^2*log(((c + d*
x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - ((a*(-c)
^(3/2)*d^2)/(32*c^(9/2)) + (a*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)
/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*a*(-c)^(3/2)*d^2*((c +
 d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*
x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2)
- c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*d^2*((c + d*x)^(1/2) -
c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

3.366 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}$$

```
[Out] 1/3*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^3+1/3*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/c^4/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {465, 97}

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx=\frac{\sqrt{dx-c}\sqrt{c+dx}(2ad^2+3bc^2)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3} + \frac{1}{3}\left(3b+\frac{2ad^2}{c^2}\right)\int \frac{1}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\ &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3} + \frac{(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{-c+dx}\sqrt{c+dx}(3bc^2x^2+a(c^2+2d^2x^2))}{3c^4x^3}$$

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(3*c^4*x
^3)
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,(2a\,d^2x^2+3b\,c^2x^2+c^2a)}{3c^4x^3}$	49
default	$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\operatorname{csgn}(d)^2\,(2a\,d^2x^2+3b\,c^2x^2+c^2a)}{3c^4x^3}$	53
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\,(2a\,d^2x^2+3b\,c^2x^2+c^2a)}{3x^3c^4\sqrt{dx-c}}$	55

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/c^4/x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{(3\,bc^2d+2\,ad^3)x^3+(ac^2+(3\,bc^2+2\,ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^4x^3}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.27

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{bdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} - \frac{ibdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2
, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2))
, ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b
*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d^2x^2-c^2}b}{c^2x} + \frac{2\sqrt{d^2x^2-c^2}ad^2}{3\,c^4x} + \frac{\sqrt{d^2x^2-c^2}a}{3\,c^2x^3}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/3
*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(63) = 126$.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c
) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^
4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c}+\frac{x^2\,\left(3\,b\,c^3+2\,a\,c\,d^2\right)}{3\,c^4}+\frac{x^3\,\left(3\,b\,c^2\,d+2\,a\,d^3\right)}{3\,c^4}+\frac{a\,d\,x}{3\,c^2}\right)}{x^3\,\sqrt{c+d\,x}}$$

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*a
*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^(1/2))
```

3.367 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 123

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4}+\frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{d^2(4bc^2+3ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}$$

```
[Out] 1/8*d^2*(3*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/4*a*(
d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^4+1/8*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x
+c)^(1/2)/c^4/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {465, 105, 12, 94, 211}

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{d^2(3ad^2+4bc^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5} + \frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{3ad^2}{c^2}\right)\int\frac{1}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{(4bc^2+3ad^2)\int\frac{d^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8c^4}$$

$$
\begin{aligned}
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} \\
&\quad + \frac{(d^2(4bc^2+3ad^2))\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8c^4} \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} \\
&\quad + \frac{(d^3(4bc^2+3ad^2))\,\mathrm{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)}{8c^4} \\
&= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} \\
&\quad + \frac{d^2(4bc^2+3ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\
&= \frac{c\sqrt{-c+dx}\sqrt{c+dx}(2ac^2+4bc^2x^2+3ad^2x^2)+2d^2(4bc^2+3ad^2)\,x^4\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{8c^5x^4}
\end{aligned}
$$

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2) + 2*d
^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*c^5*x^4
)
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{dx+c}\,(-dx+c)\left(3a\,d^2x^2+4b\,c^2x^2+2c^2a\right)}{8c^4x^4\sqrt{dx-c}}-\frac{d^2\left(3a\,d^2+4b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)\sqrt{(dx-c)(dx+c)}}{8c^4\sqrt{-c^2}\,\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4+4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4-3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2x^2-4\sqrt{}\right.}{8c^4\sqrt{d^2x^2-c^2}\,x^4\sqrt{-c^2}}$

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x+c)^(1/2)*(-d*x+c)*(3*a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/c^4/x^4/(d*x-
c)^(1/2)-1/8*d^2*(3*a*d^2+4*b*c^2)/c^4/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/
2)*(d^2*x^2-c^2)^(1/2))/x)*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{2\,(4\,bc^2d^2+3\,ad^4)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+(2\,ac^3+(4\,bc^3+3\,acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^5x^4}$$

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x -
c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))
/(c^5*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}-\frac{3\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^5}+\frac{\sqrt{d^2x^2-c^2}b}{2\,c^2x^2}+\frac{3\,\sqrt{d^2x^2-c^2}ad^2}{8\,c^4x^2}+\frac{\sqrt{d^2x^2-c^2}a}{4\,c^2x^4}$$

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c^3 - 3/8*a*d^4*arcsin(c/(d*abs(x)))/c^5 +
1/2*sqrt(d^2*x^2 - c^2)*b/(c^2*x^2) + 3/8*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x^
2) + 1/4*sqrt(d^2*x^2 - c^2)*a/(c^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.64

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = -\frac{\frac{(4\,bc^2d^3+3\,ad^5)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} + \frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/
c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(
d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))
^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4
*c^4))/d
```

Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 1005, normalized size of antiderivative = 8.17

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{3\,a\sqrt{-c}\,d^4\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{11/2}} - \frac{\frac{b\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{b\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,b\,(-c)^{3/2}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}$$

$$-\frac{\frac{a\sqrt{-c}\,d^4}{1024\,c^{11/2}}-\frac{3\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{53\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{87\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{657\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{1024\,c^{11/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}\cdots}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{6\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}\cdots}$$

$$-\frac{b\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}}-\frac{3\,a\sqrt{-c}\,d^4\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{8\,c^{11/2}}$$

$$+\frac{b\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}}-\frac{7\,a\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{256\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

$$+\frac{a\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{1024\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{b\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

[In] int((a + b*x^2)/(x^5*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)

[Out] (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(11/2)) - ((b*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) - ((a*(-c)^(1/2)*d^4)/(1024*c^(11/2)) - (3*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(128*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (53*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) + (87*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) + (657*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (121*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(8*c^(11/2)) + (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(9/2)) - (7*a*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*(-c)^(1/2)*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (a*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(1024*(-c)^(1/2)*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) + (b*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)

3.368 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 161

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} +\frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6}+\frac{3c^2(5bc^2+4ad^2)\,\text{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7}$$

```
[Out] 3/4*c^2*(4*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7-1/4*(4*a
*d^2+5*b*c^2)*x^3/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*b*x^5/d^2/(d*x-c)^(1/
2)/(d*x+c)^(1/2)+3/8*(4*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {471, 100, 21, 92, 12, 65, 223, 212}

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3c^2(4ad^2+5bc^2)\,\text{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7} +\frac{3x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+5bc^2)}{8d^6}-\frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/4*((5*b*c^2 + 4*a*d^2)*x^3)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)
/(4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c +
d*x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
d*x]/Sqrt[c + d*x]])/(4*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{1}{4}\left(-4a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{x^2(-3c^2-3cdx)}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{4cd^2} \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{(3(5bc^2+4ad^2))\int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{4d^4} \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{(3(5bc^2+4ad^2))\int \frac{c^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^6} \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{(3c^2(5bc^2+4ad^2))\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8d^6} \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} \\
&\quad + \frac{(3c^2(5bc^2+4ad^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{4d^7}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} \\
&\quad + \frac{(3c^2(5bc^2+4ad^2))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7} \\
&= -\frac{(5bc^2+4ad^2)\,x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{3(5bc^2+4ad^2)\,x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{3c^2(5bc^2+4ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$
\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{4ad^3x(-3c^2+d^2x^2)+bdx(-15c^4+5c^2d^2x^2+2d^4x^4)+6c^2(5bc^2+4ad^2)\,\sqrt{}}{8d^7\sqrt{-c+dx}\sqrt{c+dx}}
$$

```
[In] Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)
 + 6*c^2*(5*b*c^2 + 4*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x]*ArcTanh[Sqrt[c +
d*x]/Sqrt[-c + d*x]])/(8*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{x(2b\,d^2x^2+4a\,d^2+7b\,c^2)(-dx+c)\sqrt{dx+c}}{8d^6\sqrt{dx-c}}+\frac{c^2\left(\frac{12a\,d^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}+\frac{15b\,c^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}-\frac{4(a\,d^2+b\,c^2)\sqrt{d^2(x-}}{d^2(x-\frac{c}{d}}\right.}{8d^6\sqrt{dx-c}\sqrt{dx}}$
default	$\frac{\sqrt{dx-c}\left(-2\ \text{csgn}(d)b\,d^5x^5\sqrt{d^2x^2-c^2}-4\ \text{csgn}(d)a\,d^5x^3\sqrt{d^2x^2-c^2}-5\ \text{csgn}(d)b\,c^2d^3x^3\sqrt{d^2x^2-c^2}-12\ln\left(\left(\sqrt{d^2x^2-c^2}\ \text{csgn}(d)+dx\right)\right.\right.}{}$

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(2*b*d^2*x^2+4*a*d^2+7*b*c^2)*(-d*x+c)*(d*x+c)^(1/2)/d^6/(d*x-c)^(1/
2)+1/8*c^2/d^6*(12*a*d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1
/2)+15*b*c^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)-4*(a*d^2
+b*c^2)/d^2/(x-c/d)*(d^2*(x-c/d)^2+2*c*d*(x-c/d))^(1/2)-4*(a*d^2+b*c^2)/d^2
```

```
/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-
c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.18

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{8\,bc^6+8\,ac^4d^2-8\,(bc^4d^2+ac^2d^4)x^2+(2\,bd^5x^5+(5\,bc^2d^3+4\,ad^5)x^3-3\,(5 \cdots}{}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5 +
 (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)
*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*x^2
)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{bx^5}{4\,\sqrt{d^2x^2-c^2}d^2}+\frac{5\,bc^2x^3}{8\,\sqrt{d^2x^2-c^2}d^4} \\ &+ \frac{ax^3}{2\,\sqrt{d^2x^2-c^2}d^2}-\frac{15\,bc^4x}{8\,\sqrt{d^2x^2-c^2}d^6}-\frac{3\,ac^2x}{2\,\sqrt{d^2x^2-c^2}d^4} \\ &+ \frac{15\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{8\,d^7}+\frac{3\,ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{2\,d^5}\end{aligned}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^
4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2
)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)
*d)/d^5
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.33

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^7}-\frac{5\,bc}{d^7}\right)+\frac{25\,bc^2d^{35}+4\,ad^{37}}{d^{42}}\right)-\frac{35\,bc^3d^{35}+12\,acd^{37}}{d^{42}}\right)}{8\,\sqrt{dx-c}}$$
$$-\frac{3\,(5\,bc^4+4\,ac^2d^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{8\,d^7}-\frac{2\,(bc^5+ac^3d^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^7}$$

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^7 - 5*b*c/d^7) + (25*b*c^2*d^35
 + 4*a*d^37)/d^42) - (35*b*c^3*d^35 + 12*a*c*d^37)/d^42)*(d*x + c) + 2*(7*b
*c^4*d^35 + 2*a*c^2*d^37)/d^42)*sqrt(d*x + c)/sqrt(d*x - c) - 3/8*(5*b*c^4
+ 4*a*c^2*d^2)*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^7 - 2*(b*c^5 + a*c^
3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{x^4\,(b\,x^2+a)}{(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.369 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 115

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx = -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4bc^2+3ad^2)\,\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}$$

```
[Out] -1/3*(3*a*d^2+4*b*c^2)*x^2/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*b*x^4/d^2/(d
*x-c)^(1/2)/(d*x+c)^(1/2)+2/3*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)
/d^6
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {471, 100, 21, 75}

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx = \frac{2\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/3*((4*b*c^2 + 3*a*d^2)*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)
/(3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{1}{3}\left(-3a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{x(-2c^2-2cdx)}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{3cd^2} \\
&= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(2\left(3a+\frac{4bc^2}{d^2}\right)\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{3d^2} \\
&= -\frac{(4bc^2+3ad^2)\,x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-8bc^4-6ac^2d^2+4bc^2d^2x^2+3ad^4x^2+bd^4x^4}{3d^6\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[In] Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^6
*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{-b\,d^4x^4-3a\,d^4x^2-4b\,c^2d^2x^2+6a\,c^2d^2+8b\,c^4}{3d^6\sqrt{dx-c}\,\sqrt{dx+c}}$	68
default	$\frac{\sqrt{dx-c}\,(-b\,d^4x^4-3a\,d^4x^2-4b\,c^2d^2x^2+6a\,c^2d^2+8b\,c^4)}{3(-dx+c)d^6\sqrt{dx+c}}$	76
risch	$-\frac{(b\,d^2x^2+3a\,d^2+5b\,c^2)(-dx+c)\sqrt{dx+c}}{3d^6\sqrt{dx-c}}-\frac{c^2\,(a\,d^2+b\,c^2)\,\sqrt{(dx-c)(dx+c)}}{d^6\,\sqrt{-(dx+c)(-dx+c)}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	115

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^
2+6*a*c^2*d^2+8*b*c^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(bd^4x^4-8\,bc^4-6\,ac^2d^2+(4\,bc^2d^2+3\,ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3\,(d^8x^2-c^2d^6)}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt(
d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^4}{3\sqrt{d^2x^2-c^2}d^2} + \frac{4\,bc^2x^2}{3\sqrt{d^2x^2-c^2}d^4} + \frac{ax^2}{\sqrt{d^2x^2-c^2}d^2} - \frac{8\,bc^4}{3\sqrt{d^2x^2-c^2}d^6} - \frac{2\,ac^2}{\sqrt{d^2x^2-c^2}d^4}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d^
4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6)
- 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.74

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\left(2\,(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6} - \frac{4\,bc}{d^6}\right) + \frac{10\,bc^2d^{24}+3\,ad^{26}}{d^{30}}\right) - \frac{3\,(9\,bc^3d^{24}+5\,acd^{26})}{d^{30}}\right)}{6\sqrt{dx-c}} + \frac{2\,(b^2c^8+2\,abc^6d^2+a^2c^4d^4)}{\left(bc^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + 2\,bc^5 + 2\,ac^3d^2\right)d^6}$$

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 1/6*(2*(d*x + c)*((d*x + c)*((d*x + c)*b/d^6 - 4*b*c/d^6) + (10*b*c^2*d^24
+ 3*a*d^26)/d^30) - 3*(9*b*c^3*d^24 + 5*a*c*d^26)/d^30)*sqrt(d*x + c)/sqrt(
d*x - c) + 2*(b^2*c^8 + 2*a*b*c^6*d^2 + a^2*c^4*d^4)/((b*c^4*(sqrt(d*x + c)
 - sqrt(d*x - c))^2 + a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^5
 + 2*a*c^3*d^2)*d^6)
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{d\,x-c}\,\left(\frac{x^2\,\left(4\,b\,c^2\,d^2+3\,a\,d^4\right)}{3\,d^7}-\frac{8\,b\,c^4+6\,a\,c^2\,d^2}{3\,d^7}+\frac{b\,x^4}{3\,d^3}\right)}{x\,\sqrt{c+d\,x}-\frac{c\,\sqrt{c+d\,x}}{d}}$$

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((x^2*(3*a*d^4 + 4*b*c^2*d^2))/(3*d^7) - (8*b*c^4 + 6*a*c^
2*d^2)/(3*d^7) + (b*x^4)/(3*d^3)))/(x*(c + d*x)^(1/2) - (c*(c + d*x)^(1/2))
/d)
```

3.370 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{(3bc^2+2ad^2)\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}$$

```
[Out] (2*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5-1/2*c*(2*a*d^2+3
*b*c^2)/d^5/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*b*x^3/d^2/(d*x-c)^(1/2)/(d*x+c)
^(1/2)-1/2*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/2)/d^5/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {471, 91, 12, 79, 65, 223, 212}

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(2ad^2+3bc^2)\operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5} - \frac{\sqrt{dx-c}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*(c*(3*b*c^2 + 2*a*d^2))/(d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(
2*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/
```

```
(2*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c
+ d*x]])/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{1}{2}\left(-2a-\frac{3bc^2}{d^2}\right)\int\frac{x^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(-2a-\frac{3bc^2}{d^2}\right)\int\frac{cd^2x}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{2cd^3}\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\int\frac{x}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{2d^3}\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2d^4}\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}\\
&\quad + \frac{(3bc^2+2ad^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d^5}\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}\\
&= -\frac{c(3bc^2+2ad^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad - \frac{(3bc^2+2ad^2)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + \frac{(3bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-3bc^2dx-2ad^3x+bd^3x^3+2(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}\operatorname{arctanh}\left(\frac{\sqrt{c+}}{\sqrt{-c}}\right.}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[In] Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + 2*(3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x
]*Sqrt[c + d*x]*ArcTanh[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^5*Sqrt[-c + d*x
]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{bx(-dx+c)\sqrt{dx+c}}{2d^4\sqrt{dx-c}}+\frac{\left(\frac{2a\,d^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}+\frac{3b\,c^2\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-c^2}\right)}{\sqrt{d^2}}-\frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\frac{c}{d}\right)}}{d^2\left(x-\frac{c}{d}\right)}-\frac{\left(a\,d^2+b\,c^2\right)\sqrt{}}{}\right.}{2d^4\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-\operatorname{csgn}(d)b\,d^3x^3\sqrt{d^2x^2-c^2}-2\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^4x^2-3\ln\left(\left(\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2d^2x\right.}{2(}$

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*x*(-d*x+c)*(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)+1/2/d^4*(2*a*d^2*ln(x*d^2
/(d^2)^(1/2)+(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)+3*b*c^2*ln(x*d^2/(d^2)^(1/2)+
(d^2*x^2-c^2)^(1/2))/(d^2)^(1/2)-(a*d^2+b*c^2)/d^2/(x-c/d)*(d^2*(x-c/d)^2+2
*c*d*(x-c/d))^(1/2)-(a*d^2+b*c^2)/d^2/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))
^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{2\,bc^4+2\,ac^2d^2-2\,(bc^2d^2+ad^4)x^2+(bd^3x^3-(3\,bc^2d+2\,ad^3)x)\sqrt{dx+c}\sqrt{d}}{2\,(d^7x^2}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b*
c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2 -
```

```
(3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^7*
x^2 - c^2*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^3}{2\sqrt{d^2x^2-c^2}d^2} - \frac{3\,bc^2x}{2\sqrt{d^2x^2-c^2}d^4} - \frac{ax}{\sqrt{d^2x^2-c^2}d^2} + \frac{3\,bc^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{2\,d^5} + \frac{a\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}d\right)}{d^3}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4)
 - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
 c^2)*d)/d^5 + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{dx+c}\left((dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{3\,bc}{d^5}\right)+\frac{bc^2d^{15}-ad^{17}}{d^{20}}\right)}{2\sqrt{dx-c}} - \frac{(3\,bc^2+2\,ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{2\,d^5} - \frac{2\,(bc^3+acd^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^5}$$

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(d*x + c)*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^15 -
a*d^17)/d^20)/sqrt(d*x - c) - 1/2*(3*b*c^2 + 2*a*d^2)*log((sqrt(d*x + c) -
sqrt(d*x - c))^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c
))^2 + 2*c)*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{x^2\,(b\,x^2+a)}{(c+d\,x)^{3/2}\,(d\,x-c)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.371 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{(2bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{c^2d^4}$$

```
[Out] -(a/c^2+b/d^2)*x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2)+(a*d^2+2*b*c^2)*(d*x-c)^(1/2
)*(d*x+c)^(1/2)/c^2/d^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {469, 75}

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+2bc^2)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
+ 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m
, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !In
tegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p
 + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}} - \left(-\frac{a}{c^2}-\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx \\ &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\sqrt{-c+dx}\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-2bc^2-ad^2+bd^2x^2}{d^4\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[In] Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{-b\,d^2x^2+a\,d^2+2b\,c^2}{d^4\sqrt{dx-c}\,\sqrt{dx+c}}$	43
default	$\frac{\sqrt{dx-c}\,\left(-b\,d^2x^2+a\,d^2+2b\,c^2\right)}{\sqrt{dx+c}\,d^4(-dx+c)}$	50
risch	$-\frac{b(-dx+c)\sqrt{dx+c}}{d^4\sqrt{dx-c}}-\frac{\left(a\,d^2+b\,c^2\right)\sqrt{(dx-c)(dx+c)}}{d^4\sqrt{-(dx+c)(-dx+c)}\,\sqrt{dx-c}\,\sqrt{dx+c}}$	92

```
[In] int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-b*d^2*x^2+a*d^2+2*b*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(bd^2x^2-2\,bc^2-ad^2)\sqrt{dx+c}\sqrt{dx-c}}{d^6x^2-c^2d^4}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] (b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d^
4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^2}{\sqrt{d^2x^2-c^2}d^2} - \frac{2\,bc^2}{\sqrt{d^2x^2-c^2}d^4} - \frac{a}{\sqrt{d^2x^2-c^2}d^2}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(sq
rt(d^2*x^2 - c^2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(68) = 136.

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\sqrt{dx+c}\left(\frac{2\,(dx+c)b}{d^4}-\frac{5\,bc^2d^8+ad^{10}}{cd^{12}}\right)}{2\,\sqrt{dx-c}} + \frac{2\,(b^2c^4+2\,abc^2d^2+a^2d^4)}{\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^3+2\,acd^2\right)d^4}$$

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^10)/(c*d^12))/sqr
t(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(sqrt(d*x + c) -
 sqrt(d*x - c))^2 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^3 + 2*a
*c*d^2)*d^4)
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a\,d^2\,\sqrt{d\,x-c}+2\,b\,c^2\,\sqrt{d\,x-c}-b\,d^2\,x^2\,\sqrt{d\,x-c}}{d^4\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

```
[In] int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/2
))/(d^4*(c + d*x)^(1/2)*(c - d*x))
```

3.372 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=-\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

```
[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3-(a/c^2+b/d^2)*x/(d*x-c)^(1/2)/
(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {394, 65, 223, 212}

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 394

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*c - a1*a2*d))*x*(a1
 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*b1*b2*n*(p + 1))),
 x] - Dist[(a1*a2*d - b1*b2*c*(n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int
[(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1,
b1, a2, b2, c, d, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (LtQ
[p, -1] || ILtQ[1/n + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{b\int\frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-\frac{2(bc^2d+ad^3)x}{c^2\sqrt{-c+dx}\sqrt{c+dx}}+4b\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2d^3}$$

```
[In] Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*(b*c^2*d + a*d^3)*x)/(c^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*b*ArcTanh[
Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*d^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.63

method	result
default	$\frac{\sqrt{dx-c}\left(-\ln\left(\left(\sqrt{-(dx+c)(-dx+c)}\ \operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)b\,c^2d^2x^2+\sqrt{d^2x^2-c^2}\ \operatorname{csgn}(d)d^3ax+\sqrt{d^2x^2-c^2}\ \operatorname{csgn}(d)db\,c^2x+\ln\left(\left(\sqrt{-}\right.\right.\right.}{(-dx+c)\sqrt{d^2x^2-c^2}\,c^2d^3\sqrt{dx+c}}$

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)*(-ln(((-(d*x+c)*(-d*x+c))^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^2*d
^2*x^2+(d^2*x^2-c^2)^(1/2)*csgn(d)*d^3*a*x+(d^2*x^2-c^2)^(1/2)*csgn(d)*d*b*
c^2*x+ln(((-(d*x+c)*(-d*x+c))^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/(-
d*x+c)/(d^2*x^2-c^2)^(1/2)/c^2/d^3/(d*x+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bc^4+ac^2d^2-(bc^2d+ad^3)\sqrt{dx+c}\sqrt{dx-c}x-(bc^2d^2+ad^4)x^2-(bc^2d^2x^2}{c^2d^5x^2-c^4d^3}$$

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*c
^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sqrt
(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{ax}{\sqrt{d^2x^2-c^2}c^2} - \frac{bx}{\sqrt{d^2x^2-c^2}d^2} + \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}d\right)}{d^3}$$

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.79

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3} - \frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)cd^3} - \frac{\left(bc^2d^3+ad^5\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^2d^6}$$

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] -b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d*x
 + c)/(sqrt(d*x - c)*c^2*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{\left(c+d\,x\right)^{3/2}\,\left(d\,x-c\right)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.373 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{a\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3}$$

```
[Out] -a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+(-a/c^2-b/d^2)/(d*x-c)^(1/2)/(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {469, 94, 211}

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{a\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3} - \frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b1*b2*
c - a1*a2*d))*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p
 + 1)/(a1*a2*b1*b2*e*n*(p + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m +
 n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p
+ 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m
, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !In
tegerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p
 + 1/2, 0] && LeQ[-1, m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{a\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(ad)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-\frac{2(bc^3+acd^2)}{d^2\sqrt{-c+dx}\sqrt{c+dx}}+4a\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^3}$$

```
[In] Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*(b*c^3 + a*c*d^2))/(d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 4*a*ArcTan[Sqr
t[c + d*x]/Sqrt[-c + d*x]])/(2*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(58) = 116$.

Time = 4.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.98

method	result
default	$\frac{\sqrt{dx-c}\left(-\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^2+\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2d^2+\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2+b\,c^2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{c^2(-dx+c)\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,d^2\sqrt{dx+c}}$

```
[In] int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x-c)^(1/2)/c^2*(-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^4*x
^2+ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+(-c^2)^(1/2)*(
d^2*x^2-c^2)^(1/2)*a*d^2+b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-d*x+c)/(
-c^2)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^3+acd^2)\sqrt{dx+c}\sqrt{dx-c}+2\left(ad^4x^2-ac^2d^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2)
*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c^3} - \frac{a}{\sqrt{d^2x^2-c^2}c^2} - \frac{b}{\sqrt{d^2x^2-c^2}d^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] a*arcsin(c/(d*abs(x)))/c^3 - a/(sqrt(d^2*x^2 - c^2)*c^2) - b/(sqrt(d^2*x^2
- c^2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3} - \frac{(bc^2+ad^2)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^3d^2} + \frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d^
2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.374 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 67

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(bc^2+2ad^2)\,x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[Out] a/c^2/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)-(2*a*d^2+b*c^2)*x/c^4/(d*x-c)^(1/2)/(d*
x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {465, 39}

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} + \left(b+\frac{2ad^2}{c^2}\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\ &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(bc^2+2ad^2)\,x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{-bc^2x^2+a(c^2-2d^2x^2)}{c^4x\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[In] Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{-2a\,d^2x^2-b\,c^2x^2+c^2a}{c^4x\sqrt{dx-c}\,\sqrt{dx+c}}$	48
default	$\frac{(2a\,d^2x^2+b\,c^2x^2-c^2a)\sqrt{dx-c}\,\operatorname{csgn}(d)^2}{c^4(-dx+c)x\sqrt{dx+c}}$	60
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{c^4x\sqrt{dx-c}}-\frac{(a\,d^2+b\,c^2)x\sqrt{(dx-c)(dx+c)}}{\sqrt{-(dx+c)(-dx+c)}\,c^4\sqrt{dx-c}\,\sqrt{dx+c}}$	95

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/c^4/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-2*a*d^2*x^2-b*c^2*x^2+a*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2d^2+2\,ad^4)x^3-(ac^2d-(bc^2d+2\,ad^3)x^2)\sqrt{dx+c}\sqrt{dx-c}-(bc^4+2\,ac^2d^2)x}{c^4d^3x^3-c^6dx}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] -((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x
+ c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{bx}{\sqrt{d^2x^2-c^2}c^2}-\frac{2\,ad^2x}{\sqrt{d^2x^2-c^2}c^4}+\frac{a}{\sqrt{d^2x^2-c^2}c^2x}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] -b*x/(sqrt(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) + a/(s
qrt(d^2*x^2 - c^2)*c^2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(59) = 118$.

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2+ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}c^4d} - \frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + 4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + 4\,bc^4 + 12\,a\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6 + 2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + 4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + 8\,c^3\right)c^3d}$$

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(d
*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*a*
c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{2\,a\,d^2\,x^2\,\sqrt{d\,x-c} - a\,c^2\,\sqrt{d\,x-c} + b\,c^2\,x^2\,\sqrt{d\,x-c}}{c^4\,x\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (2*a*d^2*x^2*(d*x - c)^(1/2) - a*c^2*(d*x - c)^(1/2) + b*c^2*x^2*(d*x - c)^
(1/2))/(c^4*x*(c + d*x)^(1/2)*(c - d*x))
```

3.375 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5}$$

```
[Out] -1/2*(3*a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/2*(-3*a*
d^2-2*b*c^2)/c^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*a/c^2/x^2/(d*x-c)^(1/2)/(d
*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {465, 106, 21, 94, 211}

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(3ad^2+2bc^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5} - \frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*(2*b*c^2 + 3*a*d^2)/(c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*
Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(2*c^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{1}{2}\left(2b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(-2b-\frac{3ad^2}{c^2}\right)\int \frac{cd+d^2x}{x\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{2c^2d}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(d(2bc^2+3ad^2))\,\mathrm{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(2bc^2+3ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\frac{-2bc^3x^2+a(c^3-3cd^2x^2)}{x^2\sqrt{-c+dx}\sqrt{c+dx}} + (4bc^2+6ad^2)\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{2c^5}$$

```
[In] Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*b*c^3*x^2 + a*(c^3 - 3*c*d^2*x^2))/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
+ (4*b*c^2 + 6*a*d^2)*ArcTan[Sqrt[c + d*x]/Sqrt[-c + d*x]])/(2*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(99) = 198.

Time = 4.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.04

method	result
risch	$\frac{a(-dx+c)\sqrt{dx+c}}{2c^4x^2\sqrt{dx-c}} - \frac{\left(-\frac{\left(3a\,d^2+2b\,c^2\right)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{\sqrt{-c^2}} + \frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\frac{c}{d}\right)}}{dc\left(x-\frac{c}{d}\right)} - \frac{\left(a\,d^2+b\,c^2\right)\sqrt{d^2\left(x+\frac{c}{d}\right)^2-2c}}{dc\left(x+\frac{c}{d}\right)}\right.}{2c^4\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^4x^4-2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^2x^4+3\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,c^2d^2\right.}{2c^4(-dx+c)\sqrt{-c^2}\,x^2\sqrt{d^2}}$

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*(-d*x+c)*(d*x+c)^(1/2)/c^4/x^2/(d*x-c)^(1/2)-1/2/c^4*(-(3*a*d^2+2*b*c
^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)+(a*d^2+b
*c^2)/d/c/(x-c/d)*(d^2*(x-c/d)^2+2*c*d*(x-c/d))^(1/2)-(a*d^2+b*c^2)/d/c/(x+
c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(
1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(ac^3-(2\,bc^3+3\,acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}-2\,((2\,bc^2d^2+3\,ad^4)x^4-(2}{2\,(c^5d^2x^4-c^7x^2)}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*((
2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x - sq
rt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c^3}+\frac{3\,ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5} - \frac{b}{\sqrt{d^2x^2-c^2}c^2}-\frac{3\,ad^2}{2\,\sqrt{d^2x^2-c^2}c^4}+\frac{a}{2\,\sqrt{d^2x^2-c^2}c^2x^2}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*arcsin(c/(d*abs(x)))/c^3 + 3/2*a*d^2*arcsin(c/(d*abs(x)))/c^5 - b/(sqrt(d
^2*x^2 - c^2)*c^2) - 3/2*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4) + 1/2*a/(sqrt(d^2*
x^2 - c^2)*c^2*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.80

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(2\,bc^2+3\,ad^2)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} - \frac{(bc^2+ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}c^5} + \frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^4} + \frac{2\left(ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^4}$$

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 - 1
/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/((
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*
x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x^3\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.376 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{2d^2(3bc^2+4ad^2)\,x}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[Out] 1/3*a/c^2/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*(4*a*d^2+3*b*c^2)/c^4/x/(d*x-
c)^(1/2)/(d*x+c)^(1/2)-2/3*d^2*(4*a*d^2+3*b*c^2)*x/c^6/(d*x-c)^(1/2)/(d*x+c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {465, 105, 12, 39}

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{1}{3}\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{1}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{2d^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} \\
&\quad + \frac{\left(2d^2\left(3b+\frac{4ad^2}{c^2}\right)\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{2d^2(3bc^2+4ad^2)\,x}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3bc^2x^2(c^2-2d^2x^2)+a(c^4+4c^2d^2x^2-8d^4x^4)}{3c^6x^3\sqrt{-c+dx}\sqrt{c+dx}}$$

```
[In] Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^
6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4}{3c^6x^3\sqrt{dx-c}\,\sqrt{dx+c}}$	73
default	$-\frac{\sqrt{dx-c}\;\operatorname{csgn}(d)^2\left(-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4\right)}{3c^6(-dx+c)x^3\sqrt{dx+c}}$	85
risch	$\frac{\sqrt{dx+c}\,(-dx+c)\left(5a\,d^2x^2+3b\,c^2x^2+c^2a\right)}{3c^6x^3\sqrt{dx-c}}-\frac{d^2\left(a\,d^2+b\,c^2\right)x\sqrt{(dx-c)(dx+c)}}{\sqrt{-(dx+c)(-dx+c)}\,c^6\sqrt{dx-c}\,\sqrt{dx+c}}$	122

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c^6/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c
^2*d^2*x^2+3*b*c^4*x^2+a*c^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\,(3\,bc^2d^3+4\,ad^5)x^5-2\,(3\,bc^4d+4\,ac^2d^3)x^3-(ac^4-2\,(3\,bc^2d^2+4\,ad^4)x^4+(3\,bc^4+4\,ac^2d^2)x^2)\sqrt{dx+c}}{3\,(c^6d^2x^5-c^8x^3)}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a*
c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2\,bd^2x}{\sqrt{d^2x^2-c^2}c^4} - \frac{8\,ad^4x}{3\,\sqrt{d^2x^2-c^2}c^6} + \frac{b}{\sqrt{d^2x^2-c^2}c^2x} + \frac{4\,ad^2}{3\,\sqrt{d^2x^2-c^2}c^4x} + \frac{a}{3\,\sqrt{d^2x^2-c^2}c^2x^3}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] -2*b*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) - 8/3*a*d^4*x/(sqrt(d^2*x^2 - c^2)*c^6
) + b/(sqrt(d^2*x^2 - c^2)*c^2*x) + 4/3*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*x) +
 1/3*a/(sqrt(d^2*x^2 - c^2)*c^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(101) = 202.

Time = 0.40 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.03

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{(bc^2d+ad^3)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^6} - \frac{2\,(bc^2d+ad^3)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^5} - \frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,ac^2d^3\right.}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3c^4}$$

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*d
^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqrt(
```

```
d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 2
4*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) -
 sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x
- c))^4 + 4*c^2)^3*c^4)
```

Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx=\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c^2\,d}+\frac{x^2\,\left(3\,b\,c^4+4\,a\,c^2\,d^2\right)}{3\,c^6\,d}-\frac{x^4\,\left(6\,b\,c^2\,d^2+8\,a\,d^4\right)}{3\,c^6\,d}\right)}{x^4\,\sqrt{c+d\,x}-\frac{c\,x^3\,\sqrt{c+d\,x}}{d}}$$

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)

[Out] ((d*x - c)^(1/2)*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) - (
x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^(1/2) - (c*x^3*(c +
 d*x)^(1/2))/d)
```

3.377 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 166

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{3d^2(4bc^2+5ad^2)\arctan\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^7}$$

```
[Out] -3/8*d^2*(5*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^7-3/8*d^
2*(5*a*d^2+4*b*c^2)/c^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*a/c^2/x^4/(d*x-c)^(
1/2)/(d*x+c)^(1/2)+1/8*(5*a*d^2+4*b*c^2)/c^4/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {465, 105, 12, 106, 21, 94, 211}

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{3d^2(5ad^2+4bc^2)\arctan\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7} - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[In] Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
```

```
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{1}{4}\left(4b+\frac{5ad^2}{c^2}\right)\int\frac{1}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\
&= \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{(4bc^2+5ad^2)\int\frac{3d^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{8c^4}\\
&= \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad+\frac{(3d^2(4bc^2+5ad^2))\int\frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{8c^4}\\
&= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{(3d(4bc^2+5ad^2))\int\frac{cd+d^2x}{x\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{8c^6}\\
&= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{(3d^2(4bc^2+5ad^2))\int\frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{8c^6}\\
&= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad-\frac{(3d^3(4bc^2+5ad^2))\,\text{Subst}\left(\int\frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\sqrt{c+dx}\right)}{8c^6}\\
&= -\frac{3d^2(4bc^2+5ad^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}\\
&\quad+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{3d^2(4bc^2+5ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{\frac{4bc^3x^2(c^2-3d^2x^2)+a(2c^5+5c^3d^2x^2-15cd^4x^4)}{x^4\sqrt{-c+dx}\sqrt{c+dx}}+6d^2(4bc^2+5ad^2)\arctan\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8c^7}$$

[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]

[Out] ((4*b*c^3*x^2*(c^2 - 3*d^2*x^2) + a*(2*c^5 + 5*c^3*d^2*x^2 - 15*c*d^4*x^4)) /(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + 6*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[Sqrt [c + d*x]/Sqrt[-c + d*x]])/(8*c^7)

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.61

method	result
risch	$\frac{\sqrt{dx+c}\,(-dx+c)(7a\,d^2x^2+4b\,c^2x^2+2c^2a)}{8c^6x^4\sqrt{dx-c}}-\frac{d^2\left(-\frac{(15a\,d^2+12b\,c^2)\ln\left(\frac{-2c^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}}{x}\right)}{\sqrt{-c^2}}+\frac{4(a\,d^2+b\,c^2)\sqrt{d^2\left(x-\frac{c}{d}\right)^2+2cd\left(x-\right.}}{dc\left(x-\frac{c}{d}\right)}\right.}{8c^6\sqrt{dx-c}\,\sqrt{dx+c}}$
default	$\frac{\sqrt{dx-c}\left(-15\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)a\,d^6x^6-12\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)b\,c^2d^4x^6+15\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)\right.}{}$

[In] int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(d*x+c)^(1/2)*(-d*x+c)*(7*a*d^2*x^2+4*b*c^2*x^2+2*a*c^2)/c^6/x^4/(d*x-c)^(1/2)-1/8/c^6*d^2*(-(15*a*d^2+12*b*c^2)/(-c^2)^(1/2)*ln((-2*c^2+2*(-c^2)^ (1/2)*(d^2*x^2-c^2)^(1/2))/x)+4*(a*d^2+b*c^2)/d/c/(x-c/d)*(d^2*(x-c/d)^2+2* c*d*(x-c/d))^(1/2)-4*(a*d^2+b*c^2)/d/c/(x+c/d)*(d^2*(x+c/d)^2-2*c*d*(x+c/d))^(1/2))*((d*x-c)*(d*x+c))^(1/2)/(d*x-c)^(1/2)/(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{(2\,ac^5-3\,(4\,bc^3d^2+5\,acd^4)x^4+(4\,bc^5+5\,ac^3d^2)x^2)\sqrt{dx+c}\sqrt{dx-c}-}{8\,(c^7d^2x^6}$$

[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")

```
[Out] 1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x
^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4
*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c
^7*d^2*x^6 - c^9*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + bx^2}{x^5(-c + dx)^{3/2}(c + dx)^{3/2}}\, dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{a + bx^2}{x^5(-c + dx)^{3/2}(c + dx)^{3/2}}\, dx &= \frac{3\, bd^2 \arcsin\left(\frac{c}{d|x|}\right)}{2\, c^5} \\ &+ \frac{15\, ad^4 \arcsin\left(\frac{c}{d|x|}\right)}{8\, c^7} - \frac{3\, bd^2}{2\, \sqrt{d^2x^2 - c^2}c^4} - \frac{15\, ad^4}{8\, \sqrt{d^2x^2 - c^2}c^6} \\ &+ \frac{b}{2\, \sqrt{d^2x^2 - c^2}c^2x^2} + \frac{5\, ad^2}{8\, \sqrt{d^2x^2 - c^2}c^4x^2} + \frac{a}{4\, \sqrt{d^2x^2 - c^2}c^2x^4}\end{aligned}$$

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6)
+ 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*
x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(142) = 284$.

Time = 0.41 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.42

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3\,(4\,bc^2d^2+5\,ad^4)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7} - \frac{(bc^2d^2+ad^4)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^7} + \frac{2\,(bc^2d^2+ad^4)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6} + \frac{4\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+7\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+60\,ac^2}{}$$

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c^2
*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^
2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4
*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*
x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^2
*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \int \frac{b\,x^2+a}{x^5\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

3.378 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 40

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\sqrt{-1+cx}\sqrt{1+cx}+\arctan\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

```
[Out] arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {471, 94, 211}

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx=\arctan\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\sqrt{cx-1}\sqrt{cx+1}$$

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{-1+cx}\sqrt{1+cx} + \int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\ &= \sqrt{-1+cx}\sqrt{1+cx} + c\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right) \\ &= \sqrt{-1+cx}\sqrt{1+cx} + \tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{-1+cx}\sqrt{1+cx} + 2\arctan\left(\sqrt{\frac{-1+cx}{1+cx}}\right)$$

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTan[Sqrt[(-1 + c*x)/(1 + c*x)]]
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\sqrt{cx-1}\,\sqrt{cx+1}}{\sqrt{c^2x^2-1}}$	53

```
[In] int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((c^2*x^2-1)^(1/2)-arctan(1/(c^2*x^2-1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{cx+1}\sqrt{cx-1}+2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)$$

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.83 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.70

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{1}{c^2x^2}}\right)}{4\pi^{\frac{3}{2}}} - \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|{\frac{1}{c^2x^2}}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{c^2x^2}}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{c^2x^2}}\right)}{4\pi^{\frac{3}{2}}}$$

```
[In] integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2,
 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1, -
3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar
(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{c^2x^2-1} - \arcsin\left(\frac{1}{c|x|}\right)$$

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(c*abs(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{cx+1}\sqrt{cx-1} - 2\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)$$

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^
2)
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \sqrt{c\,x-1}\,\sqrt{c\,x+1} - \ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i} + \ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}$$

```
[In] int((c^2*x^2 + 1)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
[Out] log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i - log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i + (c*x - 1)^(1/2)*(c*x + 1)^(1/2)
```

3.379 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 53

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\sqrt{a+bx}$$

```
[Out] (c/a^2+d/b^2)*(b*x-a)^(1/2)*(b*x+a)^(1/2)/(x^(b^2*c/(a^2*d+b^2*c)))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {461}

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

```
[In] Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[
a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rule 461

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
```

```
), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\frac{c}{a^2} + \frac{d}{b^2}\right) x^{-\frac{b^2 c}{b^2 c + a^2 d}} \sqrt{-a + bx}\sqrt{a + bx}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.60

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\,x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{1-\frac{b^2x^2}{a^2}}\left(-\left((b^2c+2a^2d)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{b^2c}{2(b^2c+a^2d)},\frac{b^2c+2a^2d}{2b^2c+2a^2d},\frac{b^2x^2}{a^2}\right)\right)+b}{b^2\,(b^2c+2a^2d)\,\sqrt{-a+bx}\sqrt{a+bx}}$$

```
[In] Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]
*Sqrt[a + b*x]),x]
```

```
[Out] ((b^2*c + a^2*d)*Sqrt[1 - (b^2*x^2)/a^2]*(-((b^2*c + 2*a^2*d)*Hypergeometri
c2F1[1/2, -1/2*(b^2*c)/(b^2*c + a^2*d), (b^2*c + 2*a^2*d)/(2*b^2*c + 2*a^2*
d), (b^2*x^2)/a^2]) + b^2*d*x^2*Hypergeometric2F1[1/2, (b^2*c + 2*a^2*d)/(2
*b^2*c + 2*a^2*d), (3*b^2*c + 4*a^2*d)/(2*b^2*c + 2*a^2*d), (b^2*x^2)/a^2])
)/(b^2*(b^2*c + 2*a^2*d)*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[a
+ b*x])
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

method	result	size
gosper	$\frac{x\sqrt{bx-a}\,\sqrt{bx+a}\,(a^2d+b^2c)\,x^{-\frac{a^2d+2b^2c}{a^2d+b^2c}}}{a^2b^2}$	66

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a^2/b^2*(b*x-a)^(1/2)*(b*x+a)^(1/2)*(a^2*d+b^2*c)/(x^((a^2*d+2*b^2*c)/(a^
2*d+b^2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\sqrt{bx+a}\sqrt{bx-a}x}{a^2b^2x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}$$

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d)
/(b^2*c + a^2*d)))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 36.68 (sec) , antiderivative size = 1867, normalized size of antiderivative = 35.23

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2
)/(b*x+a)**(1/2),x)
```

```
[Out] -a**(-a**2*d/(a**2*d + b**2*c) - 2*b**2*c/(a**2*d + b**2*c))*b**(a**2*d/(a*
*2*d + b**2*c) + 2*b**2*c/(a**2*d + b**2*c) - 1)*c*meijerg(((a**2*d/(2*a**2
*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4, a**2*d/(2*a**2*d + 2*b**2*
c) + b**2*c/(a**2*d + b**2*c) + 3/4, 1), (a**2*d/(2*a**2*d + 2*b**2*c) + b*
*2*c/(a**2*d + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d
 + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) +
 1)), ((a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c), a**2*d/(2*
a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4, a**2*d/(2*a**2*d + 2*b
**2*c) + b**2*c/(a**2*d + b**2*c) + 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**
2*c/(a**2*d + b**2*c) + 3/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d
+ b**2*c) + 1), (0,)), a**2/(b**2*x**2))/(4*pi**(3/2)) - I*a**(-a**2*d/(a**
2*d + b**2*c) - 2*b**2*c/(a**2*d + b**2*c))*b**(a**2*d/(a**2*d + b**2*c) +
2*b**2*c/(a**2*d + b**2*c) - 1)*c*meijerg(((a**2*d/(2*a**2*d + 2*b**2*c) +
b**2*c/(a**2*d + b**2*c) - 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2
*d + b**2*c) - 1/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c)
, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/4, a**2*d/(2*
a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) + 1/2, 1), ()), ((a**2*d/(2*a
**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1/4, a**2*d/(2*a**2*d + 2*b*
```

```
*2*c) + b**2*c/(a**2*d + b**2*c) + 1/4), (a**2*d/(2*a**2*d + 2*b**2*c) + b*
*2*c/(a**2*d + b**2*c) - 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d
 + b**2*c), a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c), 0)), a
**2*exp_polar(2*I*pi)/(b**2*x**2))*exp(-I*pi*a**2*d/(a**2*d + b**2*c))*exp(
-2*I*pi*b**2*c/(a**2*d + b**2*c))/(4*pi**(3/2)) - a**(-a**2*d/(a**2*d + b**
2*c) - 2*b**2*c/(a**2*d + b**2*c) + 2)*b**(a**2*d/(a**2*d + b**2*c) + 2*b**
2*c/(a**2*d + b**2*c) - 3)*d*meijerg(((a**2*d/(2*a**2*d + 2*b**2*c) + b**2*
c/(a**2*d + b**2*c) - 3/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d +
b**2*c) - 1/4, 1), (a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c)
 - 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1/2, a**2
*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c))), ((a**2*d/(2*a**2*d +
 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1, a**2*d/(2*a**2*d + 2*b**2*c) + b
**2*c/(a**2*d + b**2*c) - 3/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*
d + b**2*c) - 1/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c)
- 1/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c)), (0,)), a**
2/(b**2*x**2))/(4*pi**(3/2)) - I*a**(-a**2*d/(a**2*d + b**2*c) - 2*b**2*c/(
a**2*d + b**2*c) + 2)*b**(a**2*d/(a**2*d + b**2*c) + 2*b**2*c/(a**2*d + b**
2*c) - 3)*d*meijerg(((a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*
c) - 3/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 5/4, a*
*2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1, a**2*d/(2*a**2*d
 + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 3/4, a**2*d/(2*a**2*d + 2*b**2*c)
 + b**2*c/(a**2*d + b**2*c) - 1/2, 1), ()), ((a**2*d/(2*a**2*d + 2*b**2*c)
+ b**2*c/(a**2*d + b**2*c) - 5/4, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a*
*2*d + b**2*c) - 3/4), (a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**
2*c) - 3/2, a**2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1, a*
*2*d/(2*a**2*d + 2*b**2*c) + b**2*c/(a**2*d + b**2*c) - 1, 0)), a**2*exp_po
lar(2*I*pi)/(b**2*x**2))*exp(-I*pi*a**2*d/(a**2*d + b**2*c))*exp(-2*I*pi*b*
*2*c/(a**2*d + b**2*c))/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \frac{(b^2c+a^2d)\sqrt{bx+a}\sqrt{bx-a}\,xe^{\left(-\frac{2\,b^2c\log(x)}{b^2c+a^2d}-\frac{a^2d\log(x)}{b^2c+a^2d}\right)}}{a^2b^2}$$

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c + a
^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

Giac [F]

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = \int \frac{dx^2+c}{\sqrt{bx+a}\sqrt{bx-a}\,x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}\,dx$$

[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))), x)

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.81

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx = -\frac{\frac{x\,(d\,a^4+c\,a^2\,b^2)}{a^2\,b^2}-\frac{x^3\,(d\,a^2\,b^2+c\,b^4)}{a^2\,b^2}}{x^{\frac{d\,a^2+2\,c\,b^2}{d\,a^2+c\,b^2}}\,\sqrt{a+b\,x}\,\sqrt{b\,x-a}}$$

[In] int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2)),x)

[Out] -((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))

3.380 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 36

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\frac{\sqrt{1-x}\arcsin(x)}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}$$

```
[Out] arcsin(x)*(1-x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {533, 41, 222}

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx=\frac{\sqrt{1-x}\arcsin(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

```
[In] Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-x}\int\frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}\\ &= \frac{\sqrt{1-x}\int\frac{1}{\sqrt{1-x^2}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}\\ &= \frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int\frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = -i\log\left(-x+i\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}\right)$$

```
[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
[Out] (-I)*Log[-x + I*Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]]
```

Maple [F]

$$\int\frac{1}{\sqrt{1+x}\,\sqrt{-1-\sqrt{x}}\,\sqrt{\sqrt{x}-1}}dx$$

```
[In] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(x^(1/2)-1)^(1/2),x)
[Out] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(x^(1/2)-1)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = -i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+i\,x-1}{x}\right) + i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-i\,x-1}{x}\right)$$

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"fricas")
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*
log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"maxima")
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{x+1}}\,dx$$

```
[In] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.381 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 75

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = -\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

```
[Out] -2*arctan((-b^2*x+a^2)^(1/2)/(b^2*x+a^2)^(1/2))*(-b^2*x+a^2)^(1/2)/b^2/(a-b
*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {533, 65, 223, 209}

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = -\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

```
[In] Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2-b^2x}\int\frac{1}{\sqrt{a^2-b^2x}\sqrt{a^2+b^2x}}\,dx}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}\\
&= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{2a^2-x^2}}\,dx,x,\sqrt{a^2-b^2x}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}\\
&= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}\\
&= -\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = -\frac{2\sqrt{a^2-b^2x}\arctan\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{b^2x+a^2}\,\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}dx$$

```
[In] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = -\frac{2\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="fricas")
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a
))/(b^2*x))/b^2
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2
),x)
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \int \frac{1}{\sqrt{a+b\,\sqrt{x}}\,\sqrt{a-b\,\sqrt{x}}\,\sqrt{a^2+x\,b^2}}\,dx$$

```
[In] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)), x)
```

3.382 $\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2n},-p,-q,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q*AppellF1(1/2/n,-p,-q,1+1/2/n,b^2*
x^(2*n)/a^2,-d*x^(2*n)/c)/((1-b^2*x^(2*n)/a^2)^p)/((1+d*x^(2*n)/c)^q)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {533, 441, 440}

$$\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2n},-p,-q,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
(2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))/a
^2)^p*(1 + (d*x^(2*n))/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(c+dx^{2n}\right)^q \, dx \\
&= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(c+dx^{2n}\right)^q \, dx \\
&= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(1+\frac{dx^{2n}}{c}\right)^q \, dx \\
&= x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)
\end{aligned}$$

Mathematica [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$$

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

Maple [F]

$$\int (a-b\,x^n)^p (a+b\,x^n)^p \left(c+d\,x^{2n}\right)^q dx$$

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

Fricas [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int \left(dx^{2\,n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="fricas")
[Out] integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \text{Timed out}$$

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
[Out] Timed out
```

Maxima [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int \left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="maxima")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Giac [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int \left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="giac")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \int \left(c+d\,x^{2n}\right)^q (a+b\,x^n)^p (a-b\,x^n)^p \, dx$$

```
[In] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p,x)
[Out] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p, x)
```

3.383 $\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 87

$$\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx = x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1 - \frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n}, -p, \frac{1}{4}\left(4+\frac{1}{n}\right), \frac{b^4x^{4n}}{a^4}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p*hypergeom([-p, 1/4/n],[1+1/4/
n],b^4*x^(4*n)/a^4)/((1-b^4*x^(4*n)/a^4)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {533, 259, 252, 251}

$$\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \, dx = x(a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p \left(1 - \frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n}, -p, \frac{1}{4}\left(4+\frac{1}{n}\right), \frac{b^4x^{4n}}{a^4}\right)$$

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(a^2+b^2x^{2n}\right)^p \, dx \\
&= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(a^4-b^4x^{4n}\right)^{-p}\right) \int \left(a^4-b^4x^{4n}\right)^p \, dx \\
&= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}\right) \int \left(1-\frac{b^4x^{4n}}{a^4}\right)^p \, dx \\
&= x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4n},-p,1+\frac{1}{4n},\frac{b^4x^{4n}}{a^4}\right)$$

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, 1 + 1/(4*n), (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Maple [F]

$$\int (a-b\,x^n)^p (a+b\,x^n)^p \left(a^2+b^2x^{2n}\right)^p dx$$

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

Fricas [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int \left(b^2x^{2n}+a^2\right)^p (bx^n+a)^p (-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="fricas"
)
[Out] integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (b^2x^{2\,n}+a^2)^p(bx^n+a)^p(-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="maxima"
)
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Giac [F]

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (b^2x^{2\,n}+a^2)^p(bx^n+a)^p(-bx^n+a)^p \, dx$$

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \int (a+b\,x^n)^p\,(a-b\,x^n)^p\,\left(a^2+b^2\,x^{2\,n}\right)^p \, dx$$

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x)
[Out] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x)
```

3.384 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \frac{x(c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},1,-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[Out] x*(c+d*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,b^2*x^(2*n)/a^2,-d*x^(2*n)/c)
/a^2/((1+d*x^(2*n)/c)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {531, 441, 440}

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx = \frac{x(c+dx^{2n})^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2n},1,-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[In] Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))
/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(c+dx^{2n})^p}{a^2-b^2x^{2n}}\,dx\\
&= \left((c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\right)\int \frac{\left(1+\frac{dx^{2n}}{c}\right)^p}{a^2-b^2x^{2n}}\,dx\\
&= \frac{x(c+dx^{2n})^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(76) = 152.

Time = 0.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.39

$$
\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx
= \frac{a^2c(1+2n)x(c+dx^{2n})^p\,\text{AppellF1}\left(\frac{1}{2n},-p,1,\right.}{(a^2-b^2x^{2n})\left(2a^2dnpx^{2n}\,\text{AppellF1}\left(1+\frac{1}{2n},1-p,1,2+\frac{1}{2n},-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}\,\text{AppellF1}\left(1+\frac{1}{2n},-p,\right.\right.}
$$

```
[In] Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n),
-((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x^(
2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^
(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -
((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p,
 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

Maple [F]

$$\int \frac{(c+d\,x^{2n})^{p}}{(a-b\,x^{n})\,(a+b\,x^{n})}dx$$

```
[In] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
[Out] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = \int -\frac{(dx^{2\,n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="fricas")
[Out] integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = \int -\frac{(dx^{2\,n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="maxima")
[Out] -integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

Giac [F]

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = \int -\frac{(dx^{2\,n}+c)^p}{(bx^n+a)(bx^n-a)}\,dx$$

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^{2n})^p}{(a-bx^n)\,(a+bx^n)}\,dx = -\int -\frac{(c+d\,x^{2\,n})^p}{a^2-b^2\,x^{2\,n}}\,dx$$

```
[In] int((c + d*x^(2*n))^p/((a + b*x^n)*(a - b*x^n)),x)
[Out] -int(-(c + d*x^(2*n))^p/(a^2 - b^2*x^(2*n)), x)
```

3.385 $\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)$

Optimal result

Integrand size = 76, antiderivative size = 96

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = -\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^{1+p}\left(a+bx^{n/2}\right)^{1+p}\left(-\frac{a^2dn(1+p)}{b^2(1+n+np)}+dx^n\right)^{-\frac{1+n+np}{n}}}{a^4dn(1+p)}$$

```
[Out] -b^2*(n*p+n+1)*x*(a-b*x^(1/2*n))^(p+1)*(a+b*x^(1/2*n))^(p+1)/a^4/d/n/(p+1)/
((-a^2*d*n*(p+1)/b^2/(n*p+n+1)+d*x^n)^((n*p+n+1)/n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {533, 389}

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = -\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*
n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 +
n + n*p)/n)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \left(\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(a^2-b^2x^n\right)^{-p}\right)\int\left(a^2-b^2x^n\right)^p\left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx$$

$$=-\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(a^2-b^2x^n\right)\left(-\frac{a^2dn(1+p)}{b^2(1+n+np)}+dx^n\right)^{-\frac{1+n+np}{n}}}{a^4dn(1+p)}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx =$$
$$-\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(d\left(-\frac{a^2n(1+p)}{b^2(1+n+np)}+x^n\right)\right)^{-\frac{1+n+np}{n}}\left(a^2-b^2x^n\right)}{a^4dn(1+p)}$$

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1 +
n + n*p)/n)))
```

Maple [F]

$$\int \left(a-b\,x^{\frac{n}{2}}\right)^{p} \left(a+b\,x^{\frac{n}{2}}\right)^{p} \left(\frac{a^{2} d (1+p)}{b^{2} \left(1+\frac{-n p-2 n-1}{n}\right)}+d\,x^{n}\right)^{\frac{-n p-2 n-1}{n}}\,dx$$

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}\,dx = \frac{\left((b^4np+b^4n+b^4)xx^{2n}-(2\,a^2b^2np+2\,a^2b^2n+a^2b^2)xx \cdots\right.}{(a^4np+a^4n)\left(-\frac{a^2dnp+a^2dn-(b^2d \cdots}{b^2np+b \cdots}\right.}$$

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^2
)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/((a
^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x^n)
/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}\,dx = \text{Timed out}$$

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-n
*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p \left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Giac [F]

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p \left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \int \frac{\left(a+b\,x^{n/2}\right)^p \left(a-b\,x^{n/2}\right)^p}{\left(d\,x^n-\frac{a^2\,d\,(p+1)}{b^2\,\left(\frac{2\,n+n\,p+1}{n}-1\right)}\right)^{\frac{2\,n+n\,p+1}{n}}}\,dx$$

```
[In] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n),x)
```

```
[Out] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
    return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
  try:
    if expn.parent() is SR:
      return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
      return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
      return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

  except AttributeError as error:
    print("Exception,AttributeError in  is_atom")
    print ("cought exception" , type(error).__name__ )
    return False


def expnType(expn):

  if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
    return 1
  elif type(expn)==list:   #isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
      return 1
    else:
      return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
    if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
      return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
      if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```